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Thermal noise correlations and subtraction

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ABSTRACT

Using a generalized formulation of the fluctuation–dissipation theorem I evaluate the correlation between thermal noises. I investigate the possibility of extracting additional information about the thermal noise coupled to the measurement of a physical quantity, such as the output of an interferometric detector of gravitational waves, using a set of auxiliary beams.

1. Introduction

Thermal noise is one of the fundamental limits for the sensitivity of gravitational wave detectors such as LIGO or Virgo in the low frequency part of the observational frequency window. In order to reduce it we can "use the statistic", using larger laser beam spots to average better the mirrors fluctuations. We can "use the thermodynamics" by putting the mirror at cryogenic temperatures. Finally we can try to reduce the coupling of the mirrors to the environment by improving the materials in such a way to reduce damping effects.

The scheme of noise reduction I want to discuss is a variant of the "statistical" approach, in principle simple and general. The first step consists in monitoring the noise of an apparatus using auxiliary measurements. The results of this monitoring can be used to implement a subtraction scheme based on linear regression. The subtraction can be done "off line", processing the recorded data.

In Section 2 I describe the basic subtraction scheme and I determine the parameters which characterize the efficiency of the procedure. For a single auxiliary measurement the key quantity is the coherence between the signal we want to subtract and the monitoring channels. As expected a good subtraction efficiency requires a good coherence between signal and monitoring channel, in a sense that is made precise. When several auxiliary channels are present the relevant parameter is an appropriate generalization of the coherence which takes into account the redundancy of auxiliary channels. The role of the extra noise introduced in the monitoring channels is discussed.

The efficiency of this method can be estimated by using the fluctuation–dissipation theorem. It is in particular possible to give a formulation which generalize the approach proposed in Levin [10,11], as discussed in Section 3.

In Section 4 I give concrete examples, and I evaluate the subtraction efficiency for several kind of thermal noises for an infinite size mirror, which allows for analytical calculations.

Finally I draw some conclusions and I comment about the possible developments of the proposed approach in Section 5.

2. The subtraction scheme

Let us suppose that our objective is to reduce the level of thermal noise which contaminates the measurement of the displacement of a mirror along its optical axis. The schematization of a possible setup is represented in Fig. 1. The main beam on the right side is the one used normally for the measurement. In the general case the mirror is monitored by an additional set of auxiliary beams, each of them coupled in a different way to it. The information we get from the i-th auxiliary beam is a phase shift $\phi_i(t)$, that we want to use to "subtract" the thermal noise contribution from the phase shift $\Phi(t)$ of the main beam.

The auxiliary phase shifts will be measured with respect to some references that I suppose to give a negligible contribution to the noise. I am not concerned here about how this could be possible, as I am interested only in understanding the basic features of the method. I suppose also that the auxiliary measurements are unaffected by the signal that the cavity is supposed to detect (for example, a gravitational wave).

I assume that all the measured phase shifts are small, so a linear analysis will be sufficient. If this is the case, in a stationary condition the statistical properties of the available signals are completely described by the power spectrum $S_\Phi(\omega)$ of the main

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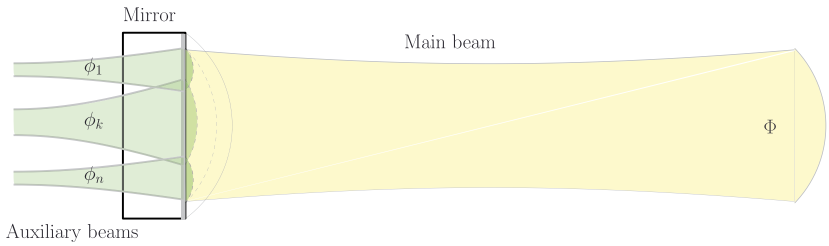

Fig. 1. Schematic setup for a basic subtraction scheme. The main beam overlaps with a set of auxiliary beams, on the left side. The reflective coating is indicated in gray.

mirror's phase, which is defined in term of the cross correlation function as

$$\left\langle \tilde{\Phi}(\omega')^* \tilde{\Phi}(\omega) \right\rangle = 2\pi\,\delta\left(\omega - \omega'\right) S_\Phi(\omega) \tag{1}$$

by the correlations $B_i(\omega)$ between the main mirror's phase and the phases of the auxiliary beams

$$\left\langle \tilde{\phi}_i(\omega)^* \tilde{\Phi}(\omega') \right\rangle = 2\pi\,\delta\left(\omega - \omega'\right) B_i(\omega) \tag{2}$$

and by the Hermitian matrix of the cross correlations $C_{ij}(\omega)$ between the ϕ_i's

$$\left\langle \tilde{\phi}_i(\omega')^* \tilde{\phi}_j(\omega) \right\rangle = 2\pi\,\delta\left(\omega - \omega'\right) C_{ij}(\omega) \tag{3}$$

