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Spectra in nested Mach–Zehnder interferometer experiments

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ABSTRACT

By the means of the standard quantum mechanics formalism I present an explicit derivation of the structure of power spectra in Danan et al. and Zhou et al. experiments with nested dynamically changing Mach–Zehnder interferometers. The analysis confirms that we observe prominent, first-order peaks on frequencies related to some of the elements of the interferometer, but not on others. However, as I shall demonstrate, there are also other, weaker effects related to all relevant elements of the setup. In case of the Danan et al. setup, there are even peaks at all frequencies of element oscillations. When confronted in an experiment, these observations shall challenge the interpretation of the experiments based on anomalous trajectories of light.

Interference of light is one of its most intriguing features, with a number applications in various areas. It reveals the wave nature of electromagnetic radiation, which coexists with particle-specific effects. Interference allows to recombine a spatially split signal (be it, a wave or individual particles) in such a way, that it can be deterministically redirected to a specific output. However, any attempt to learn a specific path taken by the signal within the interferometer immediately causes a decrease in the interference pattern depth due to the Welcher-Weg information complementarity [1].

More recently, an experiment was conducted, supposedly giving some new insight to the problem of the interference. Vaidman [2] has suggested an experimental setup, in which one Mach–Zehnder interferometer is nested in one of the arms of another. The setup was subsequently realized in Ref. [3] and more recently, the experiment was redone in a slightly modified scheme [4], but confirming results of Danan, Farfunik, Bar-Ad, and Vaidman, also another group [5] conducted a similar experiment. These results were actively discussed in recent years (Ref. [3] was chosen for *Physical Review Letters Viewpoint* at the time of its publication and by June, the 14th, 2018, it was cited 63 times, according to Web of Science®. The most noticeable critical comments include Refs. [6–13]). A very similar concept also appears in the context of “counterfactual communication” [14,15].

The details of these two experiments are described below and depicted in Figs. 1 and 2. In the version from Ref. [3], we have a large Mach–Zehnder interferometric loop. The source of coherent light is at the top in Fig. 1 and the light falls onto the first beam splitter (BS) with transmitivity 1/3. If it is transmitted, it then falls on mirror C and on the second BS, also of transmitivity 1/3. At one of the outputs of this BS there is a quad-cell detector, allowing to register the difference of intensities of light falling on its upper and lower parts (with the respect to the plane of the interferometer). If light reflects from the first BS, it is first redirected downwards (in terms of the figure) by mirror E. Then it enters a smaller Mach–Zehnder loop, with two balanced BSs and mirrors A and B at the corners.[1] Should the light leave this loop horizontally, it would encounter mirror F that redirects it onto the second BS to recombine with the initially transmitted beam. However, initially, the smaller loop is set to redirect all the light incoming from the direction of mirror F to the other output, where it is dumped.

The mirrors in this configuration oscillate with respective frequencies $f_A, \ldots, f_F$ by being slightly tilted with respect to axes in the plane of both interferometers, causing the beam to be displaced somewhat differently along each path. The experimentalists study the power spectrum of the aforementioned difference in intensities. They observe peaks at frequencies f_A, f_B, and f_C, which are obvious certificates of interaction of photons with these mirrors, but not at f_E or f_F.[2]

In the setup of Ref. [4], all the mirrors are stationary. There are no mirrors E or F, the first beamsplitter redirects a portion of light directly to the smaller loop, and one of its outputs points directly towards the recombination on the second beam-splitter. The small loop is, again, tuned destructively, and electro-optical modulators (EOMs) are placed on each path between each pair of BSs. They

[1] Ocassionally, labels A, B, C will be also used for paths.

[2] At this point, the Reader is kindly invited to examine conclusions of Refs. [3,4].

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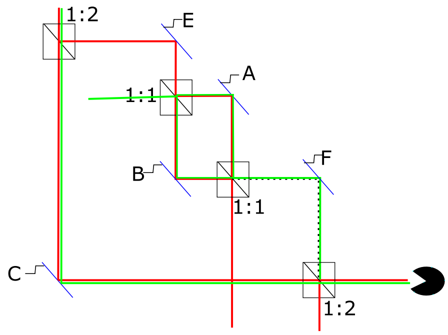

Fig. 1. The experimental setup in Ref. [3]. Mirrors A, B, C, E, F are periodically tilted with frequencies $f_A, ..., f_F$, causing a small displacement of the beam in direction perpendicular to the plane of figure. The red and the green line would correspond to the forward- and backward-evolving state in the two-state vector formalism [16, 17]. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

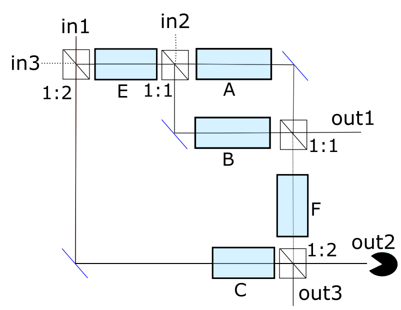

Fig. 2. The experimental setup from Ref. [4]. Electro-optical modulators modulate phases of the beams with frequencies $f_A, ..., f_F$. Input and output channels are *in*1, *in*2, *in*3, *out*1, *out*2, *out*3.

modulate the phase with very small amplitudes and frequencies $f_A, ..., f_F$. This test is meant for certification of presence of photons along paths, rather than at mirrors, but again, we observe a lack of certain peaks in the power spectrum, and similar conclusions are drawn.

In this letter I present a simple quantum-mechanical analysis of these experiments. It will be demonstrated below that, indeed, in both experiments, first-order peaks at frequencies f_E and f_F are absent, but there are other certificates of presence of light in the whole setup, namely lower-order peaks on frequencies being combinations of all frequencies involved. While these peaks may be significantly lower, they certainly prevent us from formulating strong claims about the absence of light at certain locations.

