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Modified dispersion properties of lower hybrid wave with exchange correlation potential in ultra-relativistic degenerate plasma

Prerana Sharma

Physics Department, Ujjain Engineering College, Ujjain, (M. P.) – 456010, India

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ABSTRACT

In this letter, the propagation characteristics of lower hybrid waves are investigated in electron–ion degenerate plasma with exchange effect considering non-relativistic, relativistic and ultra-relativistic regimes. The combined effect of Bohm force and exchange correlation potential are found to alter the dispersion properties of lower hybrid waves. The analytical and numerical results clearly show the influence of relativistic velocities of electrons, kinetic pressure of ions, Bohm force and exchange correlation potential on the frequency of the lower hybrid wave. The present work find its relevance for the dense astrophysical environments like white dwarfs and for laboratory fusion plasma experiments.

1. Introduction

The astrophysical and laboratory plasmas with high densities and low temperature are effectively treated as quantum plasma. When the de-Broglie wavelength of the particles becomes comparable to the average inter-particle distance, the quantum effects cannot be ignored. The dispersion properties of waves in quantum plasmas have been studied in last few decades [1–3]. Study of electron and ion waves in such plasmas provides important insights about instabilities in various laboratory, space and astrophysical objects such as white dwarfs, pulsar magnetosphere, neutron star etc. [4–8]. Several authors have studied the propagation characteristics of electromagnetic waves in quantum plasma [9–14] incorporating Bohm force. In addition to this, degeneracy appears in the system involving some novel phenomena like exchange correlation potential [8,15] and [16]. One of the simplest quantum systems exhibiting both diffraction and exchange effects is a fixed impurity in an ideal Fermi gas of electrons. The electron exchange and correlation effects embody a short-range electric potential which depends only on the number density of the Fermi particles. It has been observed that the electron exchange-correlation effects become significant for high density and low temperature plasmas such as in the ultra-small electronic devices [12]. Thus, the combine influence of both the dispersion effects and exchange potential can significantly affect the dynamics of waves in plasmas. In this regard many authors have studied different waves and instabilities in plasma considering combined influence of exchange potential and Bohm force. Ref. [17] has investigated the dispersion characteristics of extraordinary wave in electron–positron plasma with the inclusion of both the exchange potential and Bohm force and found the significant influence of both these parameters in affecting the dynamics of extraordinary waves. The instability of the upper-hybrid waves in semiconductor quantum magneto plasma is investigated by [12] considering Bohm force, exchange potential and the quantum statistical pressure of the degenerate electrons. The Jeans instability of quantum dusty plasma with exchange effects is studied by [15]. Further, [18] have investigated stimulated scattering instabilities of circularly polarized electromagnetic waves carrying orbital angular momentum with the effect of exchange potential in dense quantum plasmas with degenerate electrons and non-degenerate ions. All these studies signify that wherever quantum effects are prominent the exchange effect can no longer be ignored. Thus, it is important to study exchange effect along with Bohm force in degenerate plasma systems.

It is well established that an electron has continuous motion around the position it occupies. This motion exerts pressure on the surrounding medium, exactly as the thermal velocity of the molecules of a gas exerts its pressure. This pressure is called the electron degeneracy pressure (the electron pressure P is defined as the momentum transfer per unit area by the electrons). For degenerate electrons, the pressure is measured in terms of Fermi temperature T_{Fe} which depends on the particle density of the system [19]. But when the electrons are moving with relativistic or ultra-relativistic speed the degeneracy pressure gets modified [20,21]. Ref. [22] has presented the equation of state for dense astrophysical plasmas systems having degenerate electrons. In the limiting cases, the equation of state for nonrelativistic degener-

E-mail address: preranaiitd@rediffmail.com.

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ate electrons is defined as $p_{Fe} \sim n_e^{5/3}$ while for relativistic/ultra-relativistic degenerate electrons are given as $p_{Fe} \sim n_e^{4/3}$.

