Physics Letters A 382 (2018) 2376–2381

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Condensates in double-well potential with synthetic gauge potentials and vortex seeding

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ARTICLE INFO

Article history:
Received 1 December 2017
Received in revised form 17 April 2018
Accepted 29 May 2018
Available online 1 June 2018
Communicated by V.A. Markel

Keywords:
Bose–Einstein condensate
Artificial gauge potential
Mean-field theory
Ultra-cold atoms

ABSTRACT

We demonstrate an enhancement in the vortex generation when artificial gauge potential is introduced to condensates confined in a double well potential. This is due to the lower energy required to create a vortex in the low condensate density region within the barrier. Furthermore, we study the transport of vortices between the two wells, and show that the traverse time for vortices is longer for the lower height of the well. We also show that the critical value of synthetic magnetic field to inject vortices into the bulk of the condensate is lower in the double-well potential compared to the harmonic confining potential.

1. Introduction

Charged particles experience Lorentz force in the presence of magnetic fields, and in condensed matter systems, it is the essence for a host of fascinating phenomena like the integer quantum Hall effect [1,2], fractional quantum Hall effect [3,4], and the quantum spin Hall effect [5]. In contrast, the dilute quantum gases of atoms, which have emerged as excellent proxies of condensed matter systems, do not experience Lorentz force as these are charge neutral. This can, however, be remedied with the creation of artificial gauge fields through laser fields [6–10]. Thus, with the artificial gauge potentials it is possible to explore phenomena such as the quantum Hall effect, and the quantum spin Hall effect [7] in dilute atomic quantum gases. The introduction of synthetic magnetic field arising from artificial gauge field is also an efficient approach to generate quantized vortices in Bose–Einstein condensates (BEC) of dilute atomic gases. This method has the advantage of having time-independent trapping potentials over the other methods like rotation [11–13], topological phase imprinting [14,15], or phase engineering in two-species condensates [16,17]. In addition, it has the possibility to inject large ensembles of vortices, and an efficient scheme to nucleate vortices with synthetic magnetic fields was demonstrated in a recent work [18]. The method relies on the creation of an inhomogeneous synthetic magnetic field, which has its maxima coincident with the low density region of a spatially separated pair of BECs.

In this work we examine a scheme to nucleate vortices in BECs through synthetic magnetic field by employing the density gradient associated with a double well trapping potential. The advantages of the scheme are: vortices are generated in the bulk of the condensate; shorter relaxation time after nucleation; and higher density of vortices. In contrast, the other methods like rotating traps and phase imprinting nucleates vortices at the periphery. These then migrate to the bulk and as the process is diabatic, the relaxation times are long. Hence, the present scheme is better suited to explore phenomena associated with high vortex densities like quantum turbulence [19,20]. BECs in double well potentials were first theoretically studied to examine the physics of Josephson currents [21–23], latter observed in experiments [24–27], and studied numerically in a recent work [28]. For our study, we theoretically consider the case of a double well potential which is engineered from a harmonic potential by introducing a Gaussian barrier. For alkali metal atoms, the barrier is a blue-detuned light sheet obtained from a laser beam, and such setups have been used in experiments to observe the matter wave interference [29], Josephson effects [26], and collision of matter-wave solitons [30]. The artificial gauge potential is introduced through Raman coupling [8], and as a case study we consider the case of ^{87}Rb BEC.

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https://doi.org/10.1016/j.physleta.2018.05.051

We use Gross–Pitaevskii (GP) equation for a mean-field description of the BEC with the artificial gauge potential. In this work we quench the artificial gauge potential by increasing the Raman detuning, and simultaneously increase the height of the barrier potential. It is found that the extended low density region associated with the barrier promotes the formation of vortices. However, the quench imparts energy to the BEC and transfer it to an excited state. For comparison, we also examine the vortex generation in the case of uniform BEC [31]. Such a system, devoid of trap induced inhomogeneities, is better for quantitative comparison of experimental results with theory. This was demonstrated in a recent study on wave turbulence in uniform BECs [32]. To induce relaxation of the condensate to the ground state, we use the standard approach of introducing a dissipative term [33–35]. The presence of the dissipative term in the GP equation is consistent with the experimental observations of dissipation or damping [36, 37], which arises from the interaction between the condensate and non-condensate atoms.

