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A novel asymptotic expansion homogenization analysis for 3-D composite with relieved periodicity in the thickness direction

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ABSTRACT

A new asymptotic expansion homogenization analysis is proposed to analyze 3-D composite in which thermomechanical and finite thickness effects are considered. Finite thickness effect is captured by relieving periodic boundary condition at the top and bottom of unit-cell surfaces. The mathematical treatment yields that only 2-D periodicity (i.e. in in-plane directions) is taken into account. A unit-cell representing the whole thickness of 3-D composite is built to facilitate the present method. The equivalent in-plane thermomechanical properties of 3-D orthogonal interlock composites are calculated by present method, and the results are compared with those obtained by standard homogenization method (with 3-D periodicity). Young's modulus and Poisson's ratio obtained by present method are also compared with experiments whereby a good agreement is particularly found for the Young's modulus. Localization analysis is carried out to evaluate the stress responses within the unit-cell of 3-D composites for two cases: thermal and biaxial tensile loading. Standard finite element (FE) analysis is also performed to validate the stress responses obtained by localization analysis. It is found that present method results are in a good agreement with standard FE analysis. This fact emphasizes that relieving periodicity in the thickness direction is necessary to accurately simulate the real free-traction condition in 3-D composite.

1. Introduction

Three-dimensional (3-D) composite gains a wide interest due to its excellent mechanical performance under out-of-plane loading (e.g. impact). During the development and the application of 3-D composite, numerical analysis is generally performed to better understand its behavior under certain loading, and to further optimize its architecture. However, its heterogeneity and geometrical complexity often leads to a cumbersome analysis, and costly computational effort. Dealing with these problems, the analysis is carried out by idealizing 3-D composite with a representative volume element (RVE), i.e. unit-cell, that adequately represents the heterogeneity and the periodicity of its microstructure. To a certain extent, a unit-cell had been employed to analyze the properties of 3-D orthogonal woven composites [1] as well as their strength and damage behavior [2,3].

In the multiscale modeling approach, the unit-cell is usually assumed to be periodic in specific directions. This periodicity can be effectively and efficiently analyzed by employing asymptotic expansion analysis [4]. The asymptotic expansion analysis was developed in the framework of homogenization method, which is applicable for general composite structures [5]. The so-called asymptotic expansion homogenization (AEH) method was developed by Francfort [6] for the case of linear thermoelasticity in periodic structure. The AEH method has been employed to calculate the homogenized thermomechanical properties of composite materials (elastic moduli and coefficient of thermal expansion or CTE) [7–9].

The unit-cell in AEH method is usually implemented by considering the periodicity in three directions (i.e. *x*-, *y*- and *z*- directions). However, composite laminates, especially for the aerospace application, are very thin. The analysis of thin composite laminates necessitates the use of unit-cell that fully represents the whole thickness. Woo and Whitcomb [10] suggested the influence of finite thickness effect should be taken into account in the analysis of textile composites. In this case, the unit-cell should not be repeated infinitely in thickness direction, i.e. the unit-cell is assumed to possess only in-plane periodicity.

Several researches consider the existence of only in-plane periodicity by using plate theory. Thin composite plates reinforced with orthotropic bars were analyzed by Challagulla et al. [11] using AEH method. Rostam-Abadi et al. [12] conducted an AEH analysis

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of composite laminates by involving Kirchhoff–Love plate theory to assume the displacement field. The Kirchhoff–Love theory in AEH was also used by Buannic et al. [13] to analyze corrugated core sandwich panels. The aforementioned AEH analyses were mainly focused on the evaluation of equivalent stiffness properties of plate, while the detailed stress response of the structure was not considered. Lapeyronnie et al. [14] analyzed 3-D layer-to-layer angle-interlock composite using AEH by involving Kirchhoff–Love plate theory. Macroscopic response was represented by using homogeneous isotropic cell and compared to the response of 3-D heterogeneous cell. The existence of only in-plane periodicity of plate structure was also considered by Cai et al. [15] and it was employed to analyze the effective properties of honeycomb plate. Nevertheless, the detailed stress response of the unit-cell was not investigated. He et al. [16] proposed analytical solutions of an orthotropic multilayered rectangular plate with in-plane small periodic structure. The asymptotic expansion analysis applied the small periodicity in only one direction (i.e. x-coordinate system of rectangular plate), where the constitutive equations were based on plane-strain assumption. The paper studied the thickness effects by calculating the so-called state vectors at the top and bottom surfaces of the layered plate.

In this paper, a novel AEH method by considering the effect of finite thickness, represented by the existence of only in-plane periodicity in composite structure, is proposed for the analysis of 3-D composites. The periodicity is applied in both in-plane directions, where that of the thickness direction is relieved. The present formulation is developed to perform homogenization analysis (calculation of equivalent thermomechanical properties) and localization analysis (detailed assessment of stress response). The formulation is developed and implemented in an in-house code written using Fortran 90, and employed to specifically analyze one type of 3-D composites, namely 3-D orthogonal interlock composites.

