Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Binegativity of two qubits under noise

Sk Sazim [a,*], Natasha Awasthi [b]

[a] QIC group, Harish-Chandra Research Institute, HBNI, Allahabad, 211019, India

[b] Govind Ballabh Pant University of Agriculture and Technology, Pantnagar, Uttarakhand 263145, India

ARTICLE INFO

Article history:
Received 9 March 2018
Received in revised form 26 April 2018
Accepted 27 April 2018
Available online xxxx
Communicated by M.G.A. Paris

Keywords:
Quantum entanglement
Entanglement measures
Decoherence

ABSTRACT

Recently, it was argued that the binegativity might be a good quantifier of entanglement for two-qubit states. Like the concurrence and the negativity, the binegativity is also analytically computable quantifier for all two qubits. Based on numerical evidence, it was conjectured that it is a PPT (positive partial transposition) monotone and thus fulfills the criterion to be a good measure of entanglement. In this work, we investigate its behavior under noisy channels which indicate that the binegativity is decreasing monotonically with respect to increasing noise. We also find that the binegativity is closely connected to the negativity and has closed analytical form for arbitrary two qubits. Our study supports the conjecture that the binegativity is a monotone.

1. Introduction

Quantum entanglement is a fundamental non-classical feature of multiparticle quantum systems. It is a key resource for many quantum information processing tasks. Hence, characterizing (witnessing as well as quantification) of entanglement is of immense importance.

In the last two decades, substantial amount of progress has been made in characterizing entanglement of two-qubit systems [1]. Although the entanglement structure of pure bipartite systems is well understood, much attention is required to fully understand it for mixed two-qubit states [1]. Quantification of entangled state is related with the inconvertibility between entangled states under local operations and classical communications (LOCC), i.e., the quantities which do not increase under LOCC are the entanglement quantifiers [2–5]. Finding such measures are important for better understanding of the entangled states [2,6–8]. Out of many extant entanglement quantifiers, the concurrence [9,10] and the negativity [11] are easily computable for two-qubit mixed states. Although, negativity and concurrence coincide for pure two qubit states, they produce different ordering for mixed states [12].

One breakthrough discovery in entanglement theory is Peres–Horodecki criteria [13,14]. They found that using partial transposition operations one can detect entanglement in composite quantum systems. Let us consider a bipartite system ρ, then its partial transposition in one of the subsystems is defined as ρ^Γ. The state satisfying $\rho^\Gamma \geq 0$ are called positive under partial transposition (PPT states). It is well known that all the PPT states of two qubits are separable states. The negativity captures the degree of violation of PPTness in the two-qubit states and it is an entanglement monotone [8]. Note that there exist no known physical interpretation of partial transposition operations. The negativity can be expressed as

$$N(\rho) = 2\mathrm{Tr}[\rho^{\Gamma_-}] = \| \rho^\Gamma \|_1 - 1, \tag{1}$$

where $\| \cdot \|_1$ denotes trace-norm and we follow the notation $\rho^{\Gamma_-} = (\rho^\Gamma)_-$ to denote the negative component of ρ^Γ. (It is defined in Eq. (3).)

In Ref. [15], authors discussed a computable quantity called 'the binegativity' which may be considered as a potential entanglement measure. The concept of binegativity was first introduced in the context of relative entropy of entanglement [17]. It was shown that if $|\rho^{\Gamma_-}|^{\Gamma_-} \geq 0$, the asymptotic relative entropy of entanglement with respect to PPT states does not exceed the so-called Rains bound [16,17], where $|\rho| = \sqrt{\rho.\rho}$. This condition also guarantees that the PPT-entanglement cost for the exact preparation is given by the logarithmic negativity [18,19] which provides the operational meaning to logarithmic negativity [20]. The binegativity for two-qubit state is given by [15]

$$\begin{aligned} N_2(\rho) &= \mathrm{Tr}[\rho^{\Gamma_-}] + 2\mathrm{Tr}[\rho^{\Gamma_-\Gamma_-}] \\ &= \frac{1}{2}N(\rho) + 2\mathrm{Tr}[\rho^{\Gamma_-\Gamma_-}], \end{aligned} \tag{2}$$

* Corresponding author.
E-mail address: sk.sazimsq49@gmail.com (S. Sazim).

https://doi.org/10.1016/j.physleta.2018.04.056
0375-9601/

where $\rho^{\Gamma_-\Gamma_-} = ((\rho^{\Gamma_-})^{\Gamma})_-$. The binegativity has similar properties like negativity in two qubit systems while the former may not be a monotone under both LOCC and PPT channels [15,16,20,21]. On the basis of numerical evidence, it is conjectured that the binegativity behaves monotonically under both LOCC and PPT channels [15]. Based on this conjecture, the binegativity might be identified as a valid measure of entanglement for two qubit states. The binegativity has following properties [15]:

1. It is positive always and vanishes for two-qubit separable states.
2. It is invariant under local unitary operations.
3. For all two qubit states $N_2(\rho) \le N(\rho) \le C(\rho)$ and $N_2(\rho) = N(\rho)$ if $N(\rho) = C(\rho)$. In particular for all pure two qubit states, $|\psi\rangle$, $N_2(|\psi\rangle) = N(|\psi\rangle) = C(|\psi\rangle)$, where C denotes concurrence.

The comparison between the negativity and the concurrence have been studied extensively and these measures give different order for the two qubit states, as there exist different states with equal concurrence but different negativity and vice versa [7,12,22,23]. The binegativity also gives unique orderings of two-qubit states [15]. There exists some two qubit states with same negativity and same concurrence but have different values of binegativity. All these findings indicate that the binegativity may be a new member in the set of extant entanglement quantifier.

