Composites Science and Technology 94 (2014) 8–15

Contents lists available at ScienceDirect

Composites Science and Technology

journal homepage: www.elsevier.com/locate/compscitech

# Characterization of residual stresses in a thin-walled filament wound carbon/epoxy ring using incremental hole drilling method

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ARTICLE INFO

*Article history:*
Received 21 August 2013
Received in revised form 7 December 2013
Accepted 11 January 2014
Available online 21 January 2014

*Keywords:*
C. Residual stress
E. Filament winding
A. Polymer–matrix composites (PMCs)
A. Laminate
Incremental hole-drilling method

ABSTRACT

The incremental hole drilling method was used to assess the curing-induced residual stresses in a filament-wound carbon/epoxy ring. In the isotropic materials, the relationship between the relaxed strains in the successive depth increments and the unknown residual stresses has a trigonometric form. However, the severe anisotropic character of a filament-wound composite ring, which results from the different mechanical properties of a single layer in the longitudinal and transverse directions as well as the dissimilar orientation of the adjacent layers, necessitated the development of a more general formulation by introducing new compliance coefficients. The calculation algorithm, which solves the inverse problem of residual-stress estimation, is based on the Integral method, which assumes that the stress in each depth increment is uniform. High tensile hoop stresses (~220 MPa) were found at the inner surface of the ring. Also, a compressive hoop stress (~−140 MPa) was observed at the outer diameter. The residual hoop stresses were also measured by the slitting method. The results of the two techniques were very similar in trend.

## 1. Introduction

Three major sources of introduction of residual stresses in filament wound composite structures include the difference between the thermal and mechanical properties of fiber and matrix, the pretension applied to the fibers during the filament winding process, and the contraction of the mandrel during cooling. Interaction of these independent parameters can generate a complex stress profile containing steep gradients and discontinuities.

These stresses can adversely affect the efficiency of composite structures and result in dimensional instability, transverse cracking, delamination, and premature failure [1–3]. Thus, quantitative approximation of residual stresses is necessary for the safe performance of composite structures.

The relaxation techniques commonly used for the residual stress measurement are hole drilling, layer removal, crack compliance (or slitting), and ring core [4]. Due to its simplicity and reliability, the hole drilling method is the most widely used method. This method was initially used to measure the uniform residual stresses in homogeneous isotropic materials [5], which in this case it is named through-hole drilling method. However, in most practical cases, the residual stresses are non-uniform in depth direction. Therefore, the hole-drilling method was extended to the measurement of non-uniform residual stresses [6–8]. The incremental hole-drilling method, which involves the drilling in a series of small steps, is able to determine non-uniform stress profiles and gradients.

Although the hole drilling method has been widely used to determine uniform and non-uniform residual stresses in metallic components, very little research has so far been conducted on its application to the heterogeneous orthotropic materials. Bert et al. [9,10] extended the through-hole drilling method to determine the uniform stresses in orthotropic materials by considering three independent compliance coefficients. However, through-hole drilling method is not appropriate for laminated composites, because for such laminates, the material elastic properties change abruptly at layers boundaries and, consequently, residual stresses are always non-uniform in depth direction. For this reason, the incremental hole-drilling method was developed for laminated composites by Sicot et al. [11,12]. They applied this method to a cross-ply carbon/epoxy laminate. Extending the method proposed by Bert et al. [9,10], they related the principal stresses in each depth increment to the measured relaxed strains by three compliance coefficients.

Schajer and Yang [13] showed that the development of the through-hole drilling method for orthotropic materials proposed by Bert et al. [9,10] is not valid. In order to better correlate the residual stresses to the measured strains, they used nine

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http://dx.doi.org/10.1016/j.compscitech.2014.01.008

compliance coefficients for the application of through-hole drilling to the orthotropic materials. These coefficients can be determined by closed form, numerical, and experimental methods.

The main objective of this study is to extend the method proposed by Schajer and Yang [13] to the incremental hole-drilling measurement of non-uniform residual stresses in laminated composites. First, the general relationships between the relaxed strains and in-plane residual stresses in orthotropic materials were developed for the successive hole depth increments. Then, as a case study, the residual stress profile in a thin-walled carbon/epoxy filament wound ring was determined using this method. To provide confidence in the accuracy of the results, the hoop residual stresses were also measured using slitting method.

## 2. Theoretical basis of incremental hole drilling method in orthotropic materials

For the clockwise strain gauge rosette shown in Fig. 1, the positive *x*-direction lies along the axis of gauge 1, and the negative *y*-direction lies along the axis of gauge 3. Drilling a circular hole at the geometrical center of the rosette causes residual stresses to relax around the hole. The resulting released strains are recorded by the three gauges.

The relationship between the residual stresses released along the hole face and the relaxed strains measured on the specimen surface does not have a one-to-one form. It is because the relaxed strains are measured at the specimen surface, while the residual stresses of interest are in the interior [4,14]. The difference between the location of the measured strains and the location of the residual stresses causes the relationship between the residual stresses as a function of depth and the measured strain data to have the following integral form:

$$\varepsilon_g(a_i) = \int_0^{a_i} \left( \sum_{s=1}^{3} C_{gs}(z, a_i)\sigma_s(z) \right) dz \quad s, g = 1, 2, 3, \quad 1 \le i \le n \qquad (1)$$

where $\varepsilon_g(a_i)$ is the measured strain by the gauge number $g$ when the hole depth is $a_i$. $\sigma_1$, $\sigma_2$ and $\sigma_3$ correspond to $\sigma_x$, $\sigma_{xy}$ and $\sigma_y$, respectively. $C_{gs}(z,a_i)$ is the "kernel function" that relates the measure strains to the residual stresses. $n$ is the total number of depth increments.

