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Higher-order nonclassicalities of finite dimensional coherent states: A comparative study

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ABSTRACT

Conventional coherent states (CSs) are defined in various ways. For example, CS is defined as an infinite Poissonian expansion in Fock states, as displaced vacuum state, or as an eigenket of annihilation operator. In the infinite dimensional Hilbert space, these definitions are equivalent. However, these definitions are not equivalent for the finite dimensional systems. In this work, we present a comparative description of the lower- and higher-order nonclassical properties of the finite dimensional CSs which are also referred to as qudit CSs (QCSs). For the comparison, nonclassical properties of two types of QCSs are used: (i) nonlinear QCS produced by applying a truncated displacement operator on the vacuum and (ii) linear QCS produced by the Poissonian expansion in Fock states of the CS truncated at $(d-1)$-photon Fock state. The comparison is performed using a set of nonclassicality witnesses (e.g., higher order antibunching, higher order sub-Poissonian statistics, higher order squeezing, Agarwal–Tara parameter, Klyshko's criterion) and a set of quantitative measures of nonclassicality (e.g., negativity potential, concurrence potential and anticlassicality). The higher order nonclassicality witness have found to reveal the existence of higher order nonclassical properties of QCS for the first time.

1. Introduction

Coherent states drew considerable attention of the quantum optics and atom optics community for various reasons. For example, a CS is known to be a quasi-classical state or the most classical state among the quantum states [1], and it has applications in almost all fields of physics [2,3]. In quantum optics, CS has been traditionally defined in various ways, such as displacement of vacuum state, eigenket of annihilation operator, or infinite Poissonian superposition of Fock states [1,4]. In the infinite dimensional Hilbert space, these different definitions of CS are equivalent. However, in the finite dimensional Hilbert space, different definitions lead to different finite dimensional coherent states which are referred to as qudit coherent states [5]. In general, a qudit may be viewed as a d-dimensional quantum state that can be expanded in Fock-state ($|n\rangle$) basis as

$$|\psi\rangle_d = \sum_{n=0}^{d-1} c_n |n\rangle. \quad (1)$$

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With the recent developments in quantum state engineering [6–9] and quantum computing and communication [see Ref. [10] and references therein], production and manipulation of these types of quantum states have become very important. Further, in the recent past, several applications of nonclassicality [10–12] and a few experimental demonstrations of higher order nonclassicality [13–16] have been reported. Specifically, in the Laser Interferometer Gravitational-Wave Observatory (LIGO), squeezed vacuum state has been successfully used for the detection of the gravitational waves [17,18] by reducing the noise [19,20]. Squeezed state is also used in continuous variable quantum cryptography [11], teleportation of coherent state [21], etc. Further, a higher-order counterpart of squeezing- amplitude-squared squeezing, can be transformed to the standard lower-order squeezing using an interaction in which the square of the field amplitude is coupled to the amplitude of another field mode (e.g., second harmonic generation and certain kinds of four-wave mixing). Due to this, amplitude-squared squeezed states are useful in obtaining the noise reduction in the output of nonlinear optical devices [22]. Anti-bunching is used for characterizing single photon sources [23] which are essential for the realization of various schemes for secure quantum communication. Along the same line, the physical meaning of the higher-order antibunching (HOA) can be obtained by viewing it as a nonclassical phenomenon that ensures that the probability of getting a bunch of n-photon is always less than that of getting a bunch of m photon in the radiation field if $n > m$ [24]. In other word, the probability of getting the single photon is greater than the two or more photons which implies that a potential single photon source should satisfy the criterion of HOA [25]. Another higher-order nonclassical phenomenon is higher-order sub-Poissonian photon statistics (HOSPS). Earlier, it has been established that HOSPS is independent of HOA and can be observed in various physical systems. Further, HOSPS was found to be useful in detecting Hong–Mandel type higher-order squeezing (HOS) and amplitude-powered Hillery type HOS [26]. Here it may be noted that in the earlier studies presence of HOA or HOSPS has been observed in the absence of the corresponding lower-order phenomenon [25]. These observations motivated us to study HOA, HOSPS and HOS. In addition, entangled states have been established to be useful for various quantum information processing tasks ([10] and references therein). For example, entangled states are essential for quantum teleportation [27], densecoding [28], quantum cryptography [29], etc. In this paper, we have not studied multi-mode (multi-partite) entangled states, but it would be apt to note that every multi-partite entangled state are higher order nonclassical state and such states have many applications which further establishes the relevance of higher-order nonclassicality. In addition to the nonclassical states having the above mentioned applications, QCSs (being a finite superposition of Fock states, which are always nonclassical) have also drawn considerable attention of the quantum optics community in its own merit. In fact, various aspects related to the properties (mostly nonclassical properties), possibilities of generation and potential applications of finite dimensional optical states have been carefully investigated in last three decades [5,30–39] with specific attention to the QCSs [5,30–32,36]. Specifically, generation possibilities of finite dimensional states of light have been discussed in Refs. [5,31,38,39] and their properties have been studied in Refs. [5,32,35–37]. To obtain a QCS in particular or a finite dimensional Fock superposition state in general, we would require a mechanism to truncate the infinite dimensional conventional Fock-state expansion of a driving field. In this context, a set of closely connected and extremely interesting concepts have been developed. Such concepts include quantum scissors [5,9,40,41], which aims to truncate (cut the dimensions of) an infinite dimensional Hilbert space into a finite dimension, and photon blockades [42,43] which can be used as a tool for nonlinear optical-state truncation (i.e., as a nonlinear quantum scissors) [42]. However, to the best of our knowledge, no effort has yet been made to investigate the higher order nonclassical properties of QCSs. Motivated by the above facts, in addition to the conventional lower-order nonclassical properties, here we also aim to investigate higher order nonclassical properties of two QCSs. The idea of the first type of QCS which is usually referred to as the nonlinear QCS was developed by [30,32] using one of the definitions of the infinite dimensional coherent state. Specifically, this type of QCSs were prepared by applying a truncated displacement operator on the vacuum state as follows

