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Monogamy relations of nonclassical correlations for multi-qubit states

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ABSTRACT

Nonclassical correlations have been found useful in many quantum information processing tasks, and various measures have been proposed to quantify these correlations. In this work, we mainly study one of nonclassical correlations, called measurement-induced nonlocality (MIN). First, we establish a close connection between this nonlocal effect and the Bell nonlocality for two-qubit states. Then, we derive a tight monogamy relation of MIN for any pure three-qubit state and provide an alternative way to obtain similar monogamy relations for other nonclassical correlation measures, including squared negativity, quantum discord, and geometric quantum discord. Finally, we find that the tight monogamy relation of MIN is violated by some mixed three-qubit states, however, a weaker monogamy relation of MIN for mixed states and even multi-qubit states is still obtained.

1. Introduction

Quantum correlations have been not only recognized as fundamental properties in the quantum regime that depart from the classical world, but also regarded as useful resources in numerous quantum information and quantum computation tasks [1]. Thus, it is a prime task in quantum information theory to characterize and quantify these nonclassical resources [2–4].

One of fundamental features in quantum world is the existence of quantum nonlocality, especially Bell nonlocality [4], signaling distinct incompatibility between quantum mechanics and local realism [5]. In particular, Bell nonlocality could be revealed in the very simple scenario of two-qubit systems, shared by distant observers, where each observer chooses one of two dichotomic measurements on each qubit [6]. Moreover, it was found that the Bell nonlocal states, such as the Einstein–Podolsky–Rosen state [7], are capable of accomplishing jobs impossible in the classical world, such as device-independent quantum key distribution (QKD) [8,9], quantum teleportation [10], and super-dense coding [11].

Recently, another nonlocal effect, called measurement-induced nonlocality (MIN), was introduced by Luo and Fu in [12]. This nonlocal effect is more general than Bell nonlocality and describes the global effects caused by the local measurements on one side [12]. In this work, we explore the potential relationships between these two kinds of nonlocality. Furthermore, they are also compared to the well-known quantum entanglement, and to nonclassical correlations beyond entanglement, such as quantum discord [13,14] and geometric quantum discord [15,16].

Another peculiar quantum feature is the monogamy of quantum correlations which constrains the distribution of quantum correlations among multiparty systems. For example, when Alice and Bob share a maximally entangled state $\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)$, then each party can only be classically correlated with the third party with no entanglement at all. This phenomenon is termed monogamy, and for Bell nonlocality ensures the security of quantum cryptographic protocols [8]. Monogamy relations are already known for concurrence [17,18], negativity [19], Bell nonlocality [20–22], quantum steering [23–26], and quantum discord [27,28].

Along this line, we are interested in whether MIN obeys a similar monogamy relation. We give an affirmative answer for pure three-qubit states, and thus disprove claims in Ref. [29] that MIN does not satisfy such a monogamy relation. Generally, we show that three-qubit states and arbitrary n-qubit states obey another kind of monogamy relations of MIN.

This paper is structured as follows. In Sec. 2, we introduce the basic definitions required in the two-qubit scenario and show that MIN is no larger than the Horodecki parameter [30], which quantifies the maximal violation of a Bell inequality. Then, we derive a tight monogamy relation of MIN for pure three-qubit states in Sec. 3, and also recover known monogamy relations for negativity, quantum discord, and geometric quantum discord as byproducts. In Sec. 4, a counterexample is constructed to disprove the uni-

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versality of monogamy of MIN for more general cases, including mixed three-qubit states and multi-qubit states, but an alternative form of monogamy relations for general states is still obtained. Finally, we conclude with discussions in Sec. 5.

