Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

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Temperature gradient driven Alfvén instability producing inward energy flux in stellarators

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ARTICLE INFO

Article history:
Received 12 April 2018
Received in revised form 4 June 2018
Accepted 6 June 2018
Available online xxxx
Communicated by F. Porcelli

Keywords:
Stellarator
Alfvén eigenmodes
Destabilization
Energy flux
Inward spatial channelling

ABSTRACT

Destabilizing influence of plasma inhomogeneity on Alfvén eigenmodes in stellarators is considered. It is found that the diamagnetic frequency can strongly increase due to the resonance interaction of particles and Alfvén eigenmodes. This occurs when the particle resonance velocity exceeds the thermal velocity, in which case the role of superthermal particles enlarges. Then Alfvén eigenmodes can be destabilized even in the absence of the energetic ion population. It is shown that in the case of the temperature distribution with a large gradient at the periphery, the destabilized mode can channel the energy from the peripheral plasma region to the inner region. A stability analysis employing a model temperature profile of the ions was carried out for the Wendelstein 7-X stellarator. It indicates that the considered mechanism could lead to an Alfvén instability accompanied with the inward energy flux in the first W7-X experiments where long-lasting high-frequency oscillations were observed.

1. Introduction

The energy and momentum transfer across the magnetic field realized by the destabilized Alfvénic Eigenmodes (the phenomenon named the Spatial Channelling, SC) can be an important factor affecting the plasma performance [1]. Presumably, it was responsible for the degradation of plasma heating by neutral beam injection (NBI) in the NSTX spherical torus, by delivering a part of the NBI power from the plasma core to the periphery (an alternative explanation of the experiment was that the destabilized Alfvén eigenmodes strongly deteriorated the electron energy confinement [2]). It seems possible that the outward SC took place, although not identified, also in other experiments where degradation of the plasma heating was associated with the destabilization of Alfvén modes. On the other hand, if the SC were directed inwards, it would have a positive effect on the plasma energy balance. One can suppose that this took place in the first experiments on the Wendelstein 7-X stellarator: First, the ion temperature profile, $T_i(r)$, was rather flat in the plasma core but steep at the periphery [3–5]; second, long-lasting high frequency oscillations were observed, which were identified as an Ellipticity induced Alfvén Eigenmode (EAE) located in the region $r/a \gtrsim 0.5$, with a the plasma radius [6]. Third, there was no energetic ion sources, and the ion heating was solely due to ion-electron collisions. These facts suggest that the inhomogeneity of the ion temperature at the periphery drove an Alfvén instability which adjusted the ion temperature profile in a way that a destabilizing effect of the peripheral region was compensated by the damping in the region where $T_i(r) \approx \text{const}$.

The purposes of this work are to see whether Alfvén eigenmodes can indeed be destabilized in a stellarator plasma with Maxwellian velocity distribution (usually Alfvén instabilities are driven by fast ions produced by NBI or other sources of plasma heating) and whether instabilities caused by the temperature gradient can lead to the inward SC. In addition to a general analysis, a specific example relevant to the W7-X experiment described in reference [6] will be considered.

We will employ the fact that the so-called non-axisymmetric resonances arising because of the lack of axial symmetry in stellarators can provide the interaction of Alfvén modes and ions having considerably smaller velocities than the resonance velocities in tokamaks [7]. Due to this, particles with $\mathcal{E} \gtrsim T_i$ can interact with the modes.

Note that Alfvénic oscillations in the absence of energetic ions can be destabilized in tokamaks, too. In particular, low frequency instabilities (like Beta-induced Alfvén Eigenmodes, BAE) can be caused by the ion temperature gradient and the presence of magnetic islands [8,9], see also overview [10].

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https://doi.org/10.1016/j.physleta.2018.06.014

2. Enhanced destabilization of Alfvén modes by the temperature gradient

In the absence of the energetic ions, the growth/damping rate of Alfvén instabilities can be described by

$$\gamma = \frac{1}{2\mathcal{W}} \mathrm{Re} \int d^3x \tilde{j}_\perp^{kin} \cdot \nabla_\perp \tilde{\Phi}, \tag{1}$$

where $\gamma = \sum_{\sigma=e,i} \gamma_\sigma$, $\tilde{j}_\perp^{kin} = \sum_{\sigma=e,i} \tilde{j}_{\sigma,\perp}^{kin}$ is the transverse current, $\mathcal{W} = \int d^3x c^2 (\nabla_\perp \tilde{\Phi})^2/(4\pi v_A^2)$ is the mode energy, Φ is a scalar potential of the electromagnetic field, tilde labels perturbed quantities, v_A is Alfvén velocity, the subscripts e and i label electrons and ions, respectively. When the transverse current is associated with the particle drift in the equilibrium magnetic field and effects of the trapped particles are negligible, calculations lead to [7,11]