I start with analyzing the scheme of Ref. [4], which is more straight-forward. Between beam-splitters we place EOMs, which simply introduce phase factors. I assume light entering the setup through the first input is either a heralded single photon or a simply a bright coherent state. I shall also assume temporarily assume perfect detectors without dark counts. The transformation between input and output mode annihilation operators realized by the interferometer is

$$\begin{pmatrix}\hat{a}_1^{\text{out}}\\ \hat{a}_2^{\text{out}}\\ \hat{a}_3^{\text{out}}\end{pmatrix} \leftarrow \begin{pmatrix}1 & 0 & 0\\ 0 & \sqrt{2/3} & \sqrt{1/3}\\ 0 & \sqrt{1/3} & -\sqrt{2/3}\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\phi_F(t)} & 0\\ 0 & 0 & 1\end{pmatrix}$$
$$\times\begin{pmatrix}\sqrt{1/2} & \sqrt{1/2} & 0\\ \sqrt{1/2} & -\sqrt{1/2} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\phi_A(t)} & 0 & 0\\ 0 & e^{i\phi_B(t)} & 0\\ 0 & 0 & 1\end{pmatrix}$$
$$\times\begin{pmatrix}\sqrt{1/2} & 1/\sqrt{2} & 0\\ \sqrt{1/2} & -\sqrt{1/2} & 0\\ 0 & 0 & 1\end{pmatrix}\begin{pmatrix}e^{i\phi_E(t)} & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & e^{i\phi_C(t)}\end{pmatrix}$$
$$\times\begin{pmatrix}\sqrt{2/3} & 0 & \sqrt{1/3}\\ 0 & 1 & 0\\ \sqrt{1/3} & 0 & -\sqrt{2/3}\end{pmatrix}\begin{pmatrix}\hat{a}_1^{\text{in}}\\ \hat{a}_2^{\text{in}}\\ \hat{a}_3^{\text{in}}\end{pmatrix}, \qquad (1)$$

where $\phi_X = A_0 \sin(2\pi f_X t)$, $X = A, B, C, E, F$, and A_0 is the common amplitude of the phase change. The signal is fed by the first input. Consider the probability amplitude that a photon entering the setup reaches the detector, or the amplitude of the coherent light at the second output.

$$V_2(t) = \frac{1}{3}\Big(e^{iA_0\sin(2\pi f_C t)} + e^{iA_0(\sin(2\pi f_A t)+\sin(2\pi f_E t)+\sin(2\pi f_F t))} - e^{iA_0(\sin(2\pi f_B t)+\sin(2\pi f_E t)+\sin(2\pi f_F t))}\Big), \qquad (2)$$

where A_0 is the common amplitude of phase change. The result of the experiment analyzed in Ref. [4] is the power spectrum of this signal, which describes its decomposition into oscillating terms. For convenience, I choose all frequencies to be multiples of a certain base frequency, so that it is possible to make a decomposition into discrete frequencies only. This is the (renormalized) discrete energy spectrum over a single period, which would be equivalent to the power spectrum in the limit of the experiment running infinitely long. In a real-life experiment, it would mean that the peaks are very narrow and well-defined. The figure of merit would hence be

$$G(f) = \left|\int_0^1 e^{-2\pi i f t} V_2(t)dt\right|^2 \quad (f = 0, 1, 2, ...). \qquad (3)$$

This integral can be treated analytically in the following fashion. First, we use the Maclaurin expansion of the exponent, $\exp(x) = \sum_{k=0}^{\infty}\frac{x^k}{k!}$. Then we iteratively use identities $\sin(x)\sin(y) = \frac{1}{2}[-\cos(x+y)+\cos(x-y)]$, $\cos(x)\cos(y) = \frac{1}{2}[\cos(x+y)+\cos(x-y)]$, and $\sin(x)\cos(y) = \frac{1}{2}[\cos(x+y)+sin(x-y)]$ we see that apart from a peak of magnitude n (an arbitrary constant) at 0, $G(f)$ will have peaks of magnitude $\frac{1}{3}nA_0^2$ at f_A, f_B and f_C, as well as peaks of magnitude $\frac{1}{12}nA_0^4$ at $2f_A$, $2f_B$, $2f_C$, and $f_{A/B} \pm f_{E/F}$. In general, peaks will be present at frequencies $n_1 f_C$, $n_2 f_A + n_3 f_E + n_4 f_F$, and $n_2 f_B + n_3 f_E + n_4 f_F$, where n_1, n_2, $n_3 \neq 0$, and $n_4 \neq 0$ are integers. An example of $G(f)$ is shown in Fig. 3. If we take into account the dark counts of the detectors, we shall assume that they happen spontaneously, in accordance to the Poissonian distribution. This would overlay the spectrum onto the spectrum of the (pink) noise, which is characterized by the exponential decay for high frequencies. Nevertheless, the frequencies discussed here should still be distinguished for amounts of dark counts attainable with modern detectors.

I now pass to deriving a similar result for the Danan-Farfunik-Bar-Ad-Vaidman experiment [3]. Therein, the mirrors establishing the corners of the interferometer were slightly tilted, effectively causing a small displacement of the beam in direction d, perpendicular to the plane of the experiment. Let me assume that a displacement caused by each mirror is of a common magnitude, δ, but the result could easily be rewritten for arbitrary shifts.[3] For simplicity, I shall also assume the Gaussian profile of the incoming

[3] Ref. [3] quotes these displacements as approximately 1/5000 of the width of the beam.