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The Carnot efficiency enabled by complete degeneracies

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ABSTRACT

Quantum-size effects unavoidably produce imperfect-regeneration heat losses in irreversible isothermal expansion/compression cycles, leading to the less efficiency of micro engines. Here, we design a smallest quantum Stirling-like heat engine using a single trapped electron as the working substance. The quantum probabilities to determine the electronic position are constructed from the incoherent mixed ensemble. When the quantum well expands isothermally to double its size and an infinite delta-function potential barrier is inserted in the middle, the complete degeneracies enable the heat engine to work reversibly and achieve the Carnot efficiency. The proposed theoretical model can open up new avenues for building practical nano-energy devices.

1. Introduction

The particle in a bound state exhibits quantized energy levels and quantum degeneracies, providing an automated platform for interpreting the quantum size effects [1–3]. The emergent scale phenomena may make a nanomachine become inefficient to operate the energy harvesting. In particular, the discrete eigenstates will give rise to non-perfect regeneration when a quantum thermodynamic cycle involves two isochoric processes with the regenerative heat [4–6].

Non-equilibrium effects and the exchange of entropy with information reservoirs are non-conventional resources that can be used to develop quantum-mechanical engines running near the Carnot limit. Bringing quantum coherence into Photo-Carnot engines [7,8] and photocells [9–11] has the potential to deliver useful work with more reliable performance and faster speed. Campisi and Ma suggested a different route based on the quantum phase transition [12,13]. Roßnagel et al. proposed an experimental feasible scheme to design efficient Otto engine cycles using the quantum fuel with the degree of squeezing [14,15]. The concept of fluctuating efficiency was proposed to characterize the performance of small-scale systems [16,17]. For the Szilard engine, Maxwell's demon can significantly improve the power-extraction capability in the context of information processing [18–20].

The question arises as to whether the heat machine can be manipulated in an effective way without preparing the working substance or the heat bath in any specific status. Here, we present a reversible heat engine, relying on the systemic inherent band structure. Our main results show that quantum degeneracies due to the boundary layer effects will be of great benefit to generating entropy-free work. Using a suitable trapping potential well, one is capable of fabricating quantum heat engines that in principle attain the Carnot efficiency.

In the following, we construct a quantum Stirling-like heat engine with a single trapped electron. The cycle consists of two quantum isothermal and two quantum isochoric processes. We will give a precise description for the statistical properties of the four stroke engine. The heat transfers between the working substance and the heat baths, the amount of work done during a cycle, and the efficiency of the heat engine will be derived analytically. We are especially interested in revealing the necessary conditions for the perfect regeneration and the reversible operation by separating the isothermal process into two individual steps.

2. Quantum isothermal process

The partition function is a measure of the thermodynamic behaviors of a system in thermodynamic equilibrium. In this study, the canonical partition function for any distinct mixed state will be applied to perform a statistical analysis of a semiconductor quantum-well system. The quantum isothermal process starts from a thermal equilibrium state denoted by A in Fig. 1, which can be

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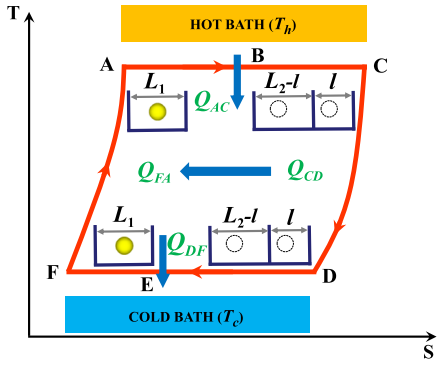

Fig. 1. The diagram of a quantum Stirling-like cycle based on a single trapped electron in a layered semiconductor structure.

treated as a single electron free to move in a narrow space surrounded by impenetrable barriers. Its position wave function is expected to take the following form

$$\langle x\,|\psi_n(L_1)\rangle = \sqrt{\frac{2}{L_1}}\sin\left(n\pi x/L_1\right), \tag{1}$$

where L_1 is the width of the space and n is a positive integer ranging from 1 to ∞. The eigenenergy corresponding to each of the permitted wavenumbers is given by $E_n(L_1) = (\hbar n\pi)^2/\left(2mL_1^2\right)$, where m is the effective electron mass and $\hbar$ is the Planck constant h divided by 2π. Writing the systemic Hamiltonian in terms of its eigenvectors, we have

$$H_A = \sum_n E_n(L_1)\,|\psi_n(L_1)\rangle\,\langle\psi_n(L_1)|\,. \tag{2}$$

These quantum well structures are particularly important in optoelectronics [21,22], and have been widely used in devices such as quantum well lasers, quantum-confined Stark effect modulators, and quantum infrared photodetectors. The initial state of the system in the energy representation is taken as

$$\rho_A = \frac{1}{Z(L_1)}\sum_n \exp\left[-\beta_h E_n(L_1)\right]|\psi_n(L_1)\rangle\,\langle\psi_n(L_1)|\,. \tag{3}$$

The partition function $Z(L_1) = \sum_n \exp\left[-\beta_h E_n(L_1)\right]$ and $\sum_n |\psi_n(L_1)\rangle\,\langle\psi_n(L_1)|$ is identically the unit operator. As usual, β_h is the inverse of the product $k_B T_h$, where $k_B = 8.617\times10^{-5}$ eV·K^{-1} is the Boltzmann constant and T_h is the equilibrium temperature of the system at state A.

