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Metaheuristic optimization-based identification of fractional-order systems under stable distribution noises

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ABSTRACT

This research investigates the identification problem of fractional-order chaotic systems under stable distribution noises. A powerful metaheuristic optimization method called composite differential evolution is used for the identification of the fractional-order Lorenz and Chen systems in the noisy environment, where the structure, parameters, orders and initial values of the systems are all unknown. The identification accuracy is examined when the noise follows the three special cases of stable distributions, i.e., Gaussian, Cauchy and Lévy distributions. In addition, the impact of the four parameters of stable distributions on the identification accuracy is discussed. The experimental results show that the identification error becomes larger when the noise switches from Gaussian to Cauchy and Lévy distributions. The results also turn out that the location of the stable distribution noise plays the most substantial role in the identification accuracy.

1. Introduction

A fractional-order system is a dynamic system that can be modeled by a fractional differential equation with derivatives of non-integer order [1]. Over the past decade, fractional-order systems have attracted much attention due to the fact that fractional dynamics have been observed in multiple systems [2–4]. Studies on fractional-order systems are helpful to understand the anomalous behavior of dynamical systems in physics, electrochemistry and biology.

In recent years, chaotic systems have gained popularity because of their successful applications in secure communication, image encryption, robotics, etc. [5–7]. In some of the fractional-order systems, chaotic behavior has been detected. For instance, Lorenz system becomes chaotic when the sum of all the orders is smaller than 3 [8]; Lü system shows chaotic behavior as the order is 0.3 [9]; Rössler equations are found to exhibit chaotic phenomenon as the order is smaller than 3 [10]. For other related work on fractional-order chaotic systems, the reader is referred to [11,12] and references therein.

The research on fractional-order chaotic systems is usually categorized into three groups: control [13], synchronization [14], and identification [15]. In these three categories, identification is the very first step before control and synchronization since it gives the system's detailed information, such as parameters, orders, etc. So far, chaotic systems are identified by means of two categories of methods: control-based or metaheuristic optimization-based methods. A control-based method needs to regulate the system to stable states beforehand [16,17]. While in real-world cases, the systems are not always easy to control; furthermore, it is possible that only limited time series data are available. Recently, metaheuristic optimization-based methods have received increasing interest when identifying chaotic systems. For example, Tang et al. used a differential evolution (DE) algorithm to identify the parameters of commensurate fractional-order chaotic systems [18]. Zhu et al. solved the identification problem of fractional-order Lorenz, Lü and Chen systems via an improved DE algorithm [15]. In [14] and [19], particle swarm optimization was successfully applied to parameter estimation of fractional-order chaotic systems. In [20], Du et al. utilized a DE variant to solve a more complicated and practical identification problem, in which the structure, parameters, orders and initial values are all unknown.

Among these studies, only Ref. [15] considered the noise perturbation (e.g., the white Gaussian noise following the distribution $\mathcal{N}(0,1)$) during the identification process. However, noise is quite common in real-world identification cases. In addition, considering noise may follow disparate distributions in different cases, it is necessary to investigate how the identification accuracy varies

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Table 1
The description of the five fractional-order chaotic systems formulated based on Eq. (3).

System	Formulation	$f(x,y,z)$	$g(x,y,z)$	$h(x,y,z)$	$\Phi(x,y,z)$
Lorenz	$\begin{cases} D^{q_1}x = a(y-x), \\ D^{q_2}y = x(b-z)-y, \\ D^{q_3}z = xy-cz. \end{cases}$	a	$x(b-z)$	x	0
Chen	$\begin{cases} D^{q_1}x = a(y-x), \\ D^{q_2}y = dx-xz+cy, \\ D^{q_3}z = xy-bz. \end{cases}$	a	$dx-xz$	x	0
Lü	$\begin{cases} D^{q_1}x = a(y-x), \\ D^{q_2}y = -xz+cy, \\ D^{q_3}z = xy-bz. \end{cases}$	a	$-xz$	x	0
Liu	$\begin{cases} D^{q_1}x = -ax-ey^2, \\ D^{q_2}y = by-kxz, \\ D^{q_3}z = -cz+mxy. \end{cases}$	$-ey$	$-kxz$	mx	0
Newton–Leipnik	$\begin{cases} D^{q_1}x = -ax+y+10yz, \\ D^{q_2}y = -x-0.4y+5xz, \\ D^{q_3}z = -5xy+bz. \end{cases}$	1	$-x+5xz$	$5x$	$10y$

while the distributions of noise are different. As a generalized distribution family, stable distributions represent a class of four-parameter probability distributions defined by specific characteristic functions; Gaussian, Cauchy and Lévy are three special cases. Moreover, stable distributions have been observed in multiple areas including engineering [21,22], economics [23,24], business [25], and so on. Therefore, it is of great practical significance to consider stable distribution noises for the identification of fractional-order chaotic systems. Furthermore, the investigation on stable distribution noises will not only enrich the research of identifying fractional-order systems, but broaden the applications of stable distributions. It is worth pointing out that system identification problems in a noisy environment have been investigated in various studies [26–28]. However, the distributions of the noises are not well discussed. In addition, these studies have not considered the identification by metaheuristic optimization-based methods. Therefore, it is meaningful to conduct the research on identifying fractional-order chaotic systems under stable distribution noises by metaheuristic optimization-based methods.

