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# Entanglement distribution in star network based on spin chain in diamond

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ABSTRACT

After star network of spins was proposed, generating entanglement directly through spin interactions between distant parties became possible. We propose an architecture which involves coupled spin chains based on nitrogen-vacancy centers and nitrogen defect spins to expand star network. The numerical analysis shows that the maximally achievable entanglement $E_m$ exponentially decays with the length of spin chains $M$ and spin noise. The entanglement capability of this configuration under the effect of disorder and spin loss is also studied. Moreover, it is shown that with this kind of architecture, star network of spins is feasible in measurement of magnetic-field gradient.





## 1. Introduction

Quantum state transfer and entanglement distribution are necessary in a solid quantum information processing system [1–3]. However, most short distance interaction strength decays rapidly on account of physical separation. Thus, it is significant to have appropriate physical system which serves as channel for quantum communication. In a star network configuration [4], one can create entangled state easily according to the number of spins. The restriction of this configuration is that the entangled particles will be created very close to each other, usually at the nanoscale. If interactions between qubits can be made long range, then this structure could be used for entanglement distributions between several distant parties.

The nitrogen-vacancy ($NV^-$) centers in diamond are excellent quantum processor in solids [5,6]. The spin qubits of $NV^-$ center can be optically initialized and read out [7] as well as the long spin-coherence time up to milliseconds even at room temperature [8,9]. Meanwhile, many quantum gates [10,11], algorithms [12,13] and simulations [14] have been demonstrated on NV scheme as well. Recently, $NV^-$ center has been used to detect AC magnetic fields [15], magnetic noises [16] and single nuclear spins [17] via dynamical decoupling. All of above features may be used for extending the star configuration of spins to practical range.

In recent years, a proposal suggested a chain which consists of implanted nitrogen impurities as a coherent quantum channel to transfer quantum states between distant nitrogen-vacancy centers at room temperature [18]. The $NV^-$ centers interact with nitrogen defects through dipole–dipole coupling. This protocol alleviated the stringent constraints currently limiting the realization of scalable quantum processors. The later research [19] extended nearest-neighbor dipole–dipole coupling to non-nearest-neighbor coupling, proved that the distribution of a finite amount of entanglement appears realistic with current systems.

In this paper, we describe and numerically simulate a feasible three-pointed star configuration of spins for entanglement distribution and the practical application of this configuration in measurement of magnetic-field gradient is also discussed. This work is inspired by the scheme of star network of spins and the rise of research on NV technologies. More details are described in Sec. 2 & 3.

## 2. Physical model and methods

First, we give an overview of the star network of spins [4]. The structure is depicted in Fig. 1, where 0 depicts the central spin. The spins 1–5 interact only with the central spin and not with each other. The ground state of this configuration is an interesting multiparticle entangled state, symmetric in the outer spins. The Hamiltonian of this system is given by

$$H = \lambda(\sigma_{0x}\sum_{\text{outer}}\sigma_{ix} + \sigma_{0y}\sum_{\text{outer}}\sigma_{iy}), \quad (1)$$

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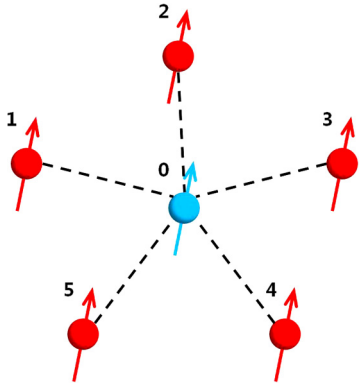


**Fig. 1.** (Color online.) Star configuration of spins, central spin is labeled 0 and spins labeled 1–5 interact with central spin.

where the summation over "outer" refers to the outer spins, $\lambda$ is coupling strength between central spin and outer spins, $\sigma_{ix}$ and $\sigma_{iy}$ denote the $\sigma_x$ and $\sigma_y$ Pauli operators for the $i$th outer spin, $\sigma_{0x}$ and $\sigma_{0y}$ denote the Pauli operators for the central spin. It is convenient to use $J_x = \sum_{\text{outer}} \sigma_{ix}$, $J_y = \sum_{\text{outer}} \sigma_{iy}$ and $J_z = \sum_{\text{outer}} \sigma_{iz}$ where $J = iJ_x + jJ_y + kJ_z$ ($\hbar = 1$). Accordingly, we can therefore rewrite the Hamiltonian in Eq. (1) with the raising and lowering operators $\sigma_\pm = (\sigma_x \pm i\sigma_y)/2$ and $J_\pm = \sum_{\text{outer}} \sigma_\pm$ as

$$H = \lambda(\sigma_{0+} J_- + \sigma_{0-} J_+). \tag{2}$$

This Hamiltonian can be seemed as a resonant interaction between a spin 1/2 and a higher spin scheme. It is readily seen that such a scheme is similar to Jaynes–Cummings Model, especially in the form of Hamiltonian. Thus the eigenstates and eigenvalues of this Hamiltonian yield the form

$$\frac{1}{\sqrt{2}}(|0\rangle|j,m\rangle \pm |1\rangle|j,m-1\rangle), \tag{3}$$

$$E = \pm\lambda\sqrt{(j+m)(j-m+1)}, \tag{4}$$

where the states $|0\rangle$ and $|1\rangle$ represent the $|-1/2\rangle$ and $|1/2\rangle$ spin states of central spin, and the second ket is an eigenstate of $J^2$. Both $m$ and $m-1$ range from $-j$ to $j$, therefore $-j+1 \le m \le j$.