In this work, we study its behavior under noisy channels, specifically, under amplitude damping (AD), phase damping (PD) and depolarizing (DP) channels and find that it is decreasing monotonically with the increasing noise. We also observe that the behavior of the binegativity is quite similar under noisy channels. All these studies indicate that the *bona fide* measure, the binegativity, might be a entanglement monotone.

In the next section, we establish a functional relation between the binegativity and the negativity. We also discuss the behavior of the binegativity under twirling operation. Then we calculate the binegativity for some class of states in section 3. In section 4, we study the behavior of the binegativity under the noisy channels. We conclude in the last section.

2. Binegativity – a LOCC monotone?

Although we do not have a proof for monotonicity of the binegativity under LOCC/PPT, we will address the issue to some extent. Mainly we will show that the binegativity contains a nontrivial term which may increase under some local operations but on average the binegativity is not increasing. Here we focus our numerical study only for twirling operations.

Binegativity of two qubit state ρ can explicitly be expressed in terms of negativity.

Lemma. *The binegativity, $N_2(\rho) = \frac{1}{2}N(\rho)\big[1 + N(\rho_\psi)\big]$, where $\rho_\psi = |\psi\rangle\langle\psi|$ with $|\psi\rangle$ being the normalized eigen vector corresponding to the negative eigen value of ρ^{Γ}.*

Proof. It is well known that the partial transposition of any two qubit entangled state has exactly one negative eigenvalue, and the eigenstate (pure) corresponding to it must be an entangled state. Hence the negative component of ρ^{Γ} is of the form

$$\rho^{\Gamma_-} = \mathrm{Tr}\big[\rho^{\Gamma_-}\big]\rho_\psi, \tag{3}$$

where $\rho_\psi = |\psi\rangle\langle\psi|$ with $|\psi\rangle$ being the normalized eigen vector corresponding to the negative eigen value of ρ^{Γ}. Now the form of $\rho^{\Gamma_-\Gamma_-}$ is given by

$$\rho^{\Gamma_-\Gamma_-} = \mathrm{Tr}\big[\rho^{\Gamma_-}\big]\rho_\psi^{\Gamma_-}. \tag{4}$$

Hence, $\mathrm{Tr}\big[\rho^{\Gamma_-\Gamma_-}\big] = \frac{1}{4}N(\rho)N(\rho_\psi)$. Therefore the binegativity can be expressed as follows

$$N_2(\rho) = \frac{1}{2}N(\rho)\big[1 + N(\rho_\psi)\big]. \tag{5}$$

Hence the proof. □

With the above expression, we can conclude that the binegativity and the negativity are related quantities. The binegativity and negativity coincide for two qubit pure states as in this case ρ_ψ is a maximally entangled state. In fact, it is true for Werner states also.

We know that the negativity is a monotone under PPT operations [8,16,20]. Having close resemblance with negativity, one might also expect that the binegativity is a monotone. However in Ref. [15], based on numerical evidence, it was conjectured that the binegativity might be a PPT monotone. Analytically, it is hard to prove the monotonicity of the binegativity because of the presence of the term like $N(\rho_\psi)$. For example, any two qubit entangled state can be transformed to a less entangled Werner state by twirling operations [24] and for the Werner state, ρ_ψ is maximally entangled i.e., $N(\rho_\psi) = 1$. Therefore, although the overall entanglement is decreasing the contribution from the term, $N(\rho_\psi)$ may increase.

In [24], Werner showed that any state ρ can be transformed to a Werner state by applying the twirling operator:

$$\rho_{Wer} = \int dU(U \otimes U)\rho(U \otimes U)^{\dagger}, \tag{6}$$

where integral is performed with respect to Haar measure on the unitary group, $U(d)$. This operation can transform any entangled state to a less entangled Werner state. Therefore, under the twirling the binegativity should also decrease for two qubit case. *We have numerically checked that the binegativity is indeed monotonically decreasing under twirling.*

Now we will compute the binegativity for some class of states.

3. Binegativity of some class of states

Here we will compute the binegativity for some two qubit mixed states. For example, we will consider the following states:

Werner state: The Werner state is $U \otimes U$ invariant state. A two qubit Werner state is given by

$$\rho_{Wer} = \frac{1-p}{4}\mathbb{I}_4 + p|\psi^-\rangle\langle\psi^-|, \tag{7}$$

where $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$ is the singlet state and $p \in [0, 1]$ is the classical mixing. The state is entangled for $p > \frac{1}{3}$. For this state the concurrence, the negativity and the binegativity are same and are equal to $\frac{3p-1}{2}$ for $p > \frac{1}{3}$.

Bell diagonal states: The Bell diagonal states can be expressed in canonical form as

$$\rho_{Bell} = \frac{1}{4}(\mathbb{I}_4 + \sum_i c_i\sigma_i \otimes \sigma_i), \tag{8}$$

where $c_i \in [-1, 1]$. The state, ρ_{Bell} is a valid density matrix if its eigen values $\lambda_{mn} \ge 0$, where $\lambda_{mn} = \frac{1}{4}[1 + (-1)^m c_1 - (-1)^{m+n} c_2 + (-1)^n c_3]$ with $m, n = 0, 1$. For this state, the concurrence, the negativity and the binegativity are equal to $2\lambda_{\max} - 1$, where $\lambda_{\max}$ is the maximum eigenvalue of ρ_{Bell}.

MEMs: The two qubit maximally entangled mixed states (MEMs) are the most entangled states for a given mixedness [25]. These states with concurrence C are