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Extreme wave formation in unidirectional sea due to stochastic wave phase dynamics

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A B S T R A C T

The authors consider a stochastic model based on the interaction and phase coupling amongst wave components that are modified envelope soliton solutions to the nonlinear Schrödinger equation. A probabilistic study is carried out and the resulting findings are compared with ocean wave field observations and laboratory experimental results. The wave height probability distribution obtained from the model is found to match well with prior data in the large wave height region. From the eigenvalue spectrum obtained from the Inverse Scattering Transform, it is revealed that the deep-water wave groups move at a speed different from the linear group speed, which justifies the inclusion of phase correction to the envelope solitary wave components. It is determined that phase synchronization amongst elementary solitary wave components can be critical for the formation of extreme waves in unidirectional sea states.

1. Introduction

Rogue waves have been described as waves that appear from nowhere and leave without a trace [1]. These extreme energy concentrations pose severe threats to maritime voyages and offshore operations [2]. Considerable work has been done on modeling and predicting rogue waves [3,4]. Related efforts include the analytical work based on modulational instability (MI) [5,6], experiments and field measurements on wave statistical properties, such as kurtosis and skewness of the underlying probability density function [7], and numerical computations of different sea state parameters [8]. Broadly speaking, there are different mechanisms that can be used to explain the occurrence of extreme waves, including nonlinear focusing, dispersive focusing, atmospheric forcing and so on (*e.g.*, the review papers by Dysthe et al. [1] and Kharif and Pelinovsky [2]). Until now, it is widely recognized that the unidirectional sea state often favors extreme wave statistics, as claimed in most of the studies [9–12].

The modulational instability (MI) is a well-recognized mechanism for generating very large waves due to energy transfer between different modes. A mathematical model for explaining MI has been developed by Shabat and Zakharov [6]. This model, known as the nonlinear Schrödinger equation (NLSE), has been used to study the interplay between nonlinearity and dispersion of water waves. NLSE is integrable in $1D + 1$ and can be solved by using the Inverse Scattering Transform (IST). Several analytical solutions, such as solitons and breathers, have been regarded as the prototypes of rogue waves. However, there is no broad agreement on which solution is the best candidate for a rogue wave, when considering different spatial and temporal periodicities [13–16].

The existence of steep solitary wave groups has been confirmed in laboratories and examined under different numerical frameworks. When transverse effect is insignificant, weakly nonlinear wave groups do exhibit structural stability without noticeable distortion in the event of collisions and these groups can propagate a long distance. Whereas in the case of large wave steepness; that is, relatively steep solitary groups, dispersion outweighs the self-focusing effect along the propagation direction. However, it has been confirmed through experiments that the envelope soliton solution to NLSE provides a rather accurate approximation to the long-time evolution of steep intense solitary wave groups up to wave steepness of 0.3 [17].

Although a single steep solitary wave group can create a freak wave event, interactions amongst multiple moderate solitary wave groups improves the likelihood of extreme waves significantly, leading to a heavy tail distribution in the wave height statistics. Soliton synchronization has been proved as an effective way to generate localized high-amplitude waves in the system governed by the NLSE [18] and the modified KdV framework [19]. In the former framework, it has been indicated with the Darboux transformation method that the solitons can be synchronized to form a

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peak at the focusing point with the magnitude equal to that of the sum of interacting solitons [20,21].

The effect of multiple soliton interactions strongly depends on the details of the collision process. Although an intersection of soliton trajectories is necessary but it is not sufficient for the efficient focusing. When approaching the focusing point, the train of solitons should be positioned with descending group velocities, which allow farther solitons to overtake the nearer ones. In addition, they should have alternating phases [19]. By simply setting position and phases to be equal amongst soliton trains, one will not have amplitude synchronization since the nonlinear interaction process makes the trajectory of each soliton bend before reaching the focusing point [18]. Although the exact synchronization of amplitude requires further details, there are two essential ingredients for soliton synchronization, phase coherence during the synchronization and different group velocities for soliton collision [22,23].

Sea waves are an example of inherently stochastic waves and they are often modeled as a combination of quasi-sinusoidal waves with independent random uniformly distributed phases, known as Gaussian sea, following earlier work [24]. Onorato et al. [11,12] have performed three-dimensional random waves water basin experiments to study the free surface profile probability distributions based on the JONSWAP spectrum. Different degrees of directionality have been considered to study the effects of wave crest length. The results indicate that the probability distributions of the surface elevation of unidirectional waves deviate most from the Gaussian or near-Gaussian sea and the occurrence of rogue waves has increased significantly compared to short-crest sea. Gramstad and Trulsen [25] have claimed a similar finding that more rogue waves are generated in unidirectional seas.