According to Eq. (1), the measured strains depend on all released stresses within the specimen thickness, and not just those at a specific depth. Obviously, residual stresses near the surface have more effects on the relaxed strains.

$$\begin{bmatrix} \begin{vmatrix} C_{1111} & C_{1112} & C_{1113} \\ C_{1121} & C_{1122} & C_{1123} \\ C_{1131} & C_{1132} & C_{1133} \end{vmatrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{vmatrix} C_{2111} & C_{2112} & C_{2113} \\ C_{2121} & C_{2122} & C_{2123} \\ C_{2131} & C_{2132} & C_{2133} \end{vmatrix} & \begin{vmatrix} C_{2211} & C_{2212} & C_{2213} \\ C_{2221} & C_{2222} & C_{2223} \\ C_{2231} & C_{2232} & C_{2233} \end{vmatrix} & \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \\ \begin{vmatrix} C_{3111} & C_{3112} & C_{3113} \\ C_{3121} & C_{3122} & C_{3123} \\ C_{3131} & C_{3132} & C_{3133} \end{vmatrix} & \begin{vmatrix} C_{3211} & C_{3212} & C_{3213} \\ C_{3221} & C_{3222} & C_{3223} \\ C_{3231} & C_{3232} & C_{3233} \end{vmatrix} & \begin{vmatrix} C_{3311} & C_{3312} & C_{3313} \\ C_{3321} & C_{3322} & C_{3323} \\ C_{3331} & C_{3332} & C_{3333} \end{vmatrix} \end{bmatrix} \left\{ \begin{matrix} \begin{vmatrix} \sigma_x^{layer,1} \\ \tau_{xy}^{layer,1} \\ \sigma_y^{layer,1} \end{vmatrix} \\ \begin{vmatrix} \sigma_x^{layer,2} \\ \tau_{xy}^{layer,2} \\ \sigma_y^{layer,2} \end{vmatrix} \\ \begin{vmatrix} \sigma_x^{layer,3} \\ \tau_{xy}^{layer,3} \\ \sigma_y^{layer,3} \end{vmatrix} \end{matrix} \right\} = \left\{ \begin{matrix} \begin{vmatrix} \varepsilon_1^{layer,1} \\ \varepsilon_2^{layer,1} \\ \varepsilon_3^{layer,1} \end{vmatrix} \\ \begin{vmatrix} \varepsilon_1^{layer,2} \\ \varepsilon_2^{layer,2} \\ \varepsilon_3^{layer,2} \end{vmatrix} \\ \begin{vmatrix} \varepsilon_1^{layer,3} \\ \varepsilon_2^{layer,3} \\ \varepsilon_3^{layer,3} \end{vmatrix} \end{matrix} \right\}$$

In order to solve Eq. (1), an initial distribution for the unknown function, $\sigma_s(z)$, should be initially considered. The most common way of estimating residual stresses in incremental hole drilling is integral method [7]. In this method, it is assumed that residual stress is constant in each depth increment. It is conceptually straightforward and is able to resolve steep stress gradients within the range of the hole depth. In this method, unknown residual stresses are therefore expressed as:

$$\sigma_s(z) = \sum_{j=1}^{i} \sigma_{js} U_j(z) \quad 0 \le z \le a_i \qquad (2)$$

$\sigma_{js}$ is the value of $s$ component of residual stress at *j'th* depth increment. $U_j'$s or pulse functions are defined as follows:

$$U_j(z) = \begin{cases} 1 & a_{j-1} \le z \le a_j \\ 0 & z < a_{j-1}, \quad z > a_j \end{cases} \qquad (3)$$

Substituting Eq. (2) in Eq. (1) results in:

$$\begin{aligned} \varepsilon_g(a_i) &= \int_0^{a_i} \left( \sum_{s=1}^{3} C_{gs}(z, a_i) \sum_{j=1}^{i} \sigma_{js} U_j(z) \right) dz \\ &= \sum_{s=1}^{3} \sum_{j=1}^{i} \sigma_{js} \int_0^{a_i} C_{gs}(z, a_i) U_j(z) dz = \sum_{s=1}^{3} \sum_{j=1}^{i} \sigma_{js} C_{ijgs} \end{aligned} \qquad (4)$$

Therefore, $C_{ijgs}$ or the elements of compliance matrix are expressed by the following equation:

$$C_{ijgs} = \int_0^{a_i} C_{gs}(z, a_i) U_j(z) dz = \int_{a_j}^{a_{j-1}} C_{gs}(z, a_i) dz \qquad (5)$$

Comparing to Eq. (1) indicates that a specific element of the compliance matrix, $C_{ijgs}$, is the measured strain by the strain gage number $g$ for a hole of depth $a_i$ when the $s$ component of the residual stress at the domain $a_{j-1} \leqslant z \leqslant a_j$ is equal to the unit load.

$$C_{ijgs} = \varepsilon_g(a = a_i, \sigma_s(z) = U_j(z)) \qquad (6)$$

Considering the superposition principle, the compliance coefficients could be determined by applying unit loads to the hole face. In this research, these coefficients were calculated using finite element simulations.

The compact form of Eq. (4) and its generalization for three depth increments is:

$$[C]\{\sigma\} = \{\varepsilon\} \qquad (7)$$

If the total number of depth increments is $n$, the number of non-zero elements of compliance matrix will be $9n(n+1)/2$. Thus, in the through-hole drilling ($n = 1$), nine compliance coefficients are needed. After each compliance coefficient was calculated, the inversion of Eq. (7) allows the user to determine the unknown residual stresses from the measured strains.