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Measurement-induced multipartite entanglement for distant four-level atoms in Markovian and non-Markovian environments ☆

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A B S T R A C T

In this paper, we propose a scheme for generating multipartite entanglement of distant four-level atoms separately trapped in individual cavities, each of which is coupled to a non-Markovian reservoir. The entanglement of atoms is generated by measuring the photons leaking from cavities. In non-Markovian environments, we derive dynamical evolution of the entanglement and obtain the condition of generating the long-living multipartite maximally entangled state. When the condition is not satisfied, by introducing a time-varying coupling strength, the maximal multipartite entangled state can also be generated.

1. Introduction

Entanglement, as a striking feature of quantum physics, is the main quantum resource in various domains and many schemes have been proposed to generate entangled states [1]. In particular, multipartite entanglement plays an important role in quantum information processing, and many theoretical and experimental studies have been carried out [2–6].

In the realistic regime, however, quantum systems unavoidably interact with surrounding environments, i.e., surrounding environments have a significant influence on the dynamical behavior of quantum systems. Therefore, it is very important to investigate the dynamical behavior of entanglement in open systems, and the dissipative dynamics of entanglement has attracted a lot of attention [7–11]. Up to now, many efforts have been devoted to the dynamical behavior of entanglement in Markovian regime [12–15], which often results in decoherence and disentanglement [16–18]. However, many systems show non-Markovian properties which lead to a lot of interesting phenomena [19–25]. In non-Markovian environments, an important phenomenon is the sudden death and revival of entanglement, which has a big impact on generating long-living entanglement. Therefore, the non-Markovian property of environments play an essential role in entanglement generation.

Recently, the Gardiner–Collett approach is widely applied in the researches of non-Markovian dynamics in cavity QED systems [26–29]. In [26], the authors study the exact entanglement dynamics of two two-level atoms in a dissipative cavity by using Gardiner–Collett approach, and introduce Zeno effect to preserve the entanglement stored in the system. The non-Markovian dynamics of the bipartite and multipartite entanglement for distant atoms is also investigated via Gardiner–Collett approach [27,29]. Here, we propose a scheme for generating the maximal multipartite entanglement of distant four-level atoms via photon detection in non-Markovian environments. In our scheme, since only the atomic ground states are used to generate the entanglement, the generated entangled state is a long-living atomic entangled state. By using Gardiner–Collett approach, we investigate the dynamics of entanglement and obtain the condition of generating the maximal atomic entanglement. In addition, we also discuss the effects of the parameter fluctuations on the entanglement generation.

2. The model of atom-cavity system

We consider a dissipative cavity, which contains a four-level atom with two excited states ($|e_\mathrm{L}\rangle$ and $|e_\mathrm{R}\rangle$) and two ground states ($|g_\mathrm{L}\rangle$ and $|g_\mathrm{R}\rangle$) (see Fig. 1(a)). The transitions $|e_\mathrm{L}\rangle \leftrightarrow |g_\mathrm{L}\rangle$ and $|e_\mathrm{R}\rangle \leftrightarrow |g_\mathrm{R}\rangle$ are coupled to left and right circularly polarized cavity modes with coupling constants g_L and g_R, respectively. The left (right) circularly polarized cavity mode interacts with a reservoir, which consists of a set of continuous harmonic oscillators. The Hamiltonian of the atom-cavity system can be written as ($\hbar = 1$) [30,27,29]

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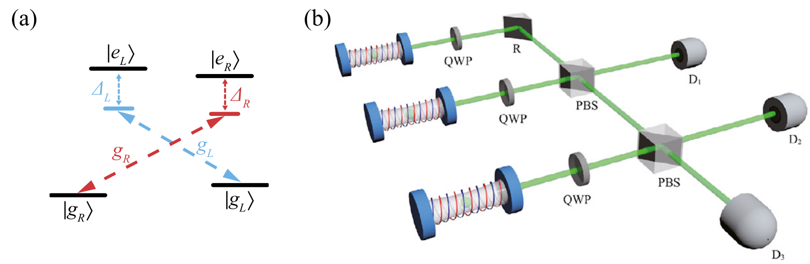

Fig. 1. (a) The level structure of the atom trapped in the dissipative cavity. It has two excited states ($|e_L\rangle$ and $|e_R\rangle$) and two ground states ($|g_L\rangle$ and $|g_R\rangle$). (b) Schematic representation of generating tripartite entanglement for atoms. QWP is a quarter wave plate and PBS is a polarization beam splitter. R is a totally reflecting mirror and D_i ($i = 1, 2, 3$) are detectors.

$$H = \sum_{j=\mathrm{L,R}} \left[\omega_{e_j}|e_j\rangle\langle e_j| + \omega_{g_j}|g_j\rangle\langle g_j| + \int \omega A_j^\dagger(\omega)A_j(\omega)\mathrm{d}\omega\right] + \sum_{j=\mathrm{L,R}} \int g_j\left[\alpha_j^*(\omega)A_j(\omega)|e_j\rangle\langle g_j| + \mathrm{H.c.}\right]\mathrm{d}\omega, \tag{1}$$

where ω_{e_j} and ω_{g_j} are the energies of the states $|e_j\rangle$ and $|g_j\rangle$ ($j = \mathrm{L, R}$), respectively. A_j is the annihilation operator of cavity mode j. $\alpha_j(\omega) = \frac{\sqrt{\kappa_j/\pi}}{\omega-\omega_c+\mathrm{i}\kappa_j}$ is used to describe the influence of environments on the cavity and g_j is the coupling strength between the atom and cavity mode j. ω_c and κ_j are the frequency and the decay rate of cavity mode j, respectively. $\tau_{g_j} = g_j^{-1}$ is related to the relaxation time of the system and $\tau_{\kappa_j} = {\kappa_j}^{-1}$ is the correlation time of the reservoir. When the correlation time of the reservoir is much longer than the relaxation time ($\tau_{\kappa_j} \gg \tau_{g_j}$), the system is coupled to a non-Markovian reservoir. Conversely, when the relaxation time is much longer than the correlation time of the reservoir ($\tau_{g_j} \gg \tau_{\kappa_j}$), the system is coupled to a Markovian reservoir. Transforming the Hamiltonian (1) into the interaction picture, we obtain