Following the discussion above, we investigate the identification problem of fractional-order chaotic systems under stable distribution noises in this research. The identification accuracy is examined when the noise follows the three special cases of stable distributions, i.e., Gaussian, Cauchy and Lévy distributions. Additionally, the impact of the four parameters of stable distributions on the identification accuracy is discussed. The contributions of our research are threefold: 1) stable distribution noises are taken into account when identifying fractional-order systems; 2) the noise is considered in the identification process when the structure, parameters, orders and initial values are all unknown; 3) the experimental results show that the distribution of the noise has a great impact on the identification accuracy.

The organization of the paper is as follows. The background information is given in Section 2. In Section 3, we formulate the identification problem under stable distribution noises. In Section 4, a series of experiments are carried out. Finally, Section 5 concludes the whole paper.

2. Background

2.1. Fractional-order chaotic systems

A number of ways are proposed to define fractional derivative. Riemann–Liouville fractional derivative is one of the most commonly used definitions, which is defined as follows:

$$D^q f(x) = \frac{1}{\Gamma(n-q)} \frac{d^n}{dt^n} \int_0^t \frac{f(\tau)}{(t-\tau)^{q-n+1}} d\tau, \tag{1}$$

where q indicates the order of the system and $n = \lceil q \rceil$; $\Gamma(\cdot)$ denotes the Gamma function:

$$\Gamma(z) = \int_0^\infty \frac{t^{z-1}}{e^t} dt. \tag{2}$$

It is assumed that the dimension size of the fractional-order chaotic systems is 3. The general formulation of 3-dimensional fractional-order chaotic system is as follows [29]:

$$\begin{cases} D^{q_1}x = y \cdot f(x,y,z) + z \cdot \Phi(x,y,z) - \alpha x, \\ D^{q_2}y = g(x,y,z) - \beta y, \\ D^{q_3}z = y \cdot h(x,y,z) - x \cdot \Phi(x,y,z) - \gamma z, \end{cases} \tag{3}$$

where $0 < q_i < 1$ $(i = 1,2,3)$; x, y, z denote the state variables of the system; $f(\cdot), g(\cdot), h(\cdot)$ and $\Phi(\cdot)$ indicate the continuation nonlinear vector functions in $\mathbb{R}^3 \to \mathbb{R}$ space. We call the system a commensurate fractional-order chaotic system when $q_1 = q_2 = q_3$; otherwise we call it an incommensurate fractional-order system. Based on Eq. (3), we list five popular fractional-order chaotic systems, that is, Lorenz, Chen, Lü, Liu and Newton–Leipnik in Table 1. We use these five systems to constitute the system candidate pool in the experiments of this research.

2.2. Stable distribution

Stable distributions, also called Lévy α-stable distributions, refer to the distributions with the following property: a linear combination of two independent copies of the random variable has the same distribution, up to location and scale parameters [30]. The stable distribution is a four-parameter family of distributions and is usually denoted by $S(\alpha, \beta, \gamma, \delta)$. The first parameter $\alpha \in (0, 2]$ is the *characteristic exponent*, which describes the tail of the distribution. The second parameter $\beta \in [-1, 1]$ is called the *skewness*, which specifies if the distribution is left- ($\beta < 0$) or right- ($\beta > 0$) skewed. The third parameter $\gamma > 0$ is the *scale*, which measures the width of the distribution. The last parameter $\delta \in (-\infty, \infty)$ is the *location*, which represents the mean value.

The probability density function of $S(\alpha, \beta, \gamma, \delta)$ with different parameter values is plotted in Fig. 1. From Fig. 1(a), as α decreases, the peak turns higher and the tail becomes heavier. In Fig. 1(b), when β is positive, the curve is skewed to the right and the right tail is thicker; when β is negative, the curve is skewed to the left and the left tail is thicker; when β is 0, the curve is symmetric. Fig. 1(c) shows that a larger γ makes the distribution more spread out, and a smaller γ makes it more concentrated. Fig. 1(d) exhibits that the value of δ determines the location of the distribution.

It is worth pointing out that the family of stable distributions is a rich class, and includes three special cases:

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