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Surface plasmons in a semi-bounded massless Dirac plasma

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ABSTRACT

The collective excitation of surface plasmons in a massless Dirac plasma (e.g., graphene) half-space (bounded by air) is investigated using a relativistic quantum fluid model. The unique features of such surface waves are discussed and compared with those in a Fermi plasma. It is found that in contrast to Fermi plasmas, the long-wavelength surface plasmon frequency (ω) in massless Dirac plasmas is explicitly nonclassical, i.e., $\omega \propto 1/\sqrt{\hbar}$, where $h = 2\pi\hbar$ is the Planck's constant. Besides some apparent similarities between the surface plasmon frequencies in massless Dirac plasmas and Fermi plasmas, several notable differences are also found and discussed. Our findings elucidate the properties of surface plasmons that may propagate in degenerate plasmas where the relativistic and quantum effects play a vital role.

1. Introduction

The collective oscillations of interacting electrons (i.e., plasmons), have attracted a considerable attention due to their potential applications, e.g., in exploring the effects of electron–electron interactions in different physical systems including optical metamaterials, in receiving light signals at the nanoscale, in ultrafast lasers, in solar cells, in photodetectors, in biochemical sensing, as well as, in transmitting antennas [1–12]. A number of theoretical works [1,2,13] on collective modes of ordinary (Schrödinger) electrons in Fermi plasmas and their experimental verifications [13–15] are already in the literature. The classical plasma frequency in three-dimensional (3D) plasmas is known to be $\omega_p = \sqrt{4\pi n_0 e^2/m}$, where n_0 is the unperturbed number density and m the mass of electrons. Though this frequency appears in Fermi plasma fluids, it may not be the same in massless Dirac plasmas, such as those in, e.g., graphene.

Because of its peculiar features and amazing electronic and optical properties, graphene has attracted a huge interest in recent years. The dense honeycomb arrangements of carbon atoms with photon-like massless energy relation have made it possible for the charge carriers in graphene to mimic both relativistic and quantum effects at the same time [16]. Such massless electrons can move with an effective Fermi speed of about $v_F \sim 10^6$ m/s, which is independent of the carrier number density. It has been shown that the dynamics of two-dimensional (2D) gas of charged particles in graphene can be described by the relativistic Dirac fluid model [17,18]. In this context, the linear-band dispersion of Dirac electrons in graphene is known to be the origin of some new features in wave dynamics that are distinctive from the ordinary 2D degenerate electron gas [17].

Dirac materials (particularly in graphene) [6,19–23] have been considered for the excitation of plasmons due to their tunable spectrum through the electrostatic control of their carrier concentration, and also their high lifetime plasmons (because of high mobility). A number of authors have proposed the theory of plasmons in Dirac systems in various forms, such as topological insulators [24,25], graphene [17,26–29], Weyl semi-metals [30], graphene microribon arrays [19], and massless Dirac plasma layers [17,31,32].

The propagation of electrostatic surface waves in semi-bounded plasmas have been studied by Ritchie [33] and the effects of finite temperature on these surface waves have also been discussed by using a hydrodynamic model. The theory of Ritchie was later extended to a quantum plasma half-space using a quantum hydrodynamic (QHD) model by Lazar et al. [34]. Furthermore, the dispersion properties of surface Langmuir oscillations have been studied by Chang et al. [35] in a semi-bounded quantum plasma using the specular reflection method. Such QHD model has been known to be one of the powerful models for the investigation of wave dynamics in quantum plasmas [37–52]. It has been shown that the propagation characteristics of surface waves can be modified by the effects of quantum tunneling [37–50], the external magnetic field [40,51,52], the particle–particle collisions [40], the relativistic factor [31], the particle spins [44,48], nonlocality [42,43,45], as

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well as, the effects of exchange-correlation of plasma particles [41, 45–48]. On the other hand, some attention has also been paid to investigate nonlinear effects in surface plasma waves. For example, Stenflo [36] showed that surface plasma solitary waves can appear in the vicinity of the interface between a plasma and the bounding medium. However, to the best of our knowledge, the theory of surface plasmons in massless Dirac plasmas has not yet been explored, and so is the subject of the present study.

In this letter, we show that the surface plasmons in massless Dirac plasmas and Fermi plasmas have several striking differences including the fact that the long-wavelength surface plasmon frequency in massless Dirac plasmas is explicitly nonclassical, whereas that in Fermi plasmas corresponds to the classical plasma frequency. The outline of this paper is as follows: An introduction is given in Sec. 1. The fundamental set of dynamical equations for massless Dirac plasmas and Fermi plasmas are presented in Sec. 2. Then, the dispersion relation of surface plasma waves is obtained in Sec. 3. Finally, Sec. 4 is left to conclude our results.

2. Hydrodynamic model for a massless Dirac plasma

We consider the propagation of surface plasma oscillations in semi-bounded massless Dirac plasmas and Fermi plasmas. To this end, we employ the quantum hydrodynamic model applicable for both Dirac and Fermi plasmas with ions forming only the neutralizing background. In the fluid equations, the appropriate pressure laws for the Dirac and Fermi fluids (to be denoted, respectively, with the subscripts 'D' and 'F') may be discussed. First of all, the assumption of a well-defined Fermi wavenumber k_F can be valid with the definition of the d-dimensional electronic density [18] $n_d = gk_F/2^d\pi^{d/2}\Gamma(1+d/2)$, where g, d, and Γ are, respectively, the degeneracy factor ($g = g_s g_v$ with $g_s = 2$ being the spin degeneracy and g_v the pseudo-spin degeneracy factor which for graphene is ~ 2), the system dimensionality and the Gamma function. We, however, consider a degenerate plasma at zero temperature in which the energy density can be obtained as $\varepsilon = \int_0^{k_F} E(k)d^dk$, where the energy dispersion relation $E(k)$ is expressed differently in each plasma system, given by, $\varepsilon_D = \hbar k v_F$ and $\varepsilon_F = \hbar^2k^2/2m$. In the case of a massive Dirac fluid we have $\varepsilon \sim \hbar\sqrt{k^2 + (\Delta/\hbar v_F)^2}v_F$, where 2Δ is the energy gap. However, this is not the case in our present theory. Next, the thermodynamical identity $P = n\partial\varepsilon/\partial n - n$ can be employed to obtain the following expressions of pressure for the Dirac and Fermi fluids in three-dimensional plasmas [53]

