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Mode-dependent mechanical losses in disc resonators

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A B S T R A C T

Mechanical spectroscopy gives information on the structure of solids and their relaxation mechanisms through the measurements of the elastic constants and the mechanical loss angle of materials. One common way to estimate these quantities is the resonant method where the frequency and the characteristic decay time of oscillations are measured. Since many solid materials can be easily found in the shape of thin disc we have investigated the mechanical loss of these resonators and we have found experimentally that the loss angle dependence on the mode is not trivial but rather follow a distribution of modes into families. We give a model that is able to justify the existence of these families and to predict the level of losses in silicon, silica and brass discs. The model considers the thermoelastic effect and the excess damping caused by the condition of the disc edge. The results of this research are relevant to the research on thin films that are deposited on thin discs like the optical coatings used on the mirrors for the gravitational wave detectors.

1. Introduction

The advent of transducers with increased sensitivity always has opened new frontiers of science and technology to mankind but also allowed studying fluctuations phenomena that were not accessible before. For example electronic amplifiers, that powered the telecommunications, opened the road to the investigation on fundamental electronic noises and, among them, the Johnson–Nyquist noise in resistors. Since then, systems with small dissipation are the solution to the quest of low noise devices, mechanical as well as electrical.

In the last decades a large effort has been put in the development of Gravitational Waves (GW) detectors. For such kilometrescale devices the fluctuations in the position of the reflecting surface of mirrors, due to thermal noise in suspensions and in the mirrors themselves, is one of the most important limitation in future developments of sensitivity. When the losses become extremely low, as in the case of the materials used in GW detectors, their measurement becomes very challenging and a profound knowledge of various loss mechanisms becomes essential.

As explained by the Fluctuation–Dissipation Theorem [1] the level of thermal noise depends on the dissipative dynamics of the observable chosen for the description of the system. As observable we choose the displacement $\vec{x}$ of a point P of a system that it is composed by N interacting parts. Following the work of Yuri Levin [2], in order to work out the thermal noise level of x one has to imagine to force the system on the point P with a periodic force $\vec{F}(\omega)$ along the direction of $\vec{x}$ and then calculate the dissipated power. The single sided Power Spectral Density S_{xx} of the thermal noise associated to the displacement is then:

$$S_{xx} = \frac{4k_B T}{\omega}\frac{1}{\mathrm{Stf}(\omega)} \cdot \phi_{\mathrm{TOT}}(\omega) \tag{1}$$

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where k_B is the Boltzmann constant, T the temperature, $\omega = 2\pi f$ the angular frequency, Stf(ω) is the stiffness defined as the ratio between the moduli of the force $\sqrt{\vec{F}(\omega)\cdot\vec{F}^*(\omega)}$ and of the displacement $\sqrt{\vec{x}(\omega)\cdot\vec{x}^*(\omega)}$ (*denotes the complex conjugate), $\phi_{\rm ToT}$ is the loss angle of the entire system defined as $\phi_{\rm ToT} = E_{\rm d}/(2\pi\, E_{\rm t})$ where $E_{\rm d}$ is the energy dissipated in one cycle and $E_{\rm t}$ is the total energy of the system. We consider small dissipations so that the total energy does not change significantly in one cycle.

As we said the system is composed by N parts and the energy can be dissipated by different mechanisms. For each of them we assume that the energy lost in one cycle is proportional to some form of stored energy and each stored energy can be associated to a different loss mechanism. For example, the energy lost in structural relaxations is proportional to the total elastic energy. Differently, one can imagine that the bulk and shear deformations are related each to different loss mechanisms. Moreover, viscous dissipation is proportional to the kinetic energy whereas thermoelastic loss is proportional to the energy stored in the volume change (therefore called here *dilatation* energy), as it will be shown in details in this work. Considering only a single part i we can express the energy lost in one cycle as a double sum, over the stored energy $s = 1, 2, \ldots$ and over the dissipation mechanisms m:

$$E_{{\rm d},i} = 2\pi \cdot \left[E_{1,i} \sum_m \phi_{m,i} + E_{2,i} \sum_m \phi_{m,i} + \ldots \right] = 2\pi \cdot \sum_s E_{s,i} \sum_m \phi_{m,i} \quad (2)$$

Replacing the previous equation into the definition of the total loss angle $\phi_{\rm ToT}$ we have:

$$\phi_{\rm ToT} = \sum_i \sum_s \mathbf{D}_{s,i}(\omega) \cdot \sum_m \phi_{m,i}(\omega) \quad (3)$$

where $\mathbf{D}_{i,s}(\omega)$, known as *dilution factor*, is the ratio between the stored energy E_s in the part i and the total energy of the system E_t, i.e. $E_{i,s}/E_t$. For each part i a stored energy of type s appears only once and it may factorize several loss angles. Equation (3) clarifies the important role of loss properties of materials to describe relaxation/dissipation processes.