2. AEH method with relieved periodicity in thickness direction

2.1. General concept

The AEH method developed in this research involves two spatial scales, namely microscopic and macroscopic scales. The macroscopic and microscopic scales are described in $\mathbf{x}$ and $\mathbf{y}$ coordinate systems, respectively. Both scales are correlated by parameter $\varepsilon = \mathbf{x}/\mathbf{y}$, which is the ratio between macroscopic and microscopic scales. The heterogeneous macrostructure shown in Fig. 1(a) consists of heterogeneous and periodic microstructure (i.e. unit-cell) shown in Fig. 1(b). The homogenization analysis enables us to regard the heterogeneous macrostructure as a homogeneous one when ε approaches zero (see Fig. 1(c)). The modeling of the whole thickness of macrostructure necessitates the relieving of periodic boundary condition in the thickness direction due to the non-repeating nature of microstructure (or the unit-cell) in the out-of-plane direction. In this case, the unit-cell is periodic, or repeated infinitely, only in the in-plane direction of the macroscopic structure (x_1 and x_2). In other words, top and bottom parts of the unit-cell are free-boundaries. As the result, the deformation of the top and bottom surfaces of the unit-cell may not be the same.

2.2. Periodic vector function

The periodicity of the unit-cell can generally be described using periodic vector function: $g^{\varepsilon}(\mathbf{x}) = g(\mathbf{x}, \mathbf{y}) = g(x_1, x_2, x_3, y_1 + Y_1, y_2 + Y_2, y_3 + Y_3)$. However, in present method, the periodic vector function is modified so that: (i) microstructural variables within the unit-cell vary in three-directions (i.e. in-plane and out-of-plane directions) when they are considered from microscopic viewpoint ($\mathbf{y}$ coordinate system); (ii) the variables vary only in the in-plane directions from the macroscopic viewpoint ($\mathbf{x}$ coordinate system). Thus, the periodic vector function is written as follows

$$g^{\varepsilon}(\mathbf{x}) = g(\mathbf{x},\mathbf{y}) = g(x_1, x_2, y_1 + Y_1, y_2 + Y_2, y_3); \quad \text{where} \quad \mathbf{y} = \frac{\mathbf{x}}{\varepsilon} \tag{1}$$

Y_1 and Y_2 are the dimensions of the unit-cell in direction −1 and −2, respectively. Derivatives of periodic vector function g with respect to the macroscopic coordinate $\mathbf{x}$, obtained by chain rule, are as follows

$$\frac{\partial g^{\varepsilon}}{\partial x_1} = \frac{\partial g}{\partial x_1} + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_1} \tag{2}$$

$$\frac{\partial g^{\varepsilon}}{\partial x_2} = \frac{\partial g}{\partial x_2} + \frac{1}{\varepsilon}\frac{\partial g}{\partial y_2} \tag{3}$$

$$\frac{\partial g^{\varepsilon}}{\partial x_3} = \frac{1}{\varepsilon}\frac{\partial g}{\partial y_3} \tag{4}$$

Homogenization analysis regards that the integration of periodic vector function exists when ε approaches zero from the positive side, which can be expressed as follows

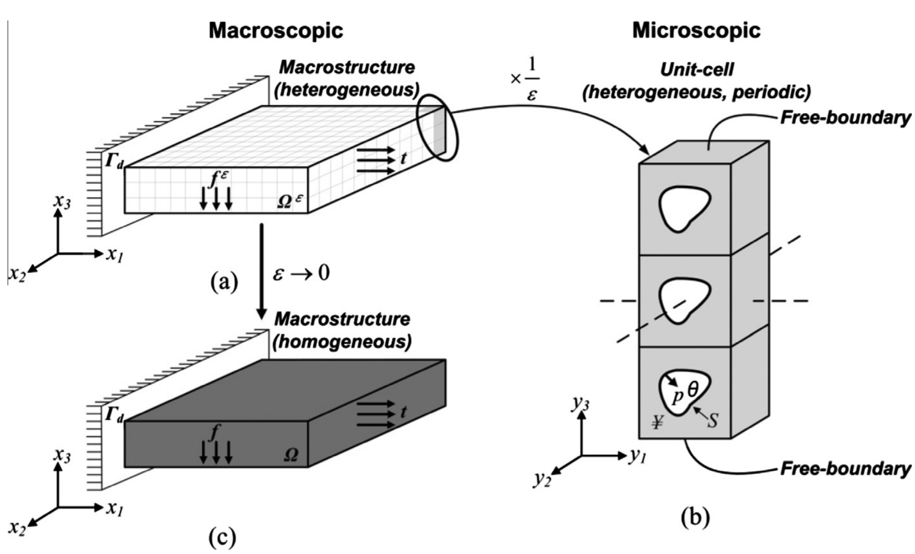

Fig. 1. (a) Heterogeneous macrostructure, (b) heterogeneous, periodic unit-cell with free-boundaries at the top and bottom surfaces, (c) homogeneous macrostructure.