To focus on the ground state, we assume $\lambda$ positive and the number of outer spins be $N$. For example, for $N = 3$, we can get these familiar $W_3$ states

$$|3/2, 1/2\rangle = \frac{1}{\sqrt{3}}(|011\rangle + |101\rangle + |110\rangle). \tag{5}$$

$$|3/2, -1/2\rangle = \frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle). \tag{6}$$

To conclude this brief introduction to this scheme, one can achieve a $N$-qubit $W$-states through once measurement on central spin.

With the rising interest on NV technologies, we can also utilize $NV^-$ center as individual spins. However, the dipole–dipole interaction between $NV^-$ centers is too weak (the coupling strength is about 26 kHz when separation between spins is 10 nm), therefore entangled particles would be created very close to each other. In this case, $NV^-$ center qubits would lose individual addressability [20]. To assure the star configuration work, spin chain structure [1] will be useful to extend the separation between spins. Nitrogen defects consist media channel in spin chain.

From Ref. [19], we derive the effective Hamiltonian for a chain consisting of $M$ spins plus two register spins located at either end of the chain (notice that in this radial pattern Fig. 2, each chain share a common end, which is, the central $NV^-$ center spin).

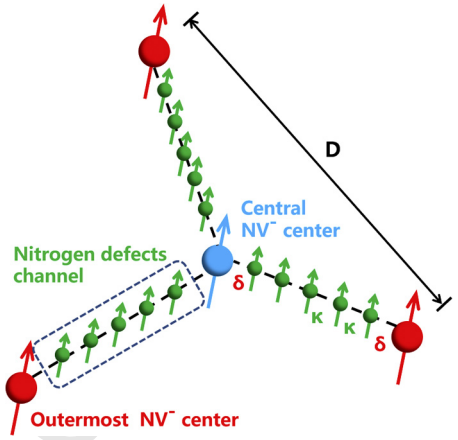


**Fig. 2.** (Color online.) Conceptual schematic of our structure. Star configuration is expanded by spin chains consisting of $M$ nitrogen defects, all spins in chain interact with each others through nearest-neighbor and second-nearest-neighbor dipole–dipole coupling. $\kappa$ and $\delta$ are the dipolar coupling strength between Nitrogen–Nitrogen and Nitrogen–$NV^-$ center, respectively. The separation between outer $NV^-$ centers is $D$.

Besides, both nearest-neighbor and second-nearest-neighbor interactions are included, therefore the effective Hamiltonian is given by

$$H_{\text{eff}} = \sum_{i=1}^{M-1} \kappa\sigma_+^i\sigma_-^{i+1} + \sum_{i=1}^{M-2} \kappa_1\sigma_+^i\sigma_-^{i+2} + \sum_{j=0,M} \delta\sigma_+^j\sigma_-^{j+1} + \text{H.c.}, \tag{7}$$

where $\sigma_\pm^i = \sigma_x^i \pm i\sigma_y^i$ denote Pauli operators acting on $i$th spin, the coupling strengths $\kappa$, $\kappa_1$ and $\delta$ (actually in our simulation, $\delta$ can be tuned to $\delta = 0.9\kappa$ for better performance Ref. [19]) are proportional to $1/r^3$. H.c. represents the Hermitian conjugate. We simulate the full spin chain dynamics by numerically integrating a Lindblad master equation using Hamiltonian in Eq. (7)

$$\dot{\rho} = -i[H_{\text{eff}}, \rho] + \sum_{i=0}^{M+1} \Gamma_i(L_i\rho L_i^\dagger - 1/2(L_i^\dagger L_i\rho + \rho L_i^\dagger L_i)), \tag{8}$$

where $\Gamma_i$ and $L_i$ denote the noise rates and Lindblad operators, respectively. Considering that phase coherence time ($T_2$) of $NV^-$ center or nitrogen defect is more significant than energy relaxation time ($T_1$), we only discuss $T_2$-like process, all noise rates are given by $\Gamma_i = 1/T_2$, and $L_i = \sigma_z^i$.

Generally speaking, spin star is used to create $W$-states with only once measurement on central spin. Unfortunately, we lack an appropriate way to evaluate the entanglement feature of mixed $W$-states [21] directly. However, after measurement on central spin, subsequent measurements can then connect any two of the outermost spins with a maximally entangled state (e.g. if a $W_3$-states has generated, we can then measure any spin in $W_3$-states and obtain the two-qubit maximally entangled state with two-thirds probability). We can therefore evaluate spin star system through evolution of two-qubit entangled state. With reduced density matrix, it is quite easy to calculate the entanglement of two-qubit system. We believe that if a two-qubit maximally entangled state can be transferred to the outermost spin without lacking much fidelity, it is reasonable to expect that the $W$-states could be created successfully as well. Finally, the entanglement feature of spin star system can be assessed indirectly.

The time at which the measurements are performed is not included in the process of entanglement evolution, and the Von Neumann entropy of the reduced two-qubit state is obtained after



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