$$\frac{\gamma^{(\sigma)}}{\omega} = \frac{\sqrt{\pi} M_\sigma}{8\delta_0 M_i} \left[\sum_{mn\mu\nu} \int_0^a dr r n_\sigma(r) \times \epsilon^{-2} \left| \mu \epsilon_{\mu\nu} \Phi'_{mn} - m \epsilon'_{\mu\nu} \Phi_{mn} \right|^2 Q(u_\sigma) \bar{k}_{res}^{-2} \right] \times \left[\sum_{mn} \int_0^a dr r^{-1} n_i(r) \left(r^2 |\Phi'_{mn}|^2 + m^2 |\Phi_{mn}|^2 \right) \right]^{-1}, \tag{2}$$

where σ labels particle species, Φ_{mn} and $\epsilon_{\mu\nu}$ are Fourier harmonics defined by $\tilde{\Phi} = \sum_{m,n} \Phi_{m,n}(r) \exp(im\vartheta - in\varphi - i\omega t)$ and $B = \bar{B}[1 + 0.5 \sum_{\mu,\nu} \epsilon_{\mu\nu} \exp(i\mu\vartheta - i\nu N\varphi)]$, $\bar{B}$ is the average magnetic field at the magnetic axis, the radial coordinate r is defined by $\psi = \bar{B} r^2/2$, ψ is the toroidal magnetic flux, ϑ and φ are the poloidal and toroidal Boozer angles, respectively, N is the number of periods of the equilibrium magnetic field, $\epsilon = r/R$, n_σ is the particle density, $\bar{k}_{res} \equiv k_{res} R = (m + \mu)\iota - (n + \nu N)$, ι is the rotational transform of the field lines, R is the major radius of the torus, prime denotes the radial derivative, $\delta_0 \gtrsim 1$ is determined by the plasma shaping [12], $Q(u)$ is defined by

$$\int d^3v (v_\parallel^2 + 0.5 v_\perp^2)^2 \delta(\omega - k_{res} v_\parallel) \hat{\Pi} F_\sigma = \frac{n_\sigma \omega}{\sqrt{\pi} k_{res}^2} Q(u_\sigma), \tag{3}$$

where F_σ is the particle distribution function, $u_\sigma \equiv |v_\parallel^{res}|/v_{T\sigma}$, $v_\parallel^{res} = \omega/k_{res}$, $v_T = \sqrt{2T/M}$ is the particle thermal velocity, M is the particle mass, $\hat{\Pi}$ in the case of a plasma with isotropic velocity distribution is

$$\hat{\Pi} = \frac{1}{v}\frac{\partial}{\partial v} + \left(\frac{\omega R}{v_\parallel} + n \right) \frac{1}{\iota \omega \omega_B} \frac{1}{r} \frac{\partial}{\partial r} \tag{4}$$

where ω_B is the gyrofrequency. Due to (4), we can write $Q = Q_v + Q_r$, where Q_v and Q_r are associated with the first term and second term in the RHS of equation (4), respectively.

It is clear that $Q_v < 0$ for a plasma with Maxwellian velocity distribution, $F_M \propto \exp\{-(\mathcal{E}_\parallel + \mathcal{E}_\perp)/T\}$, with $\mathcal{E}_\parallel$ and $\mathcal{E}_\perp$ the particle energy along and across the magnetic field. Therefore, an instability arises when $Q_r > |Q_v|$. The magnitude of Q_r essentially depends on the resonance velocity. Because

$$\left. \frac{\partial F_M}{\partial r} \right|_{v_\parallel = v_\parallel^{res}} = \frac{n_\sigma}{\pi^{3/2} v_{T\sigma}^3} e^{-u_\sigma^2 - \mathcal{E}_\perp/T_\sigma} \times \left[\frac{n'_\sigma}{n_\sigma} + \left(u_\sigma^2 + \frac{\mathcal{E}_\perp}{T_\sigma} - \frac{3}{2} \right) \frac{T'_\sigma}{T_\sigma} \right], \tag{5}$$

the ratio of the ∇T_σ term to the ∇n_σ term grows when u_σ increases, at least for $u^2 \geq 3/2$. Therefore, one can expect that the condition $Q_r > |Q_v|$ is satisfied most easily at large u. For this reason, we assume that $u > 1$. Then the integral over transverse velocities in (3) can be taken in the limits $(0, \infty)$ because the region of trapped particles lies at $\mathcal{E}_\perp \gg T_\sigma$ when $u > 1$. As a result, we have:

