Physics Letters A 382 (2018) 2181–2185

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Thermal noise computation in gravitational wave interferometers from first principles

Miroslav Tugolukov [a,*], Yuri Levin [b], Sergey Vyatchanin [a]

[a] Faculty of Physics, M.V. Lomonosov Moscow State University, Moscow, Russia
[b] Monash Center for Astrophysics and School of Physics and Astronomy, Monash University, Clayton, Australia

ARTICLE INFO

Article history:
Available online 10 July 2017
Communicated by I. Pinto

Keywords:
Thermal fluctuations
Gravitational wave interferometer
Adiabatic invariant
Fabri–Perot cavity

ABSTRACT

We propose a universal method of computation of thermal noise in mirrors of gravitational wave interferometers based on first principles. We imagine a situation where a mirror is part of a Fabry–Perot cavity. The movement of the mirror's surface produces variation of the eigen frequency of the cavity, which is computed by evaluating the variation of the energy stored in cavity. We consider two particular examples: first, the thermal noise from a dielectric slab inside the Fabry–Perot cavity, and second, the polarization-dependent thermal noise in the folded cavity.

1. Introduction

The direct detections of gravitational waves by LIGO detectors [1,2] opened the era of gravitational wave astronomy. This groundbreaking discovery is a consequence of technology breakthroughs that allowed measurement of microscopic displacements of $\sim 10^{-18}$ m. Laser gravitational wave detectors based on the Michelson scheme have proven their potential to reach this accuracy by measuring tiny phase difference between the light beams in their two arms. Among them are LIGO [3], Virgo [4] and GEO 600 [5].

V.B. Braginsky was among the pioneers of thermal noise research, who paid particular attention to the thermal noise originating from dissipation near the mirror surface and near the laser beam spot. It is this thermal noise that presents a severe limitation for the detector sensitivity in their most sensitive frequency band from 40 Hz to 2000 Hz [6–9]. In current detectors the most severe thermal noise is Brownian noise [10] in mirror coatings due to the high mechanical dissipation in the coating' materials. The thermally induced random stresses in the coatings and the substrates of the test masses lead to deformation of the mirror surfaces; it is these random deformations that are seen as thermal noise in the interferometer' output. More specifically, a passing gravitational wave produces change of an eigen frequency of the main mode in the arm cavity, which is registered by measuring a phase shift of a monochromatic optical wave reflected from the cavity. In turn, the thermal noise of the mirror's surface also randomly changes the eigen frequency of the same mode, which introduces noise to the GW signal.

In this paper we show how to compute a change in the optical mode frequency for an small deformation of the optical interfaces inside the cavity. Our approach together with the Fluctuation–Dissipation Theorem (FDT) [11–13] allows one to compute the thermal noise in a variety of complex optical configurations.

This paper follows the following format: in Sec. 2 we formulate our approach and derive the key equations. In Sec. 2.1 we consider an example of a fluctuating dielectric surface inside Fabry–Perot (FP) cavity. In Sec. 3 we apply our approach for thermal fluctuations of mirror's surface in folded cavity where we take into account the light polarization; this is a generalization of results presented in [14] that did not account for polarization.

In parallel with this work, our approach has been used in [15] for computing the thermal noise from a reflective grating (YL and SP are both co-authors on that work). In this paper we focus on the dielectric interfaces inside resonators and on the effect of polarization of the incident light.

We found out that similar approach was used in [16] to evaluate the influence of an absorption layer on the resonant frequencies and Q-factors of spherical microresonators.

2. Derivation of the readout variation

We begin by considering a simple FP cavity that is being resonantly pumped by external laser light. As a gravitational wave passes through the interferometer, it changes the physical length of the cavity. More specifically, in a transverse-traceless gauge and in a coordinate system with the input test mass as the origin, the tidal field of the gravity causes a displacement z_h of the end

* Corresponding author.
E-mail address: maeloc@yandex.ru (M. Tugolukov).

http://dx.doi.org/10.1016/j.physleta.2017.07.001

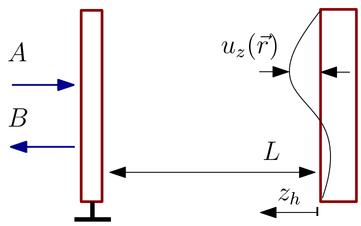

Fig. 1. FP cavity as meter of variation of eigen frequency of optical mode initiated by gravitational wave. The thermal noise produces perturbation u_z of surface, changing eigen frequency and masking gravitational signal.

mirror. The displacement z_h causes the frequency of the resonant optical mode to change linearly, changing by the amount $\Delta\omega_0$. The phase of the wave that is reflected from the cavity, is in turn affected by $\Delta\omega_0$, and thus by monitoring the phase, one monitors the resonant mode frequency and z_h. The change of frequency and the displacement are related by

$$\frac{\Delta\omega_0}{\omega_0} = \frac{z_h}{L} \tag{2.1}$$

The thermal noise is produced by small random perturbations $u_z(\vec{r})$ of the end mirror's surface, here $\vec{r}$ is marks the location of a point on the mirror surface — see Fig. 1. This perturbation, in turn, changes the eigen frequency ω_0. We have to compute how the perturbation $u_z(\vec{r})$ affects the eigen mode frequency.

