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Almost perfect transport of an entangled two-qubit state through a spin chain

Rafael Vieira, Gustavo Rigolin *

Departamento de Física, Universidade Federal de São Carlos, 13565-905, São Carlos, São Paulo, Brazil

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A B S T R A C T

We show that using a slightly modified XX model for a spin-1/2 chain, one can transmit almost perfectly a maximally entangled two-qubit state from one end of the chain to the other one. This is accomplished without external fields or modulation of the coupling constants among the qubits. We also show that this strategy works for any size of the chain and is relatively robust to imperfections in the coupling constants among the qubits belonging to the chain. Actually, under certain scenarios of small disorder, we obtain better results than those predicted by the optimal ordered and noiseless case.

1. Introduction

High fidelity transmissions of quantum states from one place to another are important ingredients needed in the implementation of several quantum information tasks [1]. Indeed, quantum communication protocols, and in particular quantum key distribution protocols [2], cannot work without a reliable transmission of a quantum state from one place (Alice) to another (Bob). Even a to-be-built quantum computer will not work without a high fidelity quantum state transfer protocol within its hardware, since quantum information must flow without much distortion among the many components of a quantum chip.

There are at least three ways by which quantum information, here a synonym to a quantum state, can be transmitted from Alice to Bob. The first one is the obvious direct transmission of the quantum state, where the original physical system (a qubit, for simplicity) encoding the quantum information is sent from Alice to Bob. This usually happens when the quantum information is encoded in the state of a photon, which is sent along an optical fiber from Alice to Bob. The second way to transmit quantum information is via the quantum teleportation protocol [3], where a highly entangled state shared between Alice and Bob is the channel through which one is able to make a qubit with Bob be described by the state originally describing Alice's qubit. A third possibility is to use spin chains connecting Alice and Bob as the channel through which the quantum state describing one end of the chain ends up after a certain time describing the other end of it [4]. This is achieved by engineering the coupling constants among the qubits such that at time $t > 0$ Bob's end of the chain is described by the state initially describing Alice's end at time $t = 0$. In this last strategy, as well as in the quantum teleportation protocol, the physical system originally encoding the information is not sent from Alice to Bob, only the quantum state (quantum information) moves from Alice to Bob.

A main advantage of using the last strategy is related to the fact that once the coupling constants among the spins of the chain are set up to achieve a high fidelity transmission, we do not need to change them or switch them on and off. Such fixed arrangements can be very practical to allow the transmission of quantum states among the several components of a quantum computer, where it is not an easy task to constantly adjust the interaction strength among its qubits [4]. In addition to that, if the quantum chip is manufactured on a solid state system, it will be an advantage to have the communication channels connecting the several logic gates of the chip built on the same physical system. In this way there will be no need to sophisticated interfacing between different physical systems as it happens, for example, when one uses photons to transmit the information and spins to process it [4].

So far the great majority of works dealing with quantum state transmission have either studied

(a) the transferring of a single excitation or an arbitrary qubit from Alice to Bob [4–8,11,13,15,17–21,23,24,26,27,29,30,32];
(b) the creation of a highly entangled state between Alice and Bob (the two ends or two specific sites of the chain) [4,11,14,17,20,24,33];

* Corresponding author.
E-mail address: rigolin@ufscar.br (G. Rigolin).

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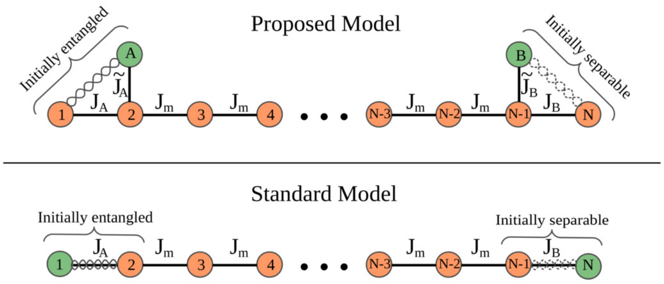

Fig. 1. Upper panel: Initially qubits A and 1 with Alice are prepared by her in the maximally entangled state $|\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ and all the other qubits are in the state $|0\rangle$. Our goal is to find the optimal constants J_A, $\tilde{J}_A$, J_m, J_B, $\tilde{J}_B$, and time t leading to the best pairwise entanglement transmission, i.e., we want the set of coupling constants and time t for which Bob's two qubits N and B become most entangled. Lower panel: Initially qubits 1 and 2 with Alice are prepared by her in the maximally entangled state $|\Psi^+\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ and all the other qubits are in the state $|0\rangle$. Our goal is to find the optimal constants J_A, J_m, J_B, and time t leading to the best pairwise entanglement transmission, i.e., we want the set of coupling constants and time t for which Bob's two qubits $N-1$ and N become most entangled.

(c) or the transferring of two (or more) excitations or two (or many)-qubit states from Alice to Bob [6,7,11,12,24,25,28].

In almost all these works the main focus was the study of a strictly one dimensional graph (spin chain-like systems), which we call the standard model (see the lower panel of Fig. 1). In the notation of the lower panel of Fig. 1, task (a) is related to transferring the state describing qubit 1 to qubit N while task (b) consists in preparing qubits 1 and 2 in a highly entangled state and wait long enough to obtain qubits 1 and N in a highly entangled state. Task (c), on the other hand, aims at transferring, for instance, a quantum state describing initially qubits 1 and 2 to qubits $N-1$ and N. See also Refs. [9,10,17,31] for chains built with continuous variable systems, i.e., systems described by pairs of canonically conjugated variables such as position and momentum.