$$P_D = \frac{(3\pi^2)^{4/3}}{12\pi^2}v_F\hbar n^{4/3}, \quad P_F = \frac{(3\pi^2)^{2/3}}{5m_e}\hbar^2 n^{5/3}. \tag{1}$$

We emphasize that the density dependencies of P_D and P_F are different. We also note that the QHD model can be employed for both the cases of non-relativistic quantum Fermi fluids and relativistic massless Dirac fluids. Furthermore, the QHD model for Dirac fluids is independent of the electron mass [54,55] for which the basic equations read

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0, \tag{2}$$

$$(P+\varepsilon)\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = enc^2\left[\nabla\phi + \boldsymbol{\beta}(\boldsymbol{\beta}\cdot\nabla)\phi\right] - \frac{c^2}{\gamma^2}\left(\nabla P + \frac{\boldsymbol{\beta}}{c}\frac{\partial P}{\partial t}\right), \tag{3}$$

$$\nabla^2\phi = 4\pi e(n - n_0), \tag{4}$$

where n and $\mathbf{u}$, respectively, denote the number density and velocity of electrons, ϕ is the electrostatic potential, n_0 is the equilibrium number density of electrons and ions, and P is the fluid pressure. Also, $\boldsymbol{\beta} = \mathbf{u}/c$ with c denoting the speed of light in vacuum and $\gamma = 1/\sqrt{1-\beta^2}$ is the relativistic factor.

In the weak relativistic limit $P \ll \varepsilon = mnc^2$, the pressure in Eq. (3) can be due to the Fermi degeneracy pressure P_F [41]. Furthermore, in unmagnetized plasmas and with $u \le v_F \ll c$ for which $\gamma \sim 1$, the following equations can be obtained for Fermi plasmas.

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0, \tag{5}$$

$$\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = \frac{e}{m}\nabla\phi - \frac{1}{mn}\nabla P_F, \tag{6}$$

$$\nabla^2\phi = 4\pi e(n - n_0), \tag{7}$$

where P_F is the Fermi pressure given by Eq. (1), and we have neglected the quantum dispersion effect associated with the Bohm potential for simplicity and also for smallness compared to the degeneracy pressure gradient (e.g., in solid density plasmas). On the other hand, in Dirac plasmas, since the Fermi speed $v_F \sim c/300$, the weakly relativistic condition ($\beta \ll 1$) can be employed, however, due to the different energy dispersion E for the Dirac fermions and the ordinary fermions (viz., $E \sim \hbar k v_F$ and $E \sim \hbar^2k^2/2m$ respectively), the weak relativistic assumption does not apply to the massless Dirac fluids, and in this case, the corresponding equations read [53,55]

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\mathbf{u}) = 0, \tag{8}$$

$$(P+\varepsilon)\left(\frac{\partial}{\partial t} + \mathbf{u}\cdot\nabla\right)\mathbf{u} = enc^2\nabla\phi - c^2\nabla P_D, \tag{9}$$

$$\nabla^2\phi = 4\pi e(n - n_0), \tag{10}$$

where the physical variables n, ϕ, $\mathbf{u}$ etc. all are functions of $\mathbf{R}$ and t with $\mathbf{R} = (\mathbf{r}, x)$ and $\mathbf{r} = (y, z)$. The pressure P_D in Eq. (9) represents the quantum fluid pressure for massless Dirac Plasmas given by Eq. (1). In what follows, we study the basic features of surface plasma oscillations at the interface of a massless Dirac plasma (e.g., graphene) and air. The theory of surface plasmon excitation in Fermi plasmas is well-known and has been studied extensively [34,41,42,44,46–48], however, we review it for Fermi plasmas and compare with that in massless Dirac plasmas.

3. Dispersion relation of surface plasmons

In order to obtain the dispersion relation for surface plasmons in a massless Dirac plasma half-space (occupying the region $x < 0$) bounded by air ($x > 0$), we linearize the relevant physical quantities about their unperturbed (with suffix 0) and perturbed (with suffix 1) values by letting $n = n_0 + n_1$, $\mathbf{u} = \mathbf{u}_1$, and $\phi = \phi_1$, where $n_1 \ll n_0$. Then applying the space–time Fourier transform formula of an arbitrary function $f(\mathbf{R}, t)$, given by

$$f(\mathbf{R}, t) = \frac{1}{(2\pi)^3}\int\int d^3k d\omega F(\mathbf{k}, \omega; x)e^{i\mathbf{k}\cdot\mathbf{r} - i\omega t}, \tag{11}$$

where $\mathbf{k} = (k_y, k_z)$, to the linearized basic equations of Eqs. (8) to (10), we obtain

$$\frac{d^2N_1(x)}{dx^2} - \gamma_j^2 N_1(x) = 0, \tag{12}$$

$$\frac{d^2\Phi_1(x)}{dx^2} - k^2\Phi_1(x) = 4\pi e N_1(x), \tag{13}$$