$$|\alpha\rangle_d = \hat{D}_d(\alpha,\alpha^*)|0\rangle = \exp(\alpha\hat{a}_d^\dagger - \alpha^*\hat{a}_d)|0\rangle, \tag{2}$$

where the truncated displacement operator $\hat{D}_d(\alpha,\alpha^*)$ operates on vacuum to generate QCS, and the qudit annihilation operator is $\hat{a}_d = \sum_{n=1}^{d-1}\sqrt{n}|n-1\rangle\langle n|$ and the corresponding commutation relation is $[\hat{a}_d, \hat{a}_d^\dagger] = d|d-1\rangle\langle d-1|$ which fundamentally differs from the standard creation and annihilation operators. The Fock-state expansion of the QCS in the form of Eq. (1) is given by [30]

$$|\alpha\rangle_d = \sum_{n=0}^{d-1} c_n^{(d)}(\alpha)|n\rangle, \tag{3}$$

where the superposition coefficients are

$$c_n^{(d)}(\alpha) = f_n^{(d)} \sum_{k=0}^{d-1} \frac{\mathrm{He}_n(x_k)}{[\mathrm{He}_{d-1}(x_k)]^2}\exp(ix_k|\alpha|), \tag{4}$$

with $f_n^{(d)} = \frac{(d-1)!}{d}(n!)^{-1/2}\exp[in\left(\phi_0 - \frac{\pi}{2}\right)]$, and the modified Hermite polynomial $\mathrm{He}_n(x)$ is related to the Hermite polynomial $H_n(x)$ as $\mathrm{He}_n(x) = 2^{-n/2}H_n\left(x/\sqrt{2}\right)$; $x_k \equiv x_k^{(d)}$ is the k th root of $\mathrm{He}_d(x)$, and $\phi_0 = \arg(\alpha)$. In the rest of the letter, we have chosen $\phi_0 = 0$. The complex parameter α (with $\phi_0 = 0$) is perfectly periodic in nature for $d = 2$, 3 and almost periodic for $d > 3$. The periods of α for $d = 2$ and 3 are $T_2 = \pi$ and $T_3 = 2\pi/\sqrt{3}$, respectively, whereas the periods for $d > 3$ are $\sqrt{4d+2}$. Due the periodic nature of α the photon number for the QCS $|\alpha\rangle$ is also periodic in nature with maximum value $|\alpha|^2 = d-1$ which corresponds to the photon number of the highest energy Fock state. In Ref. [5] and references therein, possible ways of generating this QCS and a set of its nonclassical properties have been discussed. However, no attention has yet been provided to the higher order nonclassical properties of this state.

The second type of QCSs studied here can be generated by truncating the Fock space superposition of CS. This type of QCSs for a complex amplitude β are defined as [5,34,44]

$$|\beta\rangle_d = \mathcal{N}\exp(\beta\hat{a}_d^\dagger)|0\rangle = \mathcal{N}\sum_{n=0}^{d-1}\frac{\beta^n}{\sqrt{n!}}|n\rangle, \tag{5}$$

where $\mathcal{N} = 1/(\sum_{n=0}^{d-1}\frac{\beta^{2n}}{n!})^{1/2}$ is the normalization constant. This type of QCS is referred to as the linear QCS. The QCS $|\beta\rangle_d$ can be written in the form of Eq. (1) with

$$c_n^d(\beta) = \mathcal{N}\frac{\beta^n}{\sqrt{n!}}. \tag{6}$$

This QCS is referred to as linear QCS [5] and was studied earlier in Refs. [5,34,44]. In Ref. [5], it is explicitly shown that nonclassical properties (e.g., Wigner function and nonclassical volume) of linear QCS and nonlinear QCS are different. Here, we aim to extend the observation further by comparing the nonclassical properties of these QCSs using various witnesses and measures of nonclassicality with a specific focus on the witnesses of higher order nonclassicality. The remaining part of the letter is organized as follows. In Section 2, we compare nonclassical characters of linear and nonlinear QCSs using a set of witnesses of nonclassicality which generally reflects the presence of higher order nonclassicality (except Klyshko's criterion), but does not provide any quantitative measure of nonclassicality. Specifically, in this section, we perform comparison of nonclassicality in QCSs using the criteria of HOA, HOSPS, HOS and Agarwal–Tara criterion and Klyshko's criterion. In Section 3, we compare the amount of nonclassicality present in linear and nonlinear QCSs by using a set of quantitative measures of nonclassicality (e.g., concurrence potential, negativity potential, and anticlassicality). Finally, the letter is concluded in Section 4.

2. Comparison of nonclassicality in QCSs using the witnesses of nonclassicality

A quantum state is referred to as nonclassical if its Glauber Sudarshan P-function cannot be written like a classical probability distribution. In other words, negative values of P-function implies that the state does not have classical analogue, and can be referred to as a nonclassical state. As there does not exist any general

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