Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

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Coupling induced logical stochastic resonance

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ARTICLE INFO

Article history:
Received 9 January 2018
Received in revised form 25 March 2018
Accepted 26 March 2018
Available online xxxx
Communicated by C.R. Doering

Keywords:
Logical stochastic resonance
Coupled nonlinear systems
Logic gates
Nonlinear circuits

ABSTRACT

In this work we will demonstrate the following result: when we have two coupled bistable sub-systems, each driven separately by an external logic input signal, the coupled system yields outputs that can be mapped to specific logic gate operations in a robust manner, in an optimal window of noise. So, though the individual systems receive only one logic input each, due to the interplay of coupling, nonlinearity and noise, they cooperatively respond to give a logic output that is a function of both inputs. Thus the emergent collective response of the system, due to the inherent coupling, in the presence of a noise floor, maps consistently to that of logic outputs of the two inputs, a phenomenon we term *coupling induced Logical Stochastic Resonance*. Lastly, we demonstrate our idea in proof of principle circuit experiments.

It has been shown that a single bistable system, when driven by the sum of two external signals, can consistently work as a logic gate in a window of moderate noise [1]. This phenomenon, termed *Logical Stochastic Resonance*, has drawn wide-ranging research interest. Over the years it has been successfully realized in systems ranging from electronic circuits [2] and coulomb-coupled quantum dots [3] to synthetic genetic networks [4], optical systems [5] and nanomechanical devices [6,7].

Here we will go beyond a single bistable system, and demonstrate the following result: when we have two coupled bistable sub-systems, with each driven separately by an input signal, the coupled system yields outputs that can be mapped to specific 2-input logic gate operations in a robust manner, in an optimal window of noise. So the *interplay of coupling and noise allows the system to operate as flexible logic gates*. Importantly, the two external input signals are not added into a single input signal to drive the bistable system, as in previous work on Logical Stochastic Resonance. Rather they are given separately to each sub-system, without necessitating addition of signals. The collective response of the system, due to the coupling, in the presence of a noise floor, maps consistently to that of a logic operation. We term this phenomenon *coupling induced Logical Stochastic Resonance*.

Model: We first test our idea through numerical simulations of a representative system comprised of two first order sub-systems,

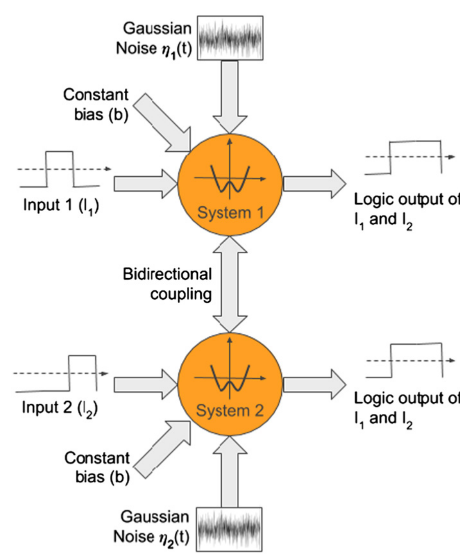

Fig. 1. Schematic representation of coupling induced Logical Stochastic Resonance.

which are coupled linearly and bidirectionally, given by the following equations,

$$\dot{x} = a_1(x - a_2x^3) + b + c(y - x) + I_1 + D\,\eta_1(t),$$
$$\dot{y} = a_1(y - a_2y^3) + b + c(x - y) + I_2 + D\,\eta_2(t). \quad (1)$$

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https://doi.org/10.1016/j.physleta.2018.03.043
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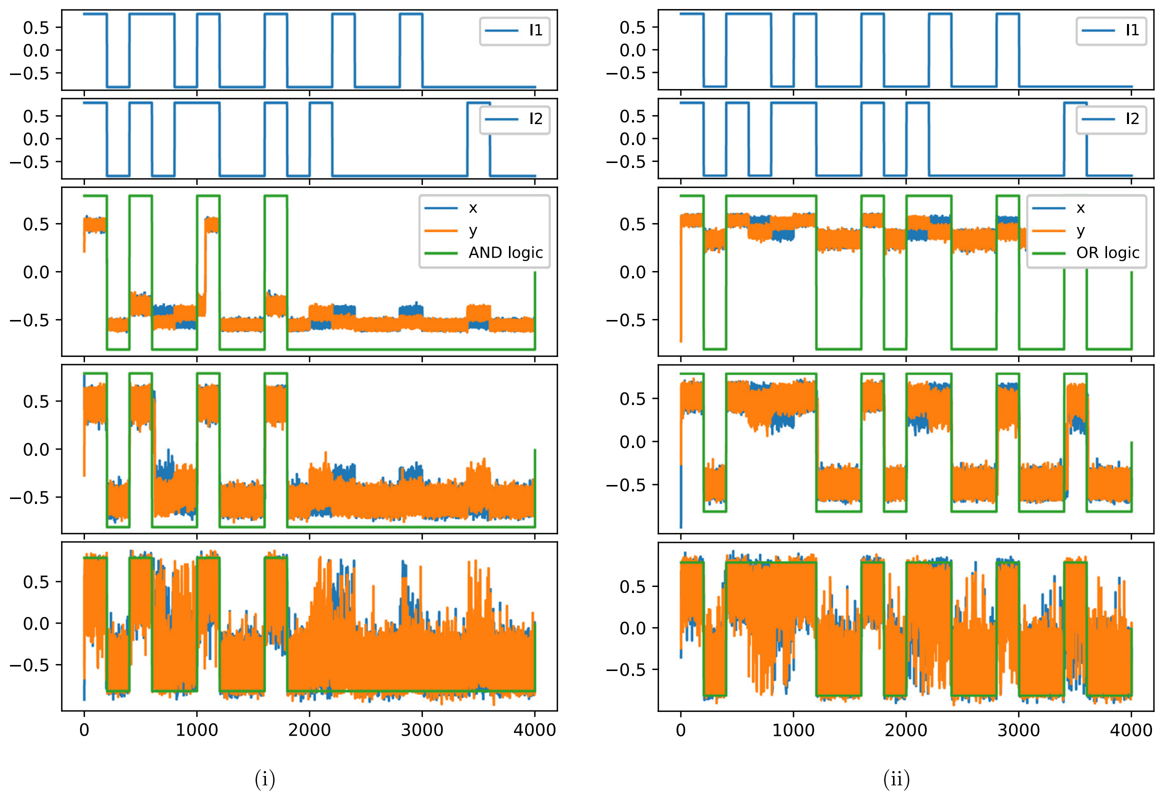