The optimal reduction of the cavity motion can be obtained by imposing that the noise power spectrum of a "subtracted" signal

$$\tilde{\Phi}_S(\omega) = \tilde{\Phi}(\omega) - \sum_i \chi_i(\omega)\tilde{\phi}_i(\omega) \tag{4}$$

is minimized. This is equivalent to impose that Φ_S is uncorrelated with each of the ϕ_i, and solving for this condition we obtain the functions $\chi_i(\omega)$

$$\chi_i(\omega) = \left[C^{-1}(\omega)\right]_{ij} B_j(\omega) \tag{5}$$

which are linear filters in the time domain that must be applied to the auxiliary signals. The reduced noise power spectrum is given by

$$\left\langle \tilde{\Phi}_S(\omega')\tilde{\Phi}_S(\omega)^* \right\rangle = 2\pi\,\delta\left(\omega - \omega'\right) S_{\Phi_S}(\omega) \tag{6}$$

which can be written as

$$S_{\Phi_S}(\omega) = \left[1 - \eta(\omega)\right] S_\Phi(\omega) \tag{7}$$

where

$$\eta(\omega) = \frac{B_i(\omega)^* \left[C^{-1}(\omega)\right]_{ij} B_j(\omega)}{S_\Phi(\omega)} \tag{8}$$

In the case of a single auxiliary channel $\eta(\omega)$ is just the squared modulus of the coherence between the main signal Φ and the auxiliary one ϕ. We obtain the intuitive result that to get a good subtraction performance, as measured by the noise power spectrum, we need a high level of coherence. The general case described by Equations (7) and (8) can be understood by noticing that a higher correlation between auxiliary signals should correspond to smaller values of C^{-1}, which is bad, while a higher correlation between auxiliary and main signal correspond to higher values of B, which is good.

As will be seen in the next section, the correlation between the phase shift of two different beams is proportional, roughly speaking, to the overlap between the beam profiles. Looking at Fig. 1, it can be seen that we should find a compromise solution with large correlations between Φ and ϕ_i's (which means that the superposition of auxiliary and the main beams must be good), but with a small one among the ϕ_i's (which means that the auxiliary beams should not overlap too much). This is intuitive because we could subtract noise proportionally to the information we get about it, and this one is increased by a larger overlap between main and auxiliary beams, but it is reduced by a redundancy between these. C^{-1} is positive semi-definite, so $\eta \geq 0$ and subtracted noise is always less (or equal) than the original one.

3. Correlation between thermal noises

In order to understand how much the subtraction technique described can be used to reduce the effective level of thermal noise in a specific situation, we need to evaluate the functions S, B_i and C_{ij} for a particular model. The basic quantity involved is given by the cross correlation between the thermally induced phase shift on two different beams, which we want now to evaluate.

A phase shift can be induced in several ways by the thermal motion of the mirror. For example the beam can be reflected by the mirror's surface. In this case we write the small deformation field of the mirror as $\vec{u}(\vec{x}, z, t)$, where z is a coordinate along the optical axis while $\vec{x}$ is in the plane orthogonal to it. The relation between the incident and the reflected field near the reflecting surface which we suppose for simplicity to be flat and at $z = 0$ is given simply by

$$E_r(\vec{x}, 0, t) = E_i(\vec{x}, 0, t)e^{2iku_z(\vec{x},0,t)}$$

and if we expand the incident and the reflected field on two suitable orthogonal basis I_{mn} and R_{pq}, each labeled by two integers, we can evaluate the transition amplitudes

$$\left\langle R_{pq} \left| e^{2iku_z} \right| I_{mn} \right\rangle = \int R_{pq}(\vec{x})^* I_{mn}(\vec{x}) e^{2iku_z(\vec{x},0,t)} d^2x \tag{9}$$

This gives us a complete characterization of the effects of mirror's motion on a beam. The two basis can be chosen in such a way that when the mirror is quiet ($\vec{u} = 0$)

$$\langle R_{pq} || I_{mn} \rangle = \delta_{pm}\delta_{qn} \tag{10}$$

which mean that the output mode is just the reflected input one. We can now write at the linear order in the variation of the mirror's position [7]

$$\delta\phi^{(I)}_{mn} = 2k \left\langle I_{mn} \left| u_z \right| I_{mn} \right\rangle = 2k \int \mathcal{I}_{mn}(\vec{x}) u_z(\vec{x}, 0, t) d^2x \tag{11}$$

where $\mathcal{I}_{mn}(\vec{x}) = |I_{mn}(\vec{x})|^2$ is the intensity profile of the mode and interpret $\delta\phi^{(I)}_{mn}$ as the phase shift induced on the mode (m, n) by the mirror's motion. More general couplings can be generated for beams which traverse the mirror's bulk, such as the auxiliary ones in Fig. 1. In this case the optical path length can be changed both by geometrical effects, such as a fluctuation of the mirror's width, and by a fluctuation of the refraction index originated by a temperature fluctuation. To proceed further the attention will be fixed on some specific mechanisms for the generation of the random fluctuations.