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Impact of local coupling on the vulnerability of 2D spatially embedded interdependent networks

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ABSTRACT

Real networks are always interdependent and spatially embedded. Considering the space constraint, dependency links between networks may be established not globally but locally. In this paper, we study how the spatial coupling will impact the robustness of interdependent scale-free networks located in a 2D square plane where dependent nodes are connected within a connecting radius $r_{connect}$. Besides the traditional assortative degree–degree coupling (*GD*) and random coupling (*GR*), some novel spatial couplings are also introduced, i.e., spatial assortative degree–degree coupling (*SD*), spatial random coupling (*SR*) and nearest neighbor coupling (*NN*). Simulation results indicate that assortative couplings, *GD* and *SD*, can improve the robustness under topological attacks while under localized attacks, *NN* coupling is the best one. In addition, for *SD* coupling under topological attacks, we find that the robustness for small $r_{connect}$ decreases with $r_{connect}$ from 0 to the critical value r_{c1}, and for larger $r_{connect}$ gradually increases with $r_{connect}$ from r_{c1} to the maximum value $(r_{connect})_{max}$. However, opposite results will be obtained under localized attacks. These findings may be helpful to understand and analyze some real interdependent infrastructures.

1. Introduction

Networks are ubiquitous in the world and support our fundamental life [1–5], such as the infrastructure, trade, society, ecology and biology. Much research has been conducted on isolated networks covering various fields [1–6] since the small-world and scale-free natures were discovered [7,8]. In recent years, researchers gradually realized that the interaction between networks is becoming more intensive, forming the dependent or interdependent networks [9]. Although connections can facilitate the interaction between individuals for well performance, it will also increase the system vulnerability [9,10]. With the dependency or interdependency relationship, failures in one system may propagate to other systems, and even cause catastrophic damages. For example, the new generation of smart grid [11–15] is a coupled system composed of power grid and information system, i.e., cyber physical power system (CPPS) [14]. The information system makes power grid more efficient, and in turn, it also needs power supply from power nodes. In CPPS, the breakdown or failure of information equipment may also cause collapse of the entire systems, for example, the Ukrainian blackout caused by cyber attacks in 2015. Taking the Italian blackout in 2013 as an illustration, Buldyrev et al. [9] proposed a mathematical framework for cascading failures of interdependent networks based on the percolation theory. Some issues were studied analytically and numerically afterwards [9,16–18], such as the dependency link, coupling preference and coupling strength.

Most of previous work merely focuses on the network topology ignoring coordinates of components, which is called non-spatially embedded network. However, some artificial infrastructures may be spatially embedded in the world [19–22], and components are not constructed randomly. Their locations will be planned rigorously considering the cost and utilization, because long-range connection may signify high cost [23,24], such as power transmission lines, water pipes and roads. Therefore, to better understand real networks, it requires modeling and analyzing spatially embedded networks taking the location information of components into consideration. For real interdependent infrastructures, dependent nodes may be connected not globally but locally, sometimes with certain range constraint. By constructing interdependent lattice networks, Li et al. [25] found that there exists a critical threshold r_c of dependency link length, which makes systems the most vulnerable, and the percolation below or above r_c will exhibit different transitions. Other researchers also studied the relationship between r_c and other parameters [19,20,26,27], such as the

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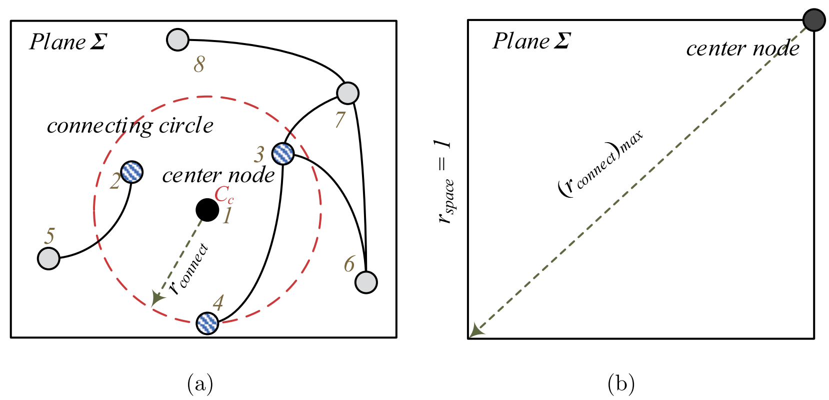