The paper is organized as follows. In Section 2 we provide a description of the theory on how to generate artificial gauge potential in BECs using Raman coupling. Then, we incorporate the gauge potential in the Gross–Pitaevskii equation to arrive at a mean field description of BEC. In Section 3, we present the results of numerical computations, and discuss the implications. We, then, conclude with the key observations.

2. BEC in artificial gauge potentials

To study the vortex formation in double well with synthetic magnetic field in BECs theoretically, we consider the scheme based on light induced gauge potential proposed in Ref. [8]. In particular, we consider a quasi-2D BEC along the xy-plane of two level atoms, which in the present work is taken as the $F = 1$ ground state of ^{87}Rb atoms. To generate spatial inhomogeneity an external magnetic field $B(y) = B_0 + \Delta B(y)$ is applied along the y direction. Here B_0 is the static magnetic field which introduces a linear Zeeman splitting of the ground state manifold. The energy levels are separated by $\Delta_z = g\mu_B B_0$, and $\delta(y) = g\mu_B \Delta B(y)$ is the measure of detuning from Raman resonance. The constants g and μ_B are the atomic Landé factor and Bohr magneton, respectively. The two levels in the ground state are Raman coupled through two counter-propagating laser beams passing through the BEC along $\pm x$ directions [38]. The momentum transferred to the atoms through interactions with the Raman lasers induces a change in the kinetic energy part of the Hamiltonian through the vector potential term A_x. The modified Hamiltonian, however, remains gauge invariant, and there is a corresponding synthetic magnetic field $B_z = -\partial A_x/\partial x$.

2.1. Modified Gross–Pitaevskii equation

In the absence of Raman coupling, the Hamiltonian of the quasi-2D BEC confined in a harmonic trapping potential $\hat{V}_{\rm trap}$ is

$$\hat{H} = \hat{H}_x + \hat{H}_y + \hat{V}_{\rm trap} + \hat{H}_{\rm int}, \tag{1}$$

where $\hat{H}_x$, $\hat{H}_y$ represent the kinetic energy part of the Hamiltonian term along x, y directions respectively, and $\hat{H}_{\rm int}$ denotes the interaction energy between the atoms. Let $|1\rangle = |1, 0\rangle$ and $|2\rangle = |1, -1\rangle$ denote the two states in the ground state manifold of the atoms. The Raman lasers are along the x direction, and hence, the addition of the atom-light coupling term modifies H_x to

$$\hat{H}_x = E_{\rm r}\begin{pmatrix} \left(\tilde{k}_x + 1\right)^2 - \frac{\hbar\delta}{2E_{\rm r}} & \frac{\hbar\Omega}{2E_{\rm r}} \\ \frac{\hbar\Omega}{2E_{\rm r}} & \left(\tilde{k}_x - 1\right)^2 + \frac{\hbar\delta}{2E_{\rm r}} \end{pmatrix}, \tag{2}$$

where $E_{\rm r} = \left(\hbar^2 k_{\rm r}^2/2m\right)$ is the recoil energy, and $\tilde{k}_x = (k_x/k_{\rm r})$ with k_x as the x-component of the wave-vector, Ω is the Raman coupling between two levels, and δ is the Raman detuning.