2. Measurement-induced nonlocality v.s. Bell nonlocality

A two-qubit state ρ_{AB} shared by Alice and Bob can be written as

$$\rho_{AB} = \frac{1}{4}\left(I_A \otimes I_B + \mathbf{a}\cdot\boldsymbol{\sigma} \otimes I_B + I_A \otimes \mathbf{b}\cdot\boldsymbol{\sigma} + \sum_{j,k=1}^{3} T_{jk}\sigma_j \otimes \sigma_k\right). \tag{1}$$

Here, $\boldsymbol{\sigma} \equiv (\sigma_1, \sigma_2, \sigma_3)$ refers to the vector of Pauli spin operators. I_A and I_B are identity operators. $\mathbf{a}$ and $\mathbf{b}$ correspond to the Bloch vectors of Alice's and Bob's reduced states, and T is the spin correlation matrix with $T_{jk} = \langle \sigma_j \otimes \sigma_k \rangle$. Complementary to the locally accessible information $\mathbf{a}$ and $\mathbf{b}$, the spin correlation matrix T is of great importance in encoding the global information and the strength of the quantum correlations of the qubits [2–4].

To measure the nonlocal effects induced by local measurements on one side, Luo and Fu proposed the measurement-induced nonlocality [12]

$$\mathcal{D}_M^{A\to B} := \max_{\Pi^A} ||\rho_{AB} - \Pi^A(\rho_{AB})||^2, \tag{2}$$

where the maximum is taken over all von Neumann measurements $\{\Pi_j^A\}$ that preserve Alice's local state, i.e., $\Pi^A(\rho_A) := \sum_j \Pi_j^A \rho_A \Pi_j^A = \rho_A$. $||X|| := (\mathrm{Tr}\left[X^\dagger X\right])^{1/2}$ denotes the Hilbert–Schmidt norm, and the notation $A \to B$ specifies Alice as the measuring party. Similarly, the nonlocality induced by Bob's local measurements $\mathcal{D}_M^{B\to A}$ could also be defined. Interestingly, it was proven in [12] that MIN has asymmetric property, i.e., $\mathcal{D}_M^{A\to B} \neq \mathcal{D}_M^{B\to A}$.

For an arbitrary two-qubit state ρ_{AB}, the MIN admits an explicit form [12]

$$\mathcal{D}_M^{A\to B} = \begin{cases} \frac{1}{4}\left(\mathrm{Tr}\left[TT^\top\right] - \frac{1}{a^2}\mathbf{a}^\top TT^\top \mathbf{a}\right) & \mathbf{a} \neq \mathbf{0}, \quad \text{(a)} \\ \frac{1}{4}\left(\mathrm{Tr}\left[TT^\top\right] - s_3\right) & \mathbf{a} = \mathbf{0}. \quad \text{(b)} \end{cases} \tag{3}$$

Three eigenvalues s_1, s_2, s_3 of the symmetric matrix $TT^\top$ are arranged in descending order, i.e., $s_1 \geq s_2 \geq s_3 \geq 0$, and here and elsewhere we use $x = |\mathbf{x}|$ to represent the modulus of a vector $\mathbf{x}$. Obviously, the MIN of a state lies in the interval $[0, \frac{1}{2}]$ and achieves its maximum value if and only if the state is locally unitary equivalent to any Bell state [12]. Other basic properties of MIN have been listed in [12]. However, this MIN based on the Hilbert–Schmidt (HS) norm suffers from one weakness that it may increase under the completely-positive and trace-preserving (CPTP) maps on Bob's side [31,32]. To overcome this weak point, other MINs are proposed, such as based on the trace-norm [32], the relative entropy [33], the fidelity [34], and the two-sided projective measurements [35]. The basic properties of these MINs are referred to Refs. [32–35]. In this work, we explore the connections between the MIN based on the HS-norm and other quantum correlations and investigate the distribution of MINs for three-qubit states.

Further, Bell nonlocality characterizes whether the outcome statistics generated by local measurements on both sides could be explained by a local hidden variable theory. This nonlocal effect could be exposed in the very simple scenario of two-qubit systems, shared by distant observers, where each observer involves two dichotomic measurements on each qubit. Specifically, each party has two measurements with outcomes $+1$ or -1, and these binary measurements are assumed to be Hermitian operators: $A_1 = \mathbf{a}_1 \cdot \boldsymbol{\sigma}$, $A_2 = \mathbf{a}_2 \cdot \boldsymbol{\sigma}$ for Alice and $B_1 = \mathbf{b}_1 \cdot \boldsymbol{\sigma}$, $B_2 = \mathbf{b}_2 \cdot \boldsymbol{\sigma}$ for Bob. Then, Bell nonlocality is witnessed by violating the Bell–Clauser, Horne, Shimony, and Holt (CHSH) inequality [6]

$$\langle \mathcal{B} \rangle^2 = (\mathrm{Tr}\,[\mathcal{B}\rho])^2 \leq 4, \tag{4}$$

with the Bell operator $\mathcal{B} = A_1 \otimes B_1 + A_1 \otimes B_2 + A_2 \otimes B_1 - A_2 \otimes B_2$.