Fig. 2. From top to bottom: panels 1 and 2 show streams of inputs I_1 and I_2, which take value -0.8 when logic input is 0 and value 0.8 when logic input is 1. Panels 3, 4 and 5 show the waveforms of the state of the two sub systems, $x(t)$ (blue) and $y(t)$ (red), obtained from numerical simulations of the system given in Eqn. (1) under noise levels $D = 0.1$, $D = 0.28$, and $D = 0.6$, with coupling constant $c = 4$ and bias $b = \pm 0.28$. Clearly panel 4 (moderate noise) yields a consistent (i) AND logic output for bias $b = -0.28$, and (ii) OR logic output for bias $b = +0.28$. Note that assigning output value 1 to $x(t)/y(t) < 0$, and output value 0 to $x(t)/y(t) > 0$, yields the complementary logic operations, namely $NAND$ and NOR. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

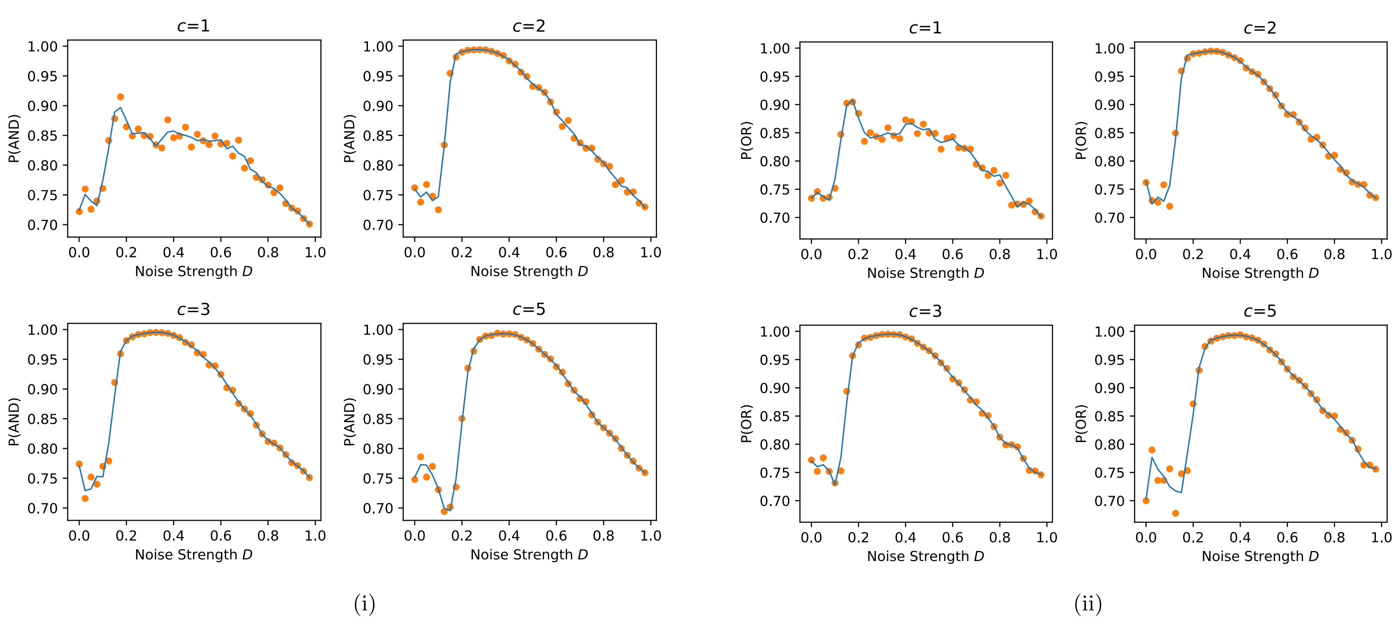

Fig. 3. Dependence of the probability of obtaining (i) AND logic operation, P(AND), and (ii) OR logic operation, P(OR), on noise strength D, for different coupling strengths c. Here bias is (i) $b = -0.28$ for AND operation, and (ii) $b = +0.28$ for OR operation.

Specifically, we consider parameter values $a_1 = 4$ and $a_2 = 5$, where each sub-system is bistable with two potential wells at $x_+ > 0$ and $x_- < 0$ (see Fig. 1 for a schematic). Here c is the coupling strength, b is a bias provided to both the sub-systems, and $\eta_1(t)$ and $\eta_2(t)$ are additive zero mean Gaussian noise with variance 1 and noise strength D. The external drive I_1 and I_2 are low-amplitude input signals encoding the two binary inputs. In particular, we consider (with no loss of generality) logic input 0 to be encoded by -0.8 and logic input 1 to be encoded by 0.8.

The output of the system is determined by the state of the system. If x or y is negative, Logic Output is 0, and if x or y is positive, Logic Output is 1. Namely, when the sub-system is in the