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Stochastic resonance in time-delayed exponential monostable system driven by weak periodic signals ☆

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Based on the exponential monostable potential, we study an exponential monostable system with time-delayed feedback driven by weak periodic signals and additive Gaussian white noises. The small delay approximation is used to deduce the steady-state probability distribution and the effective potential function is derived. The system parameters l and b, time delay τ, feedback strength β can change the shapes of the potential function. The mean first-passage time (MFPT) is calculated, which plays an extremely important role in the research of particles escape. And the signal-to-noise ratio (SNR) of the system can be obtained by using the adiabatic approximation theory. The phenomenon of stochastic resonance is investigated under different system parameters and time-delayed feedback. The amplitude of SNR can be changed by adjusting the system parameters. When the feedback strength β is positive (or negative), the time delay τ can promote (or suppress) the stochastic resonance phenomenon. The SNR versus the noise intensity D presents the stochastic resonance phenomenon. In addition, the SNR increases non-monotonically with the increasing feedback strength β and the parameter b. Also, the analysis and numerical simulation results of SNR are in good agreement with the formula simulation.

1. Introduction

Benzi and his co-workers [1] first proposed the stochastic resonance (SR) when studying the ancient meteorological glacier problem. SR, as a nonlinear method to extract weak signals under strong noise background, is widely used in various fields [2–6]. Such as physics, nonlinear mathematics, biomedical sciences, engineering and so on. Because of its potential value, SR in nonlinear systems has been investigated extensively both theoretically and experimentally. And the influence of time delay on deterministic system has become a subject of many researches [7–21]. Many useful analytical and computational methods have been proposed for deterministic systems. The delay Fokker–Planck equation as an important theoretical tool, was first introduced by Guillouzic et al. [7,8]. And the small delay approximation of stationary distributions based on the stochastic delay differential equation approach was studied. Then Frank [9,10] showed that the Novikov's theorem can be used to derive the delay Fokker–Planck equations like deriving the ordinary Fokker–Planck equations, and in this paper, a small delay approximate theory based on probability density method was introduced in detail. Moreover, using time-delayed Fokker–Planck equations, time delay and noise induced transitions to bistability were widely studied [11,12].

There are many theoretical studies on the time-delayed feedback of traditional bistable and tristable system under various noise excitation [13–20]. Recently, SR in a time-delayed bistable system driven by trichotomous noise was investigated [13]. The performance of SR in bistable systems with time-delayed feedback and three types of asymmetries were analyzed [14]. And Liu et al. discussed the simultaneous influence of potential asymmetries and time-delayed feedback on SR subject to both periodic force and additive Gaussian white noise. Shao et al. [15] studied the SR performance in time-delayed bistable systems driven by weak periodic signals. Zhang et al. [16] used Novikov theorem approximation and discussed SR with delay time and correlated Gaussian white noises in an asymmetric bistable system. Gao [17] investigated the optimal signal-to-noise ratio in a stochastic time-delayed bistable system. By contrast, it was found that the synergy of time delay and noise had a great influence on the optimal signal-to-

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noise ratio of the system output. Wu et al. [18] discussed the SR in a bistable system with time-delayed feedback and non-Gaussian noise. The influence of time delay on transient characteristics of time-delayed metastable system with cross-correlation noise was studied by means of stochastic simulation [19]. The research found that time delay increases the stability of the system. Most of the previous studies focus on the bistable system case. Hence the research and introduction of the monostable system is necessary [21, 22]. Jiao et al. [21] combined the α stable noise and power function monostable system to investigate the SR of the asymmetric monostable with the time delay. The numerical simulation study showed that the SR exists not only in the asymmetric monostable system but also in the time-delayed asymmetric monostable system. Ref. [22] studied the mean first-passage time of a multiple time-delayed power function monostable system. The influence of parameters on the mean first-passage time was analyzed. Above all, it has shown that the time delay can enhance the SR of the system, and there is very little research on the time-delayed monostable system.

In this paper, we study the exponential system with small time delay and weak periodic signals. We know that the SR system caused by time delay is a non-Markovian process. Therefore, it is difficult to obtain appropriate analysis results. Through using the small-delay approximation of the probability density [9,10,23], the time-delayed Fokker–Planck equation and the effective Langevin equation are derived. The results show that SR can be effected in the analysis results of the effective potential function, steady-state probability density and SNR. The above results are helpful for the application of the time-delayed exponential system under the strong noise background.