We consider a two-step process taking place in such a way that during each step the system remains in a state of thermal equilibrium with the heat bath at temperature T_h. At the beginning, the quantum well expands quasi-statically until the width becomes L_2 (from state A to state B). In the second process, an infinite δ function potential barrier is isothermally inserted into the potential well at position $x = l$ (from state B to state C). The potential well is then partitioned into two domains, designated simply as the left ($|L_n(l)\rangle$) and right ($|R_n(L_2\text{-}l)\rangle$) wells. The eigenfunctions is then given as

$$\langle x\,|L_n(l)\rangle = \begin{cases} \sqrt{\frac{2}{l}}\sin\left[n\pi x/l\right] & 0\le x\le l \\ 0 & l\le x\le L_2 \end{cases} \tag{4a}$$

and

$$\langle x\,|R_n(L_2\text{-}l)\rangle = \begin{cases} 0 & 0\le x\le l \\ \sqrt{\frac{2}{L_2-l}}\sin\left[\frac{n\pi(x-l)]}{(L_2-l)}\right] & l\le x\le L_2 \end{cases} \tag{4b}$$

with the corresponding eigenvalues $E_n(l)$ and $E_n(L_2-l)$. The complete Hamiltonian can be reconstructed from the nth eigenfunctions of the left ($|L_n(l)\rangle$) and right ($|R_n(L_2\text{-}l)\rangle$) wells, that is,

$$H_C = \sum_n E_n(l)\,|L_n(l)\rangle\,\langle L_n(l)| + E_n(L_2-l)\,|R_n(L_2\text{-}l)\rangle\,\langle R_n(L_2\text{-}l)|\,. \tag{5}$$

The system is still in thermodynamic equilibrium when the insertion process is complete. In the mixed ensemble, the fraction $P_h^L(l) = Z_h(l)/\mathbb{Z}_h(l)$ gives the probability of the members characterizing the state ket $|L_n(l)\rangle$. The remaining fraction $P_h^R(l) = Z_h(L_2-l)/\mathbb{Z}_h(l)$ stands for the relative population of the right domain $|R_n(L_2\text{-}l)\rangle$. The partition function $\mathbb{Z}_h(l) = Z_h(l) + Z_h(L_2-l)$ represents a collection of the various energy eigenstates. Unlike the classical ideal gas, $P_h^L(l) \ne l/L_2$ and $P_h^R(l) \ne L_2 - l/L_2$. These differences can be used to extract energy and provide the key to the perfect regeneration for reversible quantum heat engines. The density operator is regarded as an incoherent mixture of the general basis $\{|L_n(l)\rangle, |R_n(L_2\text{-}l)\rangle\}$ as

$$\rho_C = P_h^L(l)\,\rho_h^L(l) + P_h^R(l)\,\rho_h^R(L_2-l)\,, \tag{6}$$

where

$$\rho_h^L(l) = \sum_n \frac{\exp\left[-\beta_h E_n(l)\right]}{Z_h(l)}\,|L_n(l)\rangle\,\langle L_n(l)| \tag{7a}$$

and

$$\rho_h^R(L_2-l) = \sum_n \frac{\exp\left[-\beta_h E_n(L_2-l)\right]}{Z_h(L_2-l)}\,|R_n(L_2\text{-}l)\rangle\,\langle R_n(L_2\text{-}l)|\,. \tag{7b}$$

The density matrix becomes either $\rho_h^L(l)$ or $\rho_h^R(L_2-l)$ depending on the probability of finding the electron in each particular well.

We are now capable of identifying the internal energy and the entropy of the system at state C in relation to the partition function $\mathbb{Z}_h(l)$. The internal energy arises by combining Eqs. (5) and (6)

$$U_C = \mathrm{Tr}\left[\rho_C H_C\right] = -\frac{\partial}{\partial\beta_h}\ln\mathbb{Z}_h(l)\,, \tag{8}$$

which is the sum over all the accessible microstate energies weighted by their respective probabilities. In the energy representation, the density matrix at thermal equilibrium is diagonal. The thermodynamic entropy is legitimately equivalent to the von Neumann entropy and is, therefore, given by

$$S_C = k_B\mathrm{Tr}\left[-\rho_C\ln\rho_C\right] = k_B\left(1-\beta_h\frac{\partial}{\partial\beta_h}\right)\ln\mathbb{Z}_h(l)\,. \tag{9}$$

In view of Eqs. (8) and (9), the canonical partition function $\mathbb{Z}_h(l)$ gives a precise description for the statistical properties of the mixed ensemble.

In the quantum isothermal process, the system can perform work to the outside agent, and meanwhile absorb heat from the bath. Both the eigenenergies and the corresponding occupation probabilities need to change with time, so that the system remains in an equilibrium state with the heat bath at every instant. In Refs. [23–26], it has been pointed out that the quantum isothermal process can be modeled by a stair path with an infinite number of quantum adiabatic and isochoric processes. When every step of the manipulation is infinitesimal, the stair path becomes equivalent to the isothermal evolution. Koski et al. reported the experimental realization of manipulating a single electron in the isothermal process, which utilized a single-electron box consisting of two small