In addition, the study regarding the electron acceleration and the magnetic reconnection phenomena (via lower hybrid plasma waves) in laboratory, space and astrophysical plasmas is widely done by many researchers [23,24]. Reconnection is fundamental process responsible for heating and accelerating plasma in solar flares, coronas around white dwarfs, neutron stars, and black holes, and in tokamaks [23]. Lower hybrid waves are important in space [25] and fusion plasmas [23,26] and [27]. The lower hybrid waves are electrostatic waves that propagate almost perpendicular to the magnetic field and whose frequency lies in the vicinity of the lower-hybrid resonance frequency $\omega_L = \sqrt{\omega_c \Omega_c}$ (where Ω_c and ω_c are the ion and electron gyro frequencies, respectively). These waves are simultaneously in resonance with both the magnetized electrons and un-magnetized ions. The lower hybrid waves can provide the necessary electron acceleration that leads to plasma heating mechanism. The experiments on lower hybrid waves require strongly magnetized plasma. Since these waves are with a large k vector across the magnetic field which makes them attractive for radio-frequency heating of fusion plasmas. Thus, due to wide applications of electrostatic lower hybrid wave in various astrophysical and laboratory situations [23–27] in this Letter, we have investigated the instability of the lower hybrid waves in electron–ion quantum magneto plasma taking into account the quantum Bohm force and exchange-correlation potential of electrons. The obtained dispersion relation is discussed considering three different regimes i.e., non-relativistic, relativistic and ultra-relativistic regime respectively. The present investigation is done using quantum hydrodynamic (QHD) model [8,12] and [15]. This QHD model deals with the behavior of macroscopic quantities like density and current. The model has the advantage of mathematical efficiency and can be derived using Wigner–Poisson model [28].

The manuscript is organized as follows: In Sec. 2 the basic QHD set of equations governing the dynamics of electron–ion quantum plasmas are presented. In Sec. 3 linearized perturbation equations of the system along with the dispersion of lower hybrid waves for the three different cases (non-relativistic, relativistic and ultra-relativistic) is derived. In section 4, graphical discussion is presented which shows the contribution of exchange potential, magnetic field and Bohm force on the propagation characteristics of lower hybrid wave in magnetized plasmas. Finally in section 5, a brief discussion of the present work and summary is presented.

2. Model equations for degenerate plasma system

We first consider the set of QHD equations [8,12,15] for degenerate plasma system consisting of electron and ion species. The dynamics of electrons and ions is governed by the electron and ion continuity and momentum equations

$$\frac{\partial}{\partial t} n_e + n_e \nabla . \vec{v}_e = 0 \tag{1}$$

$$m_e n_e \frac{\partial}{\partial t} \vec{v}_e = q_e n_e \left(\vec{E} + \frac{1}{c} [\vec{v}_e, \vec{B}] \right) - \nabla P_e - \frac{\hbar^2}{4m_e} \nabla \left(\nabla^2 n_e \right) - V_{exc} \nabla n_e \tag{2}$$

$$V_{e,xc} = -0.985 \frac{n_e^{1/3} e^2}{\varepsilon} \left[1 + \frac{0.034}{a_{Be}^* n_e^{1/3}} \ln\left(1 + 18.37 a_{Be}^* n_e^{1/3} \right) \right] \tag{3}$$

$$\frac{\partial}{\partial t} n_i + n_i \nabla . \vec{v}_i = 0 \tag{4}$$

$$m_i n_i \frac{\partial}{\partial t} \vec{v}_i = q_i n_i \left(\vec{E} + \frac{1}{c} [\vec{v}_i, \vec{B}] \right) - \nabla P_{ti} \tag{5}$$

