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# Describing intrinsic noise in Chua's circuit

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ABSTRACT

In this Letter we demonstrate that intrinsic inevitable noise effects, existing in realistic experiments with electronic circuits, are properly described theoretically using a Gaussian noise. For this we integrate numerically the equations of motion from the Chua circuit using a fourth-order stochastic Runge–Kutta integrator. Periodic structures in parameter space, related to periodic motion, start to be destroyed when the noise intensity is increased and vanish at a critical intensity value, for which only chaotic motion remains. We find the appropriate noise intensity interval which satisfactorily reproduces the parameter space from the corresponding experiment and it is in remarkable agreement with the estimated experimental noise. Present achievements should be applicable to describe noise effects in a wide number of electronic circuits.



## 1. Introduction

From the experimental point of view, noise can promote a dual result in nonlinear systems. One can lead the system into a desirable behavior, for example in the Stochastic Resonance phenomenon [1] when a low amplitude signal can be amplified by adding white noise. On the other hand, the most common one, noise induces undesirable effect that disturbs the theoretical and/or numerical previsions of a physical phenomenon. In nonlinear systems, noise effects may destroy the periodic motion given rise to the chaotic one [2–4]. In this Letter, the aim is to explore the effect of distinct noise intensities in the numerical experiment carried out in an inductorless version of Chua's circuit model. In the canonical model [5] the circuit is constructed with three-fold piecewise Chua's diode and numerical results display, in the parameter space, characteristic *spiral hubs* [6–8] formed by connected periodic structures named *shrimps* [9]. As reported recently [10], due to features of the electronic devices in real experiments, Chua's diode is a five-fold piecewise linear function. In this paper the authors report experimental measurements in the parameter space of an inductorless Chua's circuit. As a result, the *spiral hub* presented in the three-fold piecewise Chua circuit model [7,8] is destroyed and the *shrimps* embedded in the chaotic domain become distorted, starting from their antennae towards to their central body. The authors claim that this effect is associated to thermal, electrical and analog to digital conversion noises, and to the distortion of the piecewise linear curves of the real diode and its model. Indeed, this phenomenon is similar to that reported in numerical experiments in discrete [11–13] and continuous [11] times systems under noise, where the noise intensities disturb the parameter space corroding all periodic structures, starting from their borders. For high enough noise intensities, only regimes in the parameter space remain which correspond to chaotic behavior.

Results presented in this Letter enlighten, by numerical evidences, that the noise intensities strongly contribute for the disagreements between experimental and numerical results in continuous-time systems, specifically for the inductorless Chua's circuit. Using a Gaussian noise we correctly describe qualitatively and quantitatively intrinsic effects of noise in the realistic experiment with the electronic Chua circuit. While Sec. 2 presents the model to describe experimental results, Sec. 3 shows our numerical results and concluding remarks are summarized in Sec. 4.

## 2. The model of noise

In real experiments, noise sources may arise from a combination of thermal fluctuations of the electronic devices, their inaccuracies, electromagnetic interference coming from the external environment, among others [14]. To describe the overall noise effect on an electronic circuit, we use in this Letter a Gaussian noise, whose effect is then to be compared to experimental evidences reported before [2,10]. To do so, we use the inductorless Chua's circuit with the same experimental values of the electronic device [15] with a five-fold piecewise linear function for the Chua's diode.

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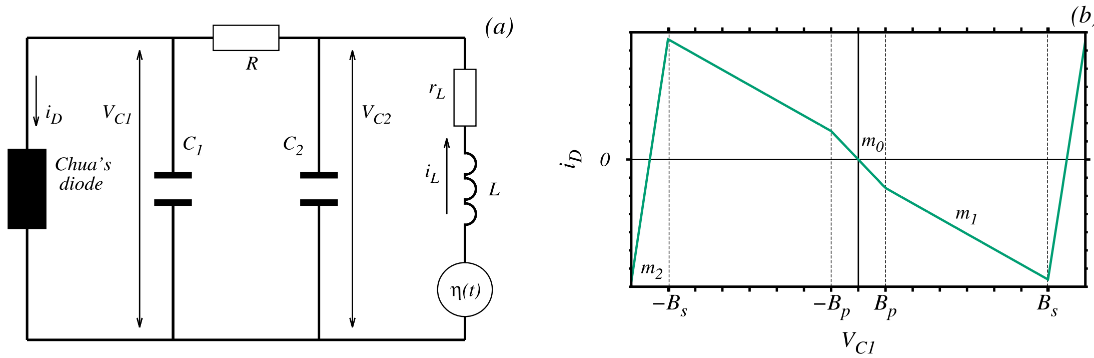


**Fig. 1.** (a) Chua's circuit model where $V_{C_1}$ and $V_{C_2}$ represent the voltage in capacitors $C_1$, $C_2$, respectively, $i_L$ and $i_D$ represent the currents through the inductor $L$ and Chua's diode, respectively and (b) the five-fold piecewise linear function for the current-voltage.