$$H_\mathrm{I} = \sum_{j=\mathrm{L,R}} \int g_j\left[\alpha_j^*(\omega)\mathrm{e}^{\mathrm{i}(\omega_j-\omega)t}A_j(\omega)|e_j\rangle\langle g_j| + \mathrm{H.c.}\right]\mathrm{d}\omega, \tag{2}$$

where $\omega_j = \omega_{e_j} - \omega_{g_j}$. Assuming the atom is initially in $|\psi(0)\rangle = [\cos(\theta/2)|e_\mathrm{L}\rangle + \sin(\theta/2)|e_\mathrm{R}\rangle]|0\rangle_\mathrm{L}|0\rangle_\mathrm{R}$, where $\theta \in [0, \pi]$ and $|0\rangle_j$ represents for the vacuum state of cavity mode j. $|1_\omega\rangle_j = A_j^\dagger(\omega)|0\rangle_j$ represents that there is one photon at frequency ω in cavity mode j. With at most only one excitation, the state at any time t can be written as

$$|\psi(t)\rangle = \sum_{j=\mathrm{L,R}} E_j(t)|e_j\rangle|0\rangle_\mathrm{L}|0\rangle_\mathrm{R} + \int U_\mathrm{L}(t,\omega)|g_\mathrm{L}\rangle|1_\omega\rangle_\mathrm{L}|0\rangle_\mathrm{R}\mathrm{d}\omega + \int U_\mathrm{R}(t,\omega)|g_\mathrm{R}\rangle|0\rangle_\mathrm{L}|1_\omega\rangle_\mathrm{R}]\mathrm{d}\omega, \tag{3}$$

where $E_\mathrm{L}(t)$, $E_\mathrm{R}(t)$, $U_\mathrm{L}(t,\omega)$, and $U_\mathrm{R}(t,\omega)$ are the time-varying probability amplitudes. Using Schrödinger equation, we obtain the equations of probabilities as follows ($j = \mathrm{L, R}$)

$$\dot{E}_j(t) = -\mathrm{i}g_j\int \alpha_j^*(\omega)\mathrm{e}^{\mathrm{i}(\omega_j-\omega)t}U_j(\omega,t)\mathrm{d}\omega, \tag{4a}$$

$$\dot{U}_j(t) = -\mathrm{i}g_j\alpha_j(\omega)\mathrm{e}^{-\mathrm{i}(\omega_j-\omega)t}E_j(t). \tag{4b}$$

The expressions of probability amplitudes can be obtained analytically by Laplace transform:

$$E_j(t) = E_j(0)\mathrm{e}^{-(\mathrm{i}\Delta_j+\kappa_j)t/2} \times \left[\cosh(\Omega_j t/2) + \frac{\mathrm{i}\Delta_j+\kappa_j}{2}\sinh(\Omega_j t/2)\right], \tag{5}$$

where $\Omega_j = \sqrt{\kappa_j^2 - \Delta_j^2 - 4g_j^2 + 2\mathrm{i}\Delta_j\kappa_j}$ and $\Delta_j = \omega_c - \omega_j$ is the detuning between the atom and cavity mode j.

3. The entanglement of the atom-cavity system

We investigate the entanglement of the atom-cavity system and linear entropy is used to quantify the amount of entanglement, which is defined as [31]

$$S_\mathrm{L}(t) = \frac{d}{d-1}\left[1 - \mathrm{Tr}\left(\rho_A^2\right)\right], \tag{6}$$

where ρ_A is the atomic reduced density matrix of the atom-cavity system and $d = 4$ is the dimension of ρ_A. From Eqs. (3) and (6), the linear entropy is calculated as follows:

$$S_\mathrm{L}(t) = \frac{4}{3}\left[1 - \left(|E_\mathrm{L}(t)|^2 + |E_\mathrm{R}(t)|^2\right)^2 - \sum_{j=\mathrm{L,R}}\left(|E_j(0)|^2 - |E_j(t)|^2\right)^2\right]. \tag{7}$$

From Eqs. (5) and (7), we know that the signs of detunings have no effect on the evolution of entanglement. Fig. 2 illustrates the time evolution of the linear entropy for different detunings in (a) non-Markovian and (b) Markovian environments. In both two environments, the linear entropy first increases to a maximum rapidly and then gradually decreases to a constant value. The constant value of linear entropy means that the atom and the cavity field are finally in an entangled steady state, which ensures the generation of long-living entanglement of distant atoms. In non-Markovian environments, the linear entropy exhibits an oscillatory behavior due to the memory effect, which can not be seen in Markovian environments. It is because that there is a reversed flow of information from the environment back to the quantum system in non-Markovian environments, and the information backflow leads to the oscillations of entanglement. In addition, the presence of detunings can suppress the interaction between the atom and the cavity field. Therefore, in both non-Markovian and Markovian environments, the atom-cavity system can reach the entangled steady state in a shorter time by decreasing the detunings.

4. The entanglement of distant atoms

Thanks to the entanglement between the atom and the cavity field, we can entangle distant atoms by measuring the photons leaking from cavities. Here, we generate the entanglement of three atoms as an example. We consider the projection operator

$$P_3 = |\varphi_3\rangle\langle\varphi_3|, \tag{8}$$

with