Fig. 1. Illustration of local connection rule in isolated network. The network is embedded in a square plane Σ, and the connection area is a circle. (a) The red dashed circle is the connection area with center C_c (black node 1) and radius $r_{connect}$. The nodes with stripe (nodes 2, 3 and 4) are possible connection nodes of C_c, constituting the candidate set Ω. (b) In the spatial network, each node can connect any nodes within certain $r_{connect}$, and thus the maximum value of connecting radius is $(r_{connect})_{max} = \sqrt{2}$ if the side length r_{space} of Σ is 1. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

proportion of autonomous nodes, network topology and component lifetime. In addition, some conclusions were also extended to n networks. However, the lattice network has extremely regular topology, and all the networks have identical topology. Furthermore, in their model, the length r does not refer to the Euclidean distance in space but the unit number between each pair of dependent nodes, which can only be an integer. In fact, components of infrastructure can locate at any site, and the area must be a three-dimensional (3D) or an approximate two-dimensional (2D) Euclidean space [23,24].

Due to the existence of dependency links, interdependent networks present some distinct properties different from what an isolated network does. Research [16,28] shows that compared with random coupling (*RC*), the assortative coupling (*AC*) can improve the network robustness. However, for some spatial networks, the space constraint of coupling should be considered, establishing the spatial (local) coupling. On the other hand, some external attacks on infrastructure may damage a fraction of components in a localized area (localized attack) [23,24,29–31], such as natural hazards and wars. Different from the traditional non-geometrical attack (topological attack), all the components within the attacking area will fail, including all kinds of nodes, dependency and connectivity links. It should be pointed that the attacking area can be any regular or irregular shape.

Therefore, in a 2D space, impacts of coupling preference (*RC* and *AC*) and connection radius on cascading failures of interdependent scale-free networks under topological and localized attacks are studied in this paper. First, by defining a local connection rule, we introduce five global and local coupling patterns. Second, we describe a localized attacking strategy and present the calculation method of failed components. In the end, based on a degree-load-based cascading failure model [18], we study the impact of connection radius of local coupling patterns on cascading failures both under topological and localized attacks. Simulation results show that *AC* is better under topological attacks while under localized attacks, the networks with short dependency links perform well.

The rest of this paper is organized as follows: in Section 2, the local connection rule of dependency links is presented, and five global and local coupling patterns are introduced; in Section 3, the degree-load-based cascading failure model is reviewed; in Section 4, the localized attacking strategy and its calculation method of failed components are explained, and the impact of connection radius of local coupling patterns on cascading failures both under topological and localized attacks is investigated; the relevant conclusion is summarized in Section 5.

2. Local connection rule of dependency links

We first illustrate the local connection rule in an isolate network (see Fig. 1). For a small area, we can ignore the distance in vertical direction and simplify the 3D space to a 2D space. As shown in Fig. 1, the network is located in a 2D square space Σ, and each node i has a 2D coordinate (x_i, y_i). Considering the space constraint, connections are more likely to be established within a specific area creating a 2D spatial network.

We assume that the connection area is a circle O_c with the center C_c and connecting radius $r_{connect}$ (see Fig. 1a), and the area can also be a square or any other shape. When $r_{connect}$ is fixed, for each node, there will be a set Ω of candidate nodes falling into O_c. To achieve this goal, we alternately regard each node as the center C_c, and respectively calculate the distance d_i $(i = 1, 2, \cdots, N-1)$ between the rest $N-1$ nodes and C_c where N is the network size. In a 2D plane coordinate system, the Euclidean distance d_{AB} between two nodes A (x_1, y_1) and B (x_2, y_2) is computed as follows.

$$d_{AB} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \quad (1)$$

Then, compare d_i with $r_{connect}$ and record node i into Ω if $d_i \le r_{connect}$. For interdependent networks, it is one way to improve the robustness by reconnecting dependency links, namely change the coupling pattern. In this paper, all the coupling patterns are constructed with one-to-one correspondence, which means each node has one and only one dependent node. Research [16,28] indicates that assortative coupling will play a role in network performance against cascading failures, for example, the degree–degree (*D-D*) assortative coupling (see Fig. 2a), which connects nodes with similar degree ranking. Similarly, considering the space constraint, dependent nodes cannot be connected globally but locally within the circle O_c mentioned above. In order to generate spatially embedded interdependent networks, we first create two networks following certain evolution model and then locate the networks in a 2D square space randomly. On this occasion, the connection of connectivity links within each network has no space constraint, and there will be some long-range connectivity links. Therefore, we just establish and study the local dependency links in this paper.

Besides the global *D-D* coupling (*GD*) shown in Fig. 2a, we also adopt another non-spatial coupling pattern, namely random coupling (*GR*), which connects nodes in networks *A* and *B* with one-to-one correspondence randomly. Following the local connection rule, one node in network *A* can only select its dependent node within O_c. To ensure the assortativity of spatial *D-D* coupling (*SD*) (see Fig. 2b), when selecting dependent nodes, the node with