In order to develop the next generation of gravitational wave detectors a novel measurement system for low loss materials has been developed. The mechanical losses can be measured with the resonant method [3] using disc resonators suspended on a sphere at the centre of their flat surface. This suspension is of the type *nodal* because several modes show one or more nodal lines passing in the centre of the disc surface. The system is called GeNS (for 'Gentle Nodal Suspension'), and is described in Section 3. More details can be found in the work of E. Cesarini et al. [4]. The loss angle of the resonator ϕ is worked out from the characteristic decay time τ of the excited mode through the simple relation $\phi^{-1} = \pi\, f\, \tau$ where f is the resonance frequency. The single point suspension used in GeNS minimises the possible loss of energy coming from the mechanical coupling between the resonant modes and the environment: on a 3″ diameter silicon wafer a loss angle as low as 4.7×10^{-9} has been measured [5] at cryogenic temperature. Considering the ultra-low excess losses, the repeatability of measurements, the density of modes associated to a 2-D oscillator with respect to a 1-D system and the easiness to procure samples in the form of wafers we believe that the combination of GeNS and discs will be the standard protocol to characterize materials for future GW detectors and in particular to characterize optical coatings deposited on discs.

For these composite resonators, made of the substrate D and coating C, the total loss angle is:

$$\phi_{\rm ToT} = \sum_s \mathbf{D}_{D,s}(\omega) \cdot \sum_m \phi_{D,m}(\omega) + \sum_s \mathbf{D}_{C,s}(\omega) \cdot \sum_m \phi_{C,m}(\omega) \quad (4)$$

The calculation of coating losses $\phi_{C,m}(\omega)$ for the different dissipation mechanisms m requires knowledge about the losses of discs D and their dilution factors. For that reason an extended research has been carried out on bare substrates showing that the disc losses have a non-trivial but reproducible dependence on the modes. These are grouped in families and these families are justified by the thermoelastic (TE) loss and the loss associated to the polishing condition of the disc edge.

2. Thermoelastic loss in cylinders

Under some particular conditions, often present in samples, the thermoelastic effect is the dominant source of mechanical energy loss at room temperature in the acoustic band. That is the case when the material has low intrinsic losses, relatively high thermal expansion coefficient and good thermal conductivity. In the following the reason for this dominance of TE damping is explained as well as the role of the physical dimensions of the sample.

In a vibrating body, the thermoelastic process consists in the coupling of the elastic strain field with the local internal energy, resulting in heating of the compressed regions and cooling of the extended regions. Consequently, an irreversible heat flow is settled along the temperature gradient, resulting in the onset of energy damping. At frequencies much higher than the inverse of a typical heat diffusion time the vibration is adiabatic, while for lower frequencies a practically isothermal process is attained. However, when the period is close to the diffusion time, the resulting lack of thermal equilibrium makes the thermoelastic damping reach its maximum effect. Since the higher is the conductivity, the shorter is the diffusion time, the peak of dissipation strength reaches the audio band for good heat conductors.

Based on these considerations, the thermoelastic loss is expected to show a peaked dependency on frequency, and, as C. Zener showed in his seminal papers [6,7], to a very good approximation, the effect follows a Debye's peak in vibrating reeds. This behaviour has been often taken as a reference plot even for bodies with aspect ratio and/or geometries significantly different from that of reeds, due to its simplicity and to the transparent relationship with thermal and mechanical parameters. Nonetheless, this interpretation must be regarded as just a first approximation.

Since the cited papers from C. Zener, the methods to work out analytic expressions for the thermoelastic damping can be traced back to one of two main approaches:

- Dynamical approach [8]: the rate of energy dissipation is related to the phase lag between the stress and the corresponding strain. Therefore, the quality factor only depends on the effective Young's modulus in the dynamical equation. To take into account the thermoelastic process, the coupled thermal and mechanical equations must be solved with a proper set of boundary conditions. The modal eigenfrequencies ω will result to be complex, so that the quality factor will be computed as:

$$Q^{-1} = \phi = 2\frac{\Re(\omega)}{\Im(\omega)} \quad (5)$$

the factor 2 being due to the fact that the mechanical energy is proportional to the square of its amplitude.
- Rate of generation of heat: the entropy rise ΔS can be used to work out the amount of energy lost per cycle. Eventually, the latter is related to the temperature field T in the body through: