Physics Letters A 382 (2018) 2275–2281

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Influence of polarization and material on Brownian thermal noise of binary grating reflectors

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ARTICLE INFO

Article history:
Available online 12 July 2017
Communicated by I. Pinto

Keywords:
Thermal noise
Grating reflectors
High precision metrology
Gravitational wave detection

ABSTRACT

Grating reflectors are a potential low-noise replacement for amorphous multilayer mirrors. We investigate the influence of polarization and refractive index on Brownian thermal noise of binary grating reflectors using Maxwell's stress tensor. Our results demonstrate that the refractive index of the grating material is a critical parameter for thermal noise in these structures. In contrast to multilayer mirrors, a low coating thickness does not necessarily lead to a low thermal noise amplitude for structures with low refractive index. We find that an improved noise performance of grating reflectors requires materials of refractive index $\gtrsim 2.5$. We present a factorized expression for the thermal noise of grating reflectors made of arbitrary materials by simply scaling the noise amplitude with the related material parameters.

1. Introduction

Brownian thermal noise poses severe limitations to the sensitivity of many applications in the field of high-precision optical metrology such as gravitational wave detectors [1,2] and optical cavities for frequency stabilization [3–6]. The work by Vladimir Braginsky pioneered this field and established fundamental calculations on thermal noise [7,8]. It is known that the high mechanical loss of amorphous materials as used for optical materials, e.g. in highly reflective multilayer coatings (Bragg mirrors), is a major source for Brownian noise of optical components [9–14]. There are several approaches to overcome the limitation set by the mechanical loss, such as material optimizations [15,16], new coating designs [17,18], coating free reflectors [19–21] and crystalline coatings based on AlGaAs/GaAs or GaP [22–24]. Grating reflectors may be another low-noise alternative to conventional multilayer mirrors [25–28]. Instead of using multiple beam interference these surface structures employ optical modes to provide high reflectivities (compare Fig. 1). This effect requires only a coating thickness of the structured material of a few hundred nanometers [25] and can be implemented even in a full monolithic way in crystalline silicon [26,29]. Thus, a superior noise performance is expected [30]. However, to make these structures competitive to multilayer based components in terms of feasible reflectivity, an optimization of fabrication processes is necessary. Since this requires extensive work, it is opportune to deliberately choose materials which are worth to undertake this challenge. Binary grating reflectors are more flexible with respect to the material choice than monolithic structures which require sophisticated grating profiles [26].

In this contribution we investigate the influence of material parameters on Brownian thermal noise of binary grating reflectors with fused silica as substrate material. Generally, the optical properties of structured surfaces depend on the polarization of light. Thus, we investigate also the influence of the different polarizations on thermal noise. We derive a semi-analytic expression for thermal noise in binary grating reflectors. It enables us to comprehensively study the influence of the following parameters: The refractive index of the grating material, the wavelength, the radius of the Gaussian beam, the mechanical angular frequency, the elastic constants (Young's modulus and Poisson's ratio), the temperature and the mechanical loss of the grating material. Due to the large uncertainties of the mechanical coating material parameters the calculated noise amplitudes give an estimate of the feasible noise level. They are meant to be a guide for the selection of materials.

The article is organized as follows: In Sec. 2 we present the formalism for the computation of Brownian thermal noise in structured surfaces and its application to binary grating reflectors. In Sec. 3 we explain how the geometric grating parameters are determined and outline how the semi-analytic expression for thermal noise is derived. The detailed derivation can be found in the appendices. Finally, in Sec. 4 we discuss the dependence of material and polarization on thermal noise.

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http://dx.doi.org/10.1016/j.physleta.2017.07.006
0375-9601/

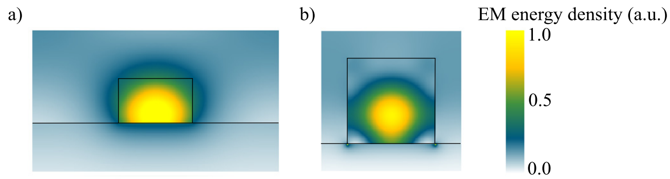

Fig. 1. Illustration of the optical modes localized in the grating ridges of binary grating reflectors. Shown is the electromagnetic energy density for high reflectivity configurations at a wavelength of 1550 nm for a) TE polarization and b) TM polarization. The substrate material is SiO_2 and the grating ridges consist of crystalline silicon. The shown fields are RCWA calculations. The structure parameters are listed in Table D.1.

2. Thermal noise in structured surfaces

In this section we briefly explain the scheme for the computation of Brownian thermal noise in nano-optical surfaces following [31] and then apply it to binary grating reflectors. The approach is based on Levin's method [9] which computes Brownian thermal noise on a planar reflecting surface. Levin's method uses a formulation of the fluctuation-dissipation theorem by Callen and Welton [32] with a virtual oscillating force. Applied to the surface of the reflector the force deforms the surface, which introduces an elastic energy density $\varepsilon(\vec{r})$ and leads to a dissipated power of:

$$P_{\text{diss}} = \omega \int \varepsilon(\vec{r})\Phi(\vec{r})\, \mathrm{d}V, \tag{1}$$

where ω is the mechanical angular frequency and $\Phi(\vec{r})$ the mechanical loss. The fluctuation-dissipation theorem [32] relates the dissipated power to the Brownian noise power spectral density [9]:

$$S_z(\omega, T) = \frac{8k_B T}{\omega^2}\frac{P_{\text{diss}}}{F_0^2}, \tag{2}$$

with Boltzmann constant k_B and the temperature T. The total applied force F_0 scales with the intensity of the laser beam light. In the case of a planar surface it may by expressed by the intensity profile of the laser beam, e.g. a Gaussian distribution. In [31,33] it is shown that the force distribution on arbitrary (corrugated) surfaces is determined by the ponderomotive light pressure p acting on the different interfaces of the surface:

$$pn_j = \sum_i \Delta\sigma_{ij} n_i, \tag{3}$$

with the components of Maxwell's stress tensor σ_{ij} and the Cartesian normal basis vectors n_i $(i \in \{x, y, z\})$. $\Delta\sigma_{ij}$ represents the difference of the stress components out- and inside the dielectric material. For an interface parallel to one of the three Cartesian coordinate planes the light pressure in direction i simplifies to:

$$p_i = \Delta\sigma_{ii}. \tag{4}$$

In SI units Maxwell's stress tensor reads:

$$\sigma_{ij} = \epsilon_0\epsilon_r E_i E_j + \frac{1}{\mu_0\mu_r} B_i B_j - \frac{1}{2}\left(\epsilon_0\epsilon_r E^2 + \frac{1}{\mu_0\mu_r} B^2\right)\delta_{ij}, \tag{5}$$

where E_i are the electric field components and B_i the magnetic flux density components, respectively. ϵ_r is the relative permittivity and μ_r is the magnetic permeability. The field distributions may be calculated with numerical tools like finite element analysis (FEA) like COMSOL [34]. The virtual force in direction i is then expressed as (A_i is the surface area perpendicular to i):

$$F_i = \int \Delta\sigma_{ii}\, \mathrm{d}A_i. \tag{6}$$

Applying these forces to a binary surface structure leads to an elastic deformation energy density of [35]:

$$\varepsilon = \frac{1}{2}\sum_{ik} \Delta\bar{\sigma}_{ik} u_{ik}, \tag{7}$$

where u_{ik} is the deformation tensor. Utilizing the arithmetic mean of the stress $\bar{\sigma}$ over the related surface yields an analytical expression for the elastic energy which is derived in Appendix B. The elastic energy is thus determined by the applied stresses σ_{ik} given by the electromagnetic field distribution and by the resulting deformations u_{ik} whose magnitudes depend on the elastic properties (Young's modulus and Poisson's ratio) of the materials. In a nano-optical surface structure, the electromagnetic field distribution depends on the polarization, wavelength and angle of the incident light. In addition, the refractive indices, $n \approx \sqrt{\epsilon_r}$, of the involved materials are critical parameters for the field distribution. The identification of relevant materials and application wavelengths thus requires the comprehensive consideration of all these parameters. To account for that, we derived the following factorized expression for Brownian thermal noise power spectral density in binary grating reflectors at normal incidence (for details see Appendix A):

$$S_z = \frac{8k_B T}{\omega}\frac{\lambda}{r_0^2}\frac{\Phi}{Y}\alpha_n\alpha_\nu. \tag{8}$$

Here, λ represents the wavelength of light, ω the mechanical angular frequency, T the temperature of the mirror, Φ the mechanical loss of the grating material, r_0 the beam radius and Y Young's modulus of the grating material. The coefficient α_n contains information about the pure optical performance of the grating with respect to the electromagnetic fields in- and outside the grating ridges. It depends on the refractive indices of the involved materials and on the polarization of light. α_ν represents the influence of transverse contractions on Brownian thermal noise whose magnitudes are defined by the applied pressure, i.e. the polarization, and the Poisson's ratio. The knowledge of the coefficients α_ν and α_n enables us to compute Brownian thermal noise for any grating material on a given low-refractive index substrate material.

3. Semi-analytical noise analysis

In the following section we briefly outline the procedure to compute thermal noise in binary grating reflectors.

3.1. Structural optimization

Grating reflectors are sub-wavelength grating structures made of high refractive index material on top of a low index substrate. As discussed in [25] it is possible to achieve high reflectivity with purely binary structures (compare Fig. 2 a)) as well as with structures on top of a solid high refractive index layer (Fig. 2 b)). To minimize the total thickness of the coating layer we aimed for purely binary structures (compare Fig. 2 a)). However, for TE-polarized light and refractive indices n smaller than 2.5 high reflectivity requires the additional solid high index layer.

As a free parameter we vary the grating refractive index n between 1.7 and 4.5 in steps of 0.1 for both transverse-magnetic (TM) and transverse-electric (TE) polarizations of the incident light. The structure optimization is carried out for a wavelength of $\lambda = 1550$ nm. The optimization results may be transferred to other wavelengths λ' by scaling the structure dimensions by a factor of λ'/λ (see Appendix C for details). The structure optimization was performed by means of one-dimensional RCWA for each refractive index and polarization with 25 basis functions and a spatial resolution of $\Lambda/250$ to meet the following requirements:

1. A reflectivity of $R > 99.99\%$ which should be feasible with state-of-the-art nanotechnology [36].
2. Fabrication tolerances as large as possible.