where $n_{e,i}, \vec{v}_{e,i}, m_{e,i}$ and $q_{e,i}$ represent the number density, fluid velocity, mass and charge of species ($e =$ electrons and $i =$ ions) respectively. Equations (1) and (2) are the continuity and momentum equations for electrons in which P_e is the electron degeneracy pressure. Equation (3) shows the expression for exchange correlation potential [17] where ε is the dielectric constant of material and $a_{Be}^* = \varepsilon \hbar^2 / m q_e^2$ is the effective Bohr atomic radius of the species. Further, (4) and (5) describe the set of continuity and momentum equations respectively for non-degenerate ions. The ion thermal pressure is given via equation of state i.e., $P_{ti} = k_B T_i n_i$ here T_i is temperature of ions and k_B is Boltzmann constant. The above equations (1)–(5) describe the governing set of equations to study electrostatic lower hybrid waves in degenerate magneto plasma. Now we would like to comment on the validity range of our model. The model is valid in the dense astrophysical region like outer regions of white dwarf, core of neutron star, pulsar magnetosphere (with temperature $T \approx 10^5$–10^7 K, magnetic field strength $B \approx 10^5$–10^{10} T and unperturbed number density $n_0 \approx 10^{29}$–10^{39} m^{-3}) [29] etc.; whereas the ions are non-relativistic and non-degenerate. The ions can be treated as cold fluid as their Fermi energy $(\hbar^2/2m_i)(3\pi^2)^{2/3} n_i^{2/3}$ is smaller than that of electrons [29].

3. Linearized perturbation equations and dispersion relation

Now in order to investigate the behavior of lower hybrid waves we make the following perturbation expansion for the fundamental quantities $n, \vec{v}, \vec{B}$ and $\vec{E}$ from their equilibrium values. The perturbation in these physical quantities can be ruled according to,

$$n = n_0 + n_1, \qquad \vec{v} = \vec{v}_0 + \vec{v}_1, \qquad \vec{B} = \vec{B}_0 + \vec{B}_1, \qquad \vec{E} = \vec{E}_0 + \vec{E}_1, \qquad P = P_0 + P_1 \tag{6}$$

where the subscript "1" denotes the perturbed part and subscript "0" denotes an unperturbed part. At equilibrium $\vec{v}_0, \vec{E}_0 = 0$. Therefore, using (6) the linearized set of equations can be given as

$$\frac{\partial}{\partial t} n_{e1} + n_{e0} \nabla . \vec{v}_{e1} = 0 \tag{7}$$

$$m_e n_{e0} \frac{\partial}{\partial t} \vec{v}_{e1} = q_e n_{e0} \left(\vec{E}_1 + \frac{1}{c} [\vec{v}_{e1}, \vec{B}_0] \right) - \nabla P_{e1} - \frac{\hbar^2}{4m_e} \nabla \left(\nabla^2 n_{e1} \right) - V_{exc} \nabla n_{e1} \tag{8}$$

$$V_{e,xc} = -0.985 \frac{n_{e1}^{1/3} e^2}{\varepsilon} \left[1 + \frac{0.034}{a_{Be}^* n_{e0}^{1/3}} \ln\left(1 + 18.37 a_{Be}^* n_{e0}^{1/3} \right) \right] \tag{9}$$

$$\frac{\partial}{\partial t} n_{i1} + n_{i0} \nabla . \vec{v}_{i1} = 0 \tag{10}$$

$$m_i n_{i0} \frac{\partial}{\partial t} \vec{v}_{i1} = q_i n_{i0} \left(\vec{E}_1 + \frac{1}{c} [\vec{v}_{i1}, \vec{B}_0] \right) - \nabla P_{ti1} \tag{11}$$

Now let the solution of the system of equations (7)–(11) be of the form $\exp[ik_x x - i\omega t]$, where ω is the frequency of perturbation and k_x is the x-component of perturbed wave vector. Here we restrict ourselves to $\vec{E} = E_x \hat{x}$ and $\vec{B}_0 = B_0 \hat{z}$. To demonstrate the combine influence of Bohm force and exchange potential in our theoretical framework, we have considered following cases.

3.1. Non-relativistic plasma

In the present subsection we have considered the simplest case, here we assume that the electrons are non-relativistic and degenerate with exchange effects. The equation of state for non-relativistic degenerate electrons [22,29] is given as