In Fig. 1(a), the schematic Chua's circuit is presented with a noise source. In Fig. 1(b), the five-fold piecewise linear function for the current-voltage characteristic of the Chua's diode is presented. By Kirchhoff's laws in the circuit of Fig. 1(a) we obtain the following set of coupled first-order differential equations,

$$\dot{V}_{C_1} = \frac{V_{C_2} - V_{C_1}}{RC_1} - \frac{i_D}{C_1}, \quad \dot{V}_{C_2} = \frac{V_{C_1} - V_{C_2}}{RC_2} + \frac{i_L}{C_2},$$
$$\dot{i}_L = -\frac{V_{C_2}}{L} - \frac{i_L . r_L}{L} - \frac{\eta(t)}{L}, \quad (1)$$

with $\dot{V}_{C_1}$, $\dot{V}_{C_2}$ being the temporal derivatives of the voltages across the capacitors $C_1$ and $C_2$, respectively, and $\dot{i}_L$ the temporal derivative of the current across the inductor $L$. $R$ and $r_L$ are resistors. $\eta(t)$ is the stochastic variable and $i_D$ is the current-voltage characteristic of the diode given by

$$i_D = \begin{cases} m_0 V_{C_1}, & |V_{C_1}| \le B_p, \\ \pm(m_0 - m_1)B_p + m_1 V_{C_1}, & B_s \ge V_{C_1} \ge B_p, \\ \pm(m_0 - m_1)B_p \pm (m_1 - m_2)B_s + m_2 V_{C_1}, & |V_{C1}| \ge B_s. \end{cases}$$

Following the experimental features presented recently [10], in the model we use the inductorless version [16], where the inductor is an electronic circuit with capacitors, resistors, and operational amplifiers, that produce an equivalent inductance $L_{eq}$. In this configuration, Eq. (1) for the current $i_L$ is transformed in an equation for the voltage $V_L$, so that the three variables become voltages. For more details about the transformations and the configuration of this electronic inductor we refer the readers to [10,16]. Finally, the new set of coupled first-order differential equations is written as

$$\frac{dX}{dT} = \alpha_1(X - Y) + i_d(X), \quad (2)$$

$$\frac{dY}{dT} = \alpha_2(Y - X) - \theta(Z - Y), \quad (3)$$

$$\frac{dZ}{dT} = \gamma(Y + \xi(T)) + \beta(Z - Y) - \alpha_2(X - Y) - \theta(Z - Y), \quad (4)$$

where $\xi(T) = \eta(t)/Bp$ is the stochastic variable which satisfies $\langle \xi(T) \rangle = 0$ and $\langle \xi(T)\xi(T') \rangle = 2Ah\delta(T - T')$, with $h$ being the time step in the stochastic integration. $A$ is the parameter which allows us to change the noise intensity. All other quantities are given explicitly by $V_{C_1} = XB_p$, $V_{C_2} = YB_p$, $V_L = ZB_p$, $t = -TC_1/m_0$, $\alpha_1 = 1/m_0R$, $\alpha_2 = C_1/m_0C_2R$, $\theta = C_1/m_0R_7C_2$, $\beta = C_1r_L/m_0L$, $\gamma = C_1R_7/m_0L$, $i_L = (V_L - V_{C_2})/R_7$, $L = 4.23 \times 10^{-4}H$, $m_0 = -4.09 \times 10^{-4}S$, $m_1 = -7.65 \times 10^{-4}S$, $m_2 = 4.75 \times 10^{-3}S$, $B_p = 1.8V$, $B_s = 5.83B_p$, $C_1 = 23.5\eta F$, $C_1 = 235\eta F$, $R_7 = 1.0K\Omega$ and $i_D = i_d(X)m_0B_p$. The set of Eqs. (2)–(4) with a Gaussian noise will be used next to model the experimental results presented in [10].

To finish this Section, we would like to call to attention that the discontinuities and symmetries which are present in five-fold piecewise linear function for the current-voltage from Fig. 1(b), do not exist in the experimental circuit. In the real experiment these discontinuities become rounded, the piecewise linear curve is asymmetric, and this may affect the dynamics, as shown in [10].

## 3. Numerical results

Using a stochastic fourth-order Runge–Kutta algorithm [17,18], we numerically solve Eqs. (2)–(4) with a time step $h = 0.1$ and an integration time $7 \times 10^5$. For each pair $(R, r_L)$ the spectrum of the Lyapunov exponent (LE) was obtained using a well known algorithm [19]. The largest LE is obtained for a discretized mesh of $600 \times 600$ parameters $(R, r_L)$. In the parameter space the largest LEs are codified by color gradients. In this work, white color refer to equilibrium points with negative largest LE, black color to periodic motion with zero largest LE, and yellow to red colors for increasing positive largest LEs, which are related to the chaotic behavior. In Fig. 2(a) the values of the largest LEs (see color bar) are plotted for the pairs $(R, r_L)$ for $A = 0$.

This is the noiseless case and displays a background with periodic motion (black color) together with a regime of chaotic motion (yellow color) with some black islands, which are the shrimps (periodic structures). Fig. 2(b) shows the same parameter space but colors indicating the number of spikes in one period of the time series of the stable attractors inside the shrimps. Each shrimp has a distinct number of spikes showing the high level of complexity as the parameters are changed. This is a typical known scenario for such continuous systems [7,8]. Even though results from Fig. 2 *satisfactorily* describe the nonlinear behavior of the electronic circuit, they certainly do not *adequately* describe the realistic experiment which includes intrinsic noise effects. Our purpose is now to analyze noise effects in such parameter space. To understand this, Fig. 3 displays the same parameter space for increasing values of the noise intensity $A$, from $A = 10^{-9}$ in Fig. 3(a) up to $A = 10^{-6}$ in Fig. 3(d). For very low noise intensities, the respective LE diagrams do not present relevant changes, as can be observed in Fig. 3(a) and roughly in Fig. 3(b). However, for intensities between $A = 10^{-7}$ and $A = 10^{-6}$ (Fig. 3(c) and (d)), evident changes are observed. A significant reduction of the size of the periodic domains embedded in the chaotic regime is observed. The noise effect is firstly visible in the destruction of the antennas of the periodic structures. After that, as the noise intensity increases, the destruction is propagated to the body (center) of the periodic structure. In addition, the periodic structures become blurred, so that is very hard to properly define their borders. At a given critical noise in-

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