In this work we are not interested in the sharing of entanglement between Alice and Bob, as described in task (b) above, or the transfer of single qubit states, i.e., task (a). Our focus is on task (c), with the following two ingredients. First, we are not concerned in the transfer of arbitrary two-qubit states. We want to study the transfer of maximally entangled two-qubit states, namely, Bell states. In other words, our main goal here is to investigate the transmission of the pairwise entanglement between two qubits with Alice to two qubits with Bob. The two qubits with Alice are prepared in a maximally entangled Bell state at time $t=0$ and our goal is to find the optimal coupling constants and time t leading to the greatest pairwise entanglement between two qubits with Bob (see Fig. 1). Second, and as depicted in the upper panel of Fig. 1, we work *slightly* beyond a one dimensional graph (spin chain). This geometry is crucial to have almost perfect transmission of a Bell state without a modulated chain [5,6,16] or external fields acting on the spins [12,22,24,28].

Indeed, as we show in the following sections, and much to our surprise, the standard model (lower panel of Fig. 1) gives very poor results in accomplishing this task for the simple unmodulated settings of Fig. 1, specially for long chains. However, the slightly modified spin chain (strictly speaking a two dimensional graph) shown in the upper panel of Fig. 1 gives extraordinary results, leading to an almost perfect pairwise entanglement transmission for spin chains of any size. Also, when studying the robustness of the proposed model, we observed that *small disorder* leads to a *greater* efficiency in many situations, a counter-intuitive result.

We should also explicitly mention the interesting work of Chen *et al.* [30], where a similar geometry to that shown in the upper panel of Fig. 1 is employed to transfer a single excitation from qubit 1 to N (task (a)). In Ref. [30] it is shown that one must couple qubits A and B with qubits 3 and $N-2$, respectively, instead of qubits 2 and $N-1$ as in our model, in order to get an efficient transfer. In Ref. [30] the authors set $J_A = J_m = J_B = J$ and $\tilde{J}_A = \tilde{J}_B = w$ and search for the optimal w/J leading to the best single excitation transfer.

Before we get to Sec. 3, where a systematic and extensive comparative study is made between the pairwise entanglement transmission efficiencies of the standard and proposed models, we give in Sec. 2 the mathematical formulation of the models studied here as well as other quantities and concepts needed to compute the efficiency of pairwise entanglement transmission. In Sec. 4 we test the proposed model against imperfections in its construction by studying how its efficiency is affected by static disorder. We show that for up to a moderately disordered system we obtain very good efficiencies and still outperform the standard model. For small disorder we can even get better results than those of the corresponding clean system. For a comprehensive analysis of the influence of noise and disorder in the functioning of the standard model we recommend Refs. [6,34–47]. In Sec. 5 we analyze, for completeness, how efficient the proposed and the standard models are in the transmission of a single excitation. In this case, the standard model is the best choice. Finally, in Sec. 6 we give our final thoughts on the subject of this manuscript and also propose further lines of research we believe might enhance our understanding of single state and pairwise entanglement transmissions along a spin chain under more realistic settings.

2. The mathematical tools

In Sec. 2.1 we give the Hamiltonian describing the systems depicted in Fig. 1 as well as how to efficiently solve numerically the Schrödinger equation that dictates their dynamics. In Sec. 2.2 we present the measure we employ to quantify the pairwise entanglement between two qubits and how we quantify the similarity between two quantum states.

2.1. The model and its time evolution

The Hamiltonian describing the proposed model is the isotropic XY model (XX model) with two extra qubits A and B coupled with qubits 2 and $N-1$, respectively. We have a total of $N+2$ qubits and the Hamiltonian reads

$$H = H_A + H_M + H_B, \tag{1}$$

where

$$\begin{aligned}
H_A &= J_A(\sigma_1^x\sigma_2^x + \sigma_1^y\sigma_2^y) + \tilde{J}_A(\sigma_A^x\sigma_2^x + \sigma_A^y\sigma_2^y), \\
H_M &= \sum_{j=2}^{N-2} J_m(\sigma_j^x\sigma_{j+1}^x + \sigma_j^y\sigma_{j+1}^y), \\
H_B &= J_B(\sigma_{N-1}^x\sigma_N^x + \sigma_{N-1}^y\sigma_N^y) + \tilde{J}_B(\sigma_{N-1}^x\sigma_B^x + \sigma_{N-1}^y\sigma_B^y).
\end{aligned} \tag{2}$$

Here $\sigma_i^\alpha\sigma_j^\alpha = \sigma_i^\alpha \otimes \sigma_j^\alpha$, with the superscript denoting a particular Pauli matrix and the subscript labeling the qubit acted by it. We also adopt the following convention usually employed in the quantum information community: $\sigma^z|0\rangle = |0\rangle, \sigma^z|1\rangle = -|1\rangle, \sigma^x|0\rangle = |1\rangle, \sigma^x|1\rangle = |0\rangle, \sigma^y|0\rangle = i|1\rangle, \sigma^y|1\rangle = -i|0\rangle$, where i is the imaginary unity and $|0\rangle$ and $|1\rangle$ are the eigenvectors of σ^z. For those used to the up and down notation of the condensed matter physics community, the relation between the latter and the present notation is $|\uparrow\rangle = |0\rangle$ and $|\downarrow\rangle = |1\rangle$. Note that if we set $\tilde{J}_A = \tilde{J}_B = 0$ we get the Hamiltonian describing the standard model.

An important property of Hamiltonian (2) is that it commutes with the operator $Z = \sigma_A^z + \sum_{j=1}^N \sigma_j^z + \sigma_B^z$, namely, $[H, Z] = 0$.

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