To derive the modified Gross–Pitaevskii (GP) equation, we diagonalize $\hat{H}_x$ and obtain the dispersion relation for the two levels in the limit of strong Rabi coupling, $\hbar\Omega \gg 4E_r$. This ensures that there is single energy minima of the system and leads to the following: there is a change in the momentum along the x direction which provides a gauge potential $eA_x/\hbar k_{\rm r} = \tilde{\delta}/(\tilde{\Omega} \pm 4)$ in the system; and from the light-atom coupling the atoms acquires an effective mass m^* defined by $m^*/m = \tilde{\Omega}/(\tilde{\Omega} \pm 4)$. Here $\pm$ denotes the two energy levels in the system and $\tilde{\delta} = \hbar\delta/E_r$, $\tilde{\Omega} = \hbar\Omega/E_r$. Based on this Hamiltonian and restricting the dynamics to only the lowest dressed state, the behaviour of such a condensate in the presence of artificial gauge fields is governed by the following dimensionless modified Gross–Pitaevskii (GP) equation

$$\begin{aligned} i\frac{\partial\phi(x,y,t)}{\partial t} = \Bigg[& -\frac{1}{2}\frac{m}{m^*}\frac{\partial^2}{\partial x^2} - \frac{1}{2}\frac{\partial^2}{\partial y^2} + \frac{i2\pi\delta'}{\lambda_{\rm L}\Omega E_{\rm r}} y \frac{\partial}{\partial x} \\ & + \frac{1}{2}x^2 + \frac{1}{2}y^2\left(1 + \frac{2C_{\rm rab}\delta'^2}{E_{\rm r}}\right) + g_{\rm 2D}|\phi(x,y,t)|^2 \\ & - \left(\frac{\Omega - 2}{2}\right)E_{\rm r}\Bigg]\phi(x,y,t). \end{aligned} \tag{3}$$

In the above equation, all the parameters having the dimensions of length, energy, and time have been scaled with respect to the oscillator length $a_{\rm osc} = \sqrt{\hbar/m\omega_x}$, energy $\hbar\omega_x$ and time $\omega_x t$ respectively. For simplicity of notations, from here on we will represent the transformed quantities $(\Omega, \delta', E_{\rm r}, \lambda_{\rm L})$ without tilde. The condensate wavefunction is represented by $\phi(x, y, t)$, $C_{\rm rab} = \left(1/\Omega(\Omega - 4) + (4 - \Omega)/4(\Omega + 4)^2\right)$, $\delta = \delta' y$, δ' is defined to be the detuning gradient, Ω is the Rabi frequency, $E_{\rm r} = \left(2\pi^2/\lambda_{\rm L}^2\right)$ is the recoil energy of electrons, $g_{\rm 2D} = 2a_s N\sqrt{2\pi\lambda}/a_{\rm osc}$ is the interaction energy with N as the total number of atoms in the condensate, and $\lambda \gg 1$ is the trap anisotropy parameter along the z direction.

2.2. Double well (DW) potential

For the present work, we consider quasi-2D BEC confined in a double well potential

$$V_{\rm dw} = V_{\rm trap} + U_0 \exp(-2y^2/\sigma^2), \tag{4}$$

where U_0 and σ are the depth and width of the double well potential respectively and $V_{\rm trap} = (1/2)m\omega_\perp^2(x^2 + y^2)$ is the harmonic potential along x and y directions, and we have considered the symmetric case $\omega_\perp = \omega_x = \omega_y$.

The presence of the double well potential modifies the density distribution, breaks the rotational symmetry of the condensate, and brings about novel effects in the dynamical evolution of the condensate which forms the main topic of the present study.

2.3. Thomas Fermi correction in the condensates density

The focus of the present work, as mentioned earlier, is to examine the formation of vortices in the quasi-2D BEC with the introduction of artificial gauge potential. It has been shown in previous works on rotated condensates that vortices are seeded at the periphery of the condensate cloud, where the low density of the condensate is energetically favourable for the formation of vortices [39]. This is due to the presence of nodeless surface excitations [40], which create instabilities in the condensate and lead to the nucleation of vortices [41]. At later times the vortices migrate and