Here, the authors focus on understanding how the introduction of phase interference and wave train modulation can enhance the possibility of extreme waves formations in the unidirectional sea states. The rest of the paper is organized as follows. In the next section, they describe the model construction as an extension of the envelope solitary wave solution to NLSE. Following that, in Section 3, the authors present the results obtained through the application of this model to North Sea Draupner events to demonstrate the validity of the described methodology. Statistical results obtained from large-scale simulations are also discussed in support of the proposed model. Finally, concluding remarks are collected and presented together in Section 4.

2. Solitary wave model approximation

2.1. Nonlinear Schrödinger equation and fundamental solitary wave solution

The leading-order theory for the description of unidirectional gravity water wave nonlinear focusing is the classic cubic NLSE written for the complex wave envelope $A(x,t)$ as

$$A_t + c_g A_x + \frac{i}{4} c_g k_0^{-1} A_{xx} + \frac{i}{2}\omega_0 k_0^2 |A|^2 A = 0. \tag{1}$$

Here, ω_0 and k_0 are dominant wave frequency and wavenumber, respectively, and $c_g = \omega_0/2k_0$ is the linear group velocity in deep water, with the dispersion relation

$$\omega_0 = \sqrt{gk_0}. \tag{2}$$

Both the surface elevation $\eta(x,t) = \mathrm{Re}\{A(x,t)e^{ik_0 x - iw_0 t}\}$ and velocity potential $\phi(x,z,t)$ are determined by the complex-valued function $A(x,t)$. The η and ϕ fields can be computed with high accuracy by including higher order nonlinear terms in NLSE, such as the Dysthe equation. The fundamental envelope soliton, a solution to equation (1), is of the form [26]

$$A = a_0 \mathrm{sech}[\sqrt{2} a_0 k_0^2 (x - c_g t)] e^{-i a_0^2 k_0^2 \omega_0 t/4}, \tag{3}$$

where a_0 is the soliton amplitude. The envelope soliton given by equation (3) is propagated with the linear group velocity c_g. Different from transient wave groups, the envelope soliton consists of coherent wave harmonics that prevent the dispersion of the wave group. The Fourier spectrum of wave group (3) may be obtained as

$$\hat{A}(k,t) = \int_{-\infty}^{+\infty} A(x,t) e^{ikx} dx = F(k) e^{i\xi(t)}, \tag{4}$$

$$F(k) = \frac{\pi A_0}{\sqrt{2} k_0^2 a_0} \sinh\left(\frac{\pi k}{2\sqrt{(2)} k_0^2 a_0}\right), \tag{5}$$

$$\xi(t) = -k c_g t - \frac{k_0^2 a_0^2 \omega_0 t}{4}. \tag{6}$$

Hence, all Fourier modes have the same phases and the Fourier amplitudes $F(k)$ do not evolve in time for a single envelope soliton. However, within the framework of NLSE, envelope solitons (3) may interact amongst each other, and also with other quasi-linear waves. It is noted that equation (1) has high-order solutions such as the Peregrine soliton, Kuznetsov–Ma breather, and Akhmediev breather [27], which are the results of interactions involving envelope solitons (3) with background waves [28]. These high-order breathers have different characteristic group velocity than c_g and they are defined by the IST spectrum [28,23]. Next, the authors revisit the IST to examine the determination of the spectrum from the complex modulation amplitude based on NLSE.

2.2. Inverse scattering transform

The authors consider the non-dimensional NLSE equation of the form

$$iA_t + A_{xx} + 2|A|^2 A = 0. \tag{7}$$

This equation satisfies the compatibility condition of the following system of linear equations:

$$\mathbf{B}_x = \begin{pmatrix} -i\lambda & A \\ -A^* & i\lambda \end{pmatrix} \mathbf{B}, \tag{8}$$

$$\mathbf{B}_t = \begin{pmatrix} -2i\lambda^2 + i|A|^2 & iA_x + 2\lambda A \\ -iA_x^* - 2\lambda A^* & 2i\lambda^2 - i|A|^2 \end{pmatrix} \mathbf{B}, \tag{9}$$

where λ is a spectral parameter, $\mathbf{B}(x,t,\lambda)$ is a vector or matrix function, and A^* represents the complex conjugate of A. In fact, if one differentiates equations (8) and (9) with respect to t and x respectively, one can find that in order to force the right hand side to be equal to each other, the complex envelope function $A(x,t)$ must satisfy equation (7). In other words, equation (8) and (9) are compatible with each other on the equation condition (7). The matrix operators in the above linear systems are called the Lax pair of equation (7) and these operators were first studied by Zakharov and Shabat [6]. Equation (8) is called the Zakharov–Shabat (ZS) scattering problem. The parameter λ, which lies in the complex plane, is such that $\lambda = \lambda_R + i\lambda_I$. Then, the λ_I can be interpreted as having the information about the amplitude of the unstable mode and λ_R can be interpreted as referring to the group velocity relative to the linear group velocity, which corresponds to λ located on the imaginary axis.

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