The perturbation u_z may have an arbitrary form, however, we assume that it is smaller than the wavelength of light, so that it impacts $\Delta\omega_0$ linearly, and occurs on much longer time scale than the round-trip of the laser light inside the cavity, so that it does not create coupling with other optical modes and changes $\Delta\omega_0$ adiabatically. The cavity is decoupled from the outside, and the light inside the cavity is concentrated in the mode ω_0. The slow displacement u_z will produce a change of optical energy $\mathcal{E}$ in the mode, and following adiabatic invariant should be conserved:

$$\frac{\mathcal{E}}{\omega_0} = \text{const}, \quad \Rightarrow \quad \frac{\Delta\omega_0}{\omega_0} = \frac{\Delta\mathcal{E}}{\mathcal{E}_0}\,. \tag{2.2}$$

We now give a simple argument of how to compute $\Delta\mathcal{E}$.

The physical reason of energy to vary is the work performed by the Lebedev's light pressure $p(\vec{r}_\perp)$ acting normally on the mirror's surface:

$$\Delta\mathcal{E} = -\int p(\vec{r}_\perp)\, u_z(\vec{r}_\perp)\, d\vec{r}_\perp\,. \tag{2.3}$$

In writing this we use the fact that the motion does not introduce coupling into other modes, and therefore, all of the optical energy remains in the same mode. The light pressure p_i acting on dielectric media surface (along *outer* i-th axis) may be calculated through the Maxwell stress tensor σ_{ij} in the media and outside [17]:

$$\sigma_{ij}^{\epsilon} = \frac{1}{4\pi}\left(\epsilon E_i E_j + H_i H_j - \frac{\epsilon|E|^2 + |H|^2}{2}\,\delta_{ij}\right). \tag{2.4}$$

We emphasize that the light pressure is calculated for the unperturbed cavity.

For a diagonal component σ_{zz} the formula may be simplified:

$$\sigma_{zz}^{\epsilon} = \frac{\epsilon\left(E_z^2 - E_x^2 - E_y^2\right) + H_z^2 - H_x^2 - H_y^2}{8\pi}\,. \tag{2.5}$$

If we are interested in the light pressure acting on a dielectric plate inside the cavity (for example, a beam splitter inside the power recycling cavity of LIGO interferometer), the normal pressure is equal to the difference of normal stress tensor components inside and outside the dielectric:

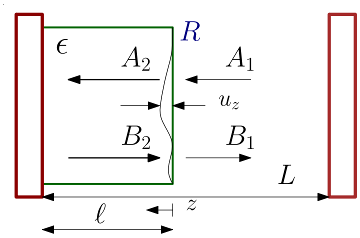

Fig. 2. The incident waves with amplitudes B_2 and A_1 of electric fields falls on the boundary of dielectric media (permeability ϵ). Boundary is covered by coating with reflection coefficient R. We derive formulas for reflected and transmitted amplitudes of electric fields A_2, B_1.

$$p_i = \sigma_{ii}^{\text{outer}} - \sigma_{ii}^{\text{inside}} \tag{2.6}$$

Equations (2.2), (2.3), (2.5) give a direct recipe for computing the generalized displacement z_{eff}, which defines variation of eigen frequency as in (2.1), using the field distribution in *the unperturbed* cavity:

$$z_{\text{eff}} = \int f_{\text{eff}}(\vec{r}_\perp)\, u_z(\vec{r}_\perp)\, d\vec{r}_\perp\,, \quad f_{\text{eff}}(\vec{r}_\perp) = \frac{L}{\mathcal{E}}\cdot p_i(\vec{r}_\perp) \tag{2.7}$$

Now we can apply the FDT [11,12] in a formulation of [13]. Following the latter, we apply a virtual pressure p_z oscillating at frequency ω to mirror surface:

$$p_z(\vec{r}) = F_0 \sin\omega t\, f_{\text{eff}}(\vec{r}) \tag{2.8}$$

and calculate the time-averaged dissipated power W_{diss}. The one-sided spectral density $S(\omega)$ of the fluctuations in the generalized coordinate z_{eff} is equal to:

$$S(\omega) = \frac{8k_B T\, W_{\text{diss}}}{\omega^2 F_0^2} \tag{2.9}$$

2.1. The case of a dielectric boundary inside a Fabry–Perot cavity

As an illustrative auxiliary example, in this subsection we consider a FP cavity with a dielectric media inside and show how a small perturbation of its surface changes the eigen frequency of the cavity. Let the permeability of the dielectric be ϵ, and assume that the magnetic permittivity is 1, so that the refractive index of the media is equal to $n = \sqrt{\epsilon}$. The surfaces of both mirrors are assumed to be fixed, while the surface of the dielectric media is perturbed by $u_z(\vec{r}_\perp)$ — see Fig. 2. We assume that the free surface of the dielectric is covered by a reflecting coating with a reflectivity R.

We now recall that we have to compute the fields in an unperturbed cavity in order to find the optical pressure on the dielectric boundary.

First of all we derive the expressions for the amplitudes of reflected and transmitted waves for the scheme shown on Fig. 2. We must take into account that the fluxes corresponding to the waves are given by

$$W_{A1} = \frac{|A_1|^2}{4\pi}c, \quad W_{B1} = \frac{|B_1|^2}{4\pi}c, \ \text{in vacuum} \tag{2.10}$$
$$W_{A2} = \frac{\sqrt{\epsilon}\,|A_2|^2}{4\pi}c, \quad W_{B2} = \frac{\sqrt{\epsilon}\,|B_2|^2}{4\pi}c, \ \text{in media}$$

Here A_i, B_i are the amplitudes of *the electric fields* and c is the speed of light in vacuum. We can write down the relations between the reflected and the transmitted waves (see notations on Fig. 2):

$$\sqrt{n}\,A_2 = T A_1 + R\sqrt{n}\,B_2, \tag{2.11a}$$
$$B_1 = T\sqrt{n}\,B_2 - R A_1, \quad R^2 + T^2 = 1 \tag{2.11b}$$