2. Delay induced transitions and stochastic resonance

We consider an over-damped exponential monostable system, and a time-delayed feedback model is introduced, thus the exponential monostable time-delayed feedback system can be described by the following Langevin equation.

$$\frac{dx(t)}{dt} = -\frac{dU(x)}{dx} + \beta x(t-\tau) + A\cos(\Omega t) + n(t) \tag{1}$$

In which, the $x(t)$ is the system output, parameters β and τ are the feedback intensity and the time-delayed parameter respectively, $A\cos(\Omega t)$ is periodic input signals with the amplitude A (usually select $A = 0.02$) and the characteristic frequency $\Omega = 2\pi f_0$ and $n(t) = \sqrt{2D}\xi(t)$ is a Gaussian white noise with $\langle \xi(t)\rangle = 0$ and $\langle \xi(t)\xi(t')\rangle = \delta(t-t')$. D is the intensity of noise. $U(x)$ is an exponential monostable potential function written as below

$$U(t) = le^{x^2/b} \tag{2}$$

where $l > 0$ and $b > 0$ are system parameters. Substituting Eq. (2) into Eq. (1), we can get

$$\frac{dx(t)}{dt} = -\frac{2lx}{b}e^{x^2/b} + \beta x(t-\tau) + A\cos(\Omega t) + \sqrt{2D}\xi(t) \tag{3}$$

where $x(t)$ indicates that it is the response of SR system, and $x(t)$ is essentially the solution to the differential equation obtained by the integral of Eq. (3). Here the function of integral is to filter out the high frequencies. So SR can be seen as a special low-pass filter. In addition, from Eq. (3), it can be found that the system parameters l and b, the time-delay τ and feedback intensity β also affect the system output and the performance of the special low-pass filter. On this basis, the next step is to study the system output laws and influence of the system parameters on the SR by virtue of the adiabatic approximation theory in the regime of small-parameter limitation. In addition, the small delay approximation when $\tau < 1$ is used to deviate the exact expression of SR [9,10]. The dynamics of Eq. (3) is a non-Markov process, and such a process can be reduced to a Markov process by using the probability density approach with approximate time-delayed feedback [10]. The approximate time-delayed Fokker–Planck equation corresponding to Langevin equation in Eq. (3) can be written as

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial[h_{eff}P(x,t)]}{\partial x} + D\frac{\partial^2 P(x,t)}{\partial^2 x} \tag{4}$$

where $P(x,t)$ represents the probability density of the stochastic process defined at time t of the Eq. (3). Then the h_{eff} is the conditional average drift that can be expressed as

$$h_{eff} = \int_{-\infty}^{+\infty} h(x, x_\tau)P(x_\tau, t-\tau\,|x,t)dx_\tau \tag{5}$$

where $x_\tau = x(t-\tau)$, $h(x, x_\tau) = x - x^3 + \beta x_\tau + A\cos(\Omega t)$, $h(x) = x - x^3 + \beta x + A\cos(\Omega t)$. $P(x_\tau, t-\tau|x,t)$ is the zeroth order approximate Markovian transition probability density [9,10,24].

$$P(x_\tau, t-\tau\,|x,t) = \sqrt{\frac{1}{4\pi D\tau}}\exp\left(-\frac{(x_\tau - x - h(x)\tau)^2}{4D\tau}\right) \tag{6}$$

Substituting Eq. (6) into Eq. (5), we obtain

$$h_{eff} = -(1+\beta\tau)\frac{2lx}{b}e^{x^2/b} + \beta(1+\beta\tau)x + (1+\beta\tau)A\cos(\Omega t) \tag{7}$$

In statistical physics, Fokker–Planck equation is an equivalent scheme to describe Langevin equation. Note that the derivation of Eq. (4) is under the condition of small-delay approximation and therefore the effective Langevin equation under small-delay condition with time-delayed feedback can be rewritten as

$$\frac{dx(t)}{dt} = -\frac{2lx}{b}e^{x^2/b} + \beta x + A\cos(\Omega t) + \beta\tau\left(-\frac{2lx}{b}e^{x^2/b} + \beta x + A\cos(\Omega t)\right) + \sqrt{2D}\xi(t) \tag{8}$$

Due to the existence of time-delayed feedback, Eq. (8) produces a coupling term $\beta\tau[-2lxe^{x^2/b}/b + \beta x + A\cos(\Omega t)]$. It can be seen that the output of the system can also be modulated by the time delay τ and feedback strength β. And the dynamic output laws of the system can be analyzed according to the effective Langevin equation.

So, the effective time-delayed potential function of Eq. (8) is written as

$$U_{eff}(x) = (1+\beta\tau)le^{x^2/b} - \frac{1}{2}\beta(1+\beta\tau)x^2 + (1+\beta\tau)A\cos(\Omega t)x \tag{9}$$

On one hand, due to the presence of the periodic force $A\cos(\Omega t)$, the effective potential $U_{\mathrm{eff}}(x)$ is modulated by the periodic force. Hence the potential is time-periodic. Assuming that the amplitude of the periodic force is small enough, i.e., $A \ll 1$, and because of the absence of noise, it is insufficient to induce a Brownian particle to jump from one well to the other. Therefore, the stable and unstable states of this system are considered to be invariable under small time-delayed approximation. On the other hand, assuming that the variation of the periodic force is slow enough, i.e., $\Omega \ll 1$ or adiabatic limit, there is enough time

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