A two-qubit state ρ_{AB} is Bell nonlocal if it violates the Bell–CHSH inequality for some set of local measurements. It is remarkable that the above statement still holds even if the state is not limited to qubit states or even does not admit a quantum description. Hence, determining whether ρ_{AB} is Bell–CHSH nonlocal or not is equivalent to check if [30]

$$\mathcal{M} := \frac{1}{4}\max_{A_1, A_2, B_1, B_2} \langle \mathcal{B} \rangle^2 = s_1 + s_2 = \mathrm{Tr}\left[TT^\top\right] - s_3 \leq 1 \tag{5}$$

Here the Horodecki parameter is denoted as $\mathcal{M}$, quantifying the maximal violation of Bell–CHSH inequality.

It follows immediately from Eqs. (3b) and (5) that there is

$$\mathcal{D}_M^{A\to B} = \frac{1}{4}\mathcal{M}, \quad \mathbf{a} = \mathbf{0}. \tag{6}$$

For $\mathbf{a} \neq \mathbf{0}$, note that $s_3 \leq \mathbf{n}^\top TT^\top \mathbf{n} \leq s_1$ for an arbitrary unit vector $\mathbf{n}$. Hence, choosing $\mathbf{n} = \mathbf{a}/a$, yields from Eq. (3a) that

$$\mathcal{D}_M^{A\to B} = \frac{1}{4}\left(\mathrm{Tr}\left[TT^\top\right] - \mathbf{n}^\top TT^\top \mathbf{n}\right) \leq \frac{1}{4}\mathcal{M}, \tag{7}$$

for two-qubit states generally. This implies that the Horodecki parameter provides an upper bound for MIN. Since $\mathcal{M}$ is symmetric under the interchange of Alice and Bob, one similarly has $\mathcal{D}_M^{B\to A} \leq \frac{1}{4}\mathcal{M}$, corresponding to the choice $\mathbf{n} = \mathbf{b}/b$.

Finally, it is of interest to also consider other nonclassical correlations. For example, in contrast to Eq. (2), geometric quantum discord is defined as [15]

$$\mathcal{D}_G^{A\to B} := \min_{\Pi^A} ||\rho_{AB} - \Pi^A(\rho_{AB})||^2, \tag{8}$$

where the minimum is taken over all von Neumann measurements. It is apparent that $\mathcal{D}_G^{A\to B} \leq \mathcal{D}_M^{A\to B}$. Additionally, for two-qubit states, it was shown in [36,37] that the geometric quantum discord further upper bounds both the computable entanglement measure $\mathcal{N}$ [38] and quantum discord $\mathcal{D}$ [13,14]. Thus, we are able to obtain an ordering chain of these nonclassical correlation measures

$$\mathcal{N}^2, (\mathcal{D}^{A\to B})^2 \leq 2\mathcal{D}_G^{A\to B} \leq 2\mathcal{D}_M^{A\to B} \leq \frac{1}{2}\mathcal{M}, \tag{9}$$

for two-qubit states. This ordering chain is useful in exploring the monogamy phenomenon for multi-party systems. In particular, if there exist monogamy relations for correlation measures in the right side of above ordering (9), such as MIN or Bell nonlocality, it is highly possible that these correlation measures in the left side also obey a similar monogamy relation. We will show it is indeed the case in the next section.

3. Monogamy relations for pure three-qubit states

Consider a general pure three-qubit state of a tripartite system held by Alice, Bob, and Charlie:

$$|\phi_{ABC}\rangle = \sum_{i,j,k=0}^{1} a_{ijk}|ijk\rangle. \tag{10}$$