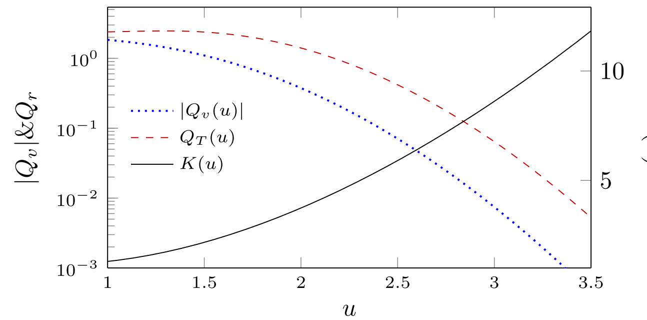

Fig. 1. Functions $Q_v(u)$, $Q_T(u)$, and the enhancement factor $\mathcal{K}$ versus $u \equiv |v_\parallel^{res}|/v_T$.

$$Q_v(u) = -\frac{1}{u}(2u^4 + 2u^2 + 1)e^{-u^2}, \tag{6}$$

$$Q_r(u) = -\frac{\rho_\sigma v_{T\sigma}}{2\iota\omega r}[(m+\mu)\iota - \nu N] \left[Q_v \frac{n'_\sigma}{n_\sigma} + Q_T \frac{T'_\sigma}{T_\sigma} \right], \tag{7}$$

where $\rho_\sigma = v_{T_\sigma}/\omega_{B\sigma}$ and

$$Q_T(u) = \frac{1}{u}(2u^6 + u^4 + 2u^2 + 1.5)e^{-u^2}. \tag{8}$$

We assume that $Q_r > 0$ (otherwise plasma is stable), which imposes a restriction on the mode numbers, the resonance numbers (μ, ν), and radial derivatives of the plasma temperature and density. Then the ratio of the driving term to the stabilizing one is negative and given by

$$\frac{Q_r}{Q_v} = -\frac{\rho_\sigma v_{T\sigma}}{2\iota\omega r}[(m+\mu)\iota - \nu N] \left(\frac{n'_\sigma}{n_\sigma} + \mathcal{K} \frac{T'_\sigma}{T_\sigma} \right), \tag{9}$$

where $\mathcal{K} \equiv Q_T/|Q_v|$ is the “enhancement factor”,

$$\mathcal{K} = \frac{2u^6 + u^4 + 2u^2 + 1.5}{2u^4 + 2u^2 + 1}. \tag{10}$$

For instance, $\mathcal{K}(2) = 3.44$, $\mathcal{K}(3) = 8.61$. On the other hand, the function $Q_T(u)$ is decreasing rather weakly in the range $1 < u < 3$, see Fig. 1. Therefore, this range of the resonance velocities may play the main role in instabilities.

Equations (6)–(10) remain valid during the instability provided that Coulomb collisions are strong enough to sustain Maxwellian velocity distribution.

3. Inward spatial channelling and its consequences

Let us assume that $\mathcal{K} \gg 1$ and the temperature gradient term in the ratio Q_r/Q_v dominates. Then $\mathcal{K} \approx u^2 \propto 1/T_\sigma$ (for $k_{res} \approx$ const within the mode width when the magnetic shear is not large), which leads to $Q_r/Q_v \propto T'_\sigma/(rT_\sigma)$. It follows from here that the instability condition $Q_r/|Q_v| > 1$, with $Q_r > 0$, is most easily satisfied at the plasma periphery where the temperature is low. For instance, when $T_\sigma = T_0(1 - r^2/a^2)^\tau$, $Q_r/Q_v \propto (1 - r^2/a^2)^{-1}$. It follows from here that it may happen that $Q_r/|Q_v| < 1$ in the region $r_1 < r < r_0$, but $Q_r/|Q_v| > 1$ in the region $r_0 < r < r_2$, where r_0 is defined by equation $Q_r/|Q_v| = 1$, $r_1 < r < r_2$ is the region where the mode is located. The instability growth rate then is $\gamma = \gamma_+ - |\gamma_-|$, where $\gamma_+ > 0$ and $\gamma_- < 0$ are the drive and damping determined by equation (2) with the integral in numerator taken in the limits $(0, r_0)$ and (r_0, a), respectively. In a steady state $\gamma_+ = |\gamma_-|$, so that $\gamma = 0$.