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# Dynamical Galam model

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ABSTRACT

We introduce a model of temporal evolution of political opinions which amounts to a dynamical extension of Galam model in which the number of inflexibles are treated as dynamical variables. We find that the critical value of inflexibles in the original Galam model now turns into a fixed point of the system whose stability controls the phase trajectory of the political opinions. The appearance of two phases is found, in which majority-preserving and regime-changing limit cycles are respectively dominant, and the phase transition between them is observed.



## 1. Introduction

We have witnessed in past two decades, the rise of a new type of mathematical models of human society, distinct from conventional microeconomics, which focus on the formation of social and political opinions in a society [1–4]. These models have revealed unexpected similarity between certain characteristics of human society and the statistical properties of condensed matter systems. We can nowadays, for example, talk about the phase transition in opinion dynamics in human society [5–7].

The Galam model is their prime example, in which heterogenous "agent types" played a critical role [8,9]. In particular, one salient trait to discriminate agents is the individual ability of an agent to eventually shift opinion from the one it had adapted earlier. An agent who skips to its initial choice is tagged as an inflexible against floaters who are susceptible of shifting opinion driven by local exchanges with other within small groups of discussion. The concept of inflexibilty in opinion forming was first introduced to study group decision making using Ising spin like modeling with quenched random local field [10].

Incorporating into opinion dynamics with moving agents within the so-called Galam dynamics model [11,12], it leads to numerous studies [13–18]. Latter, other denominations have been used like "zealots" [19] and "committed" agents [20]. While all these works consider the proportions of inflexibles as fixed external parameters, a study has investigated numerically the building of inflexibilty as an internal dynamics, which is a function of the number of times an agent found itself having the same opinion [21]. Here we extend the investigation of inflexibility considering a novel feature, which reinterprets these inflexibles as opinionated determined minority, or minority with extremist views.

Observing that in real world politics, a motivated minority is often found to be the driving force in political regime change [22]. It is natural to assume that the number of determined minority is not fixed, but can increase and decrease depending on the environment inflexibles and floaters are in. Accordingly we make inflexibles dependent on their overall local environment. A hostile environment tends to strengthen them increasing their number while their victory tends to weaken them decreasing their number.

In this paper we incorporate such a local dependance of inflexibles extending the Galam model of majoritarian dynamics which has the characteristic agent type "inflexibles" [8,12,23] by adding update rules for the production and reduction of inflexibles. Analyzing the model both numerically and analytically with linearization around the fixed points, we found the emergence of fixed points and limit cycles.

It turns out that there are two types of limit cycles, ones that preserve the majority, and the other that causes the cyclic alternation of winning opinions. We show that with the change of system parameters, a phase transition-like behavior is observed between one phase where majority-conserving cycle dominates, and the other phase where only majority-alternating cycles are present. Our model being more realistic with respect to inflexibility, which is a major ingredient of real social system, it may shed new light to understand political cycles in democratic countries with elected governments.

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**Table 1**
The $r = 3$ group agent pattern table for the system with floaters (0/1) and inflexibles (a/b). See the main text for the explanation of the items. In the fifth column, $p$ is the ratio of agents supporting $A$ party, and $a$ and $b$ are the ratio of $A$- and $B$-inflexibles among all agents, and $u$, $d$ are defined by $u = p - a$, $d = 1 - p - b$.

| $k$ | agents | update | $m_k$ | $P_k$ | $K_k^{f1}$ | $K_k^{f0}$ | $K_k^a$ | $K_k^b$ |
|---|---|---|---|---|---|---|---|---|
| 1 | 000 | 000 → 000 | 1 | $d^3$ | 0 | $\frac{3-h}{3}$ | $\frac{h}{3}$ | 0 |
| 2 | 100 | 100 → 000 | 3 | $ud^2$ | 0 | 1 | 0 | 0 |
| 3 | 110 | 110 → 111 | 3 | $u^2d$ | 1 | 0 | 0 | 0 |
| 4 | 111 | 111 → 111 | 1 | $u^3$ | $\frac{3-h}{3}$ | 0 | 0 | $\frac{h}{3}$ |
| 5 | b00 | b00 → b00 | 3 | $bd^2$ | 0 | $\frac{2+g}{3}$ | 0 | $\frac{1-g}{3}$ |
| 6 | b10 | b10 → b00 | 6 | $bud$ | 0 | $\frac{2+g}{3}$ | 0 | $\frac{1-g}{3}$ |
| 7 | b11 | b11 → b11 | 3 | $bu^2$ | $\frac{2-f}{3}$ | 0 | 0 | $\frac{1+f}{3}$ |
| 8 | bb0 | bb0 → bb0 | 3 | $b^2d$ | 0 | $\frac{1+2g}{3}$ | 0 | $\frac{2-2g}{3}$ |
| 9 | bb1 | bb1 → bb0 | 3 | $b^2u$ | 0 | $\frac{1+2g}{3}$ | 0 | $\frac{2-2g}{3}$ |
| 10 | bbb | bbb → bbb | 1 | $b^3$ | 0 | $g$ | 0 | $1-g$ |
| 11 | a00 | a00 → a00 | 3 | $ad^2$ | 0 | $\frac{2-f}{3}$ | $\frac{1+f}{3}$ | 0 |
| 12 | a10 | a10 → a11 | 6 | $aud$ | $\frac{2+g}{3}$ | 0 | $\frac{1-g}{3}$ | 0 |
| 13 | a11 | a11 → a11 | 3 | $au^2$ | $\frac{2+g}{3}$ | 0 | $\frac{1-g}{3}$ | 0 |
| 14 | ab0 | ab0 → ab0 | 6 | $abd$ | 0 | $\frac{1-f+g}{3}$ | $\frac{1+f}{3}$ | $\frac{1-g}{3}$ |
| 15 | ab1 | ab1 → ab1 | 6 | $abu$ | $\frac{1+g-f}{3}$ | 0 | $\frac{1-g}{3}$ | $\frac{1+f}{3}$ |
| 16 | abb | abb → abb | 3 | $ab^2$ | 0 | $\frac{-f+2g}{3}$ | $\frac{1+f}{3}$ | $\frac{2-2g}{3}$ |
| 17 | aa0 | aa0 → aa1 | 3 | $a^2d$ | $\frac{1+2g}{3}$ | 0 | $\frac{2-2g}{3}$ | 0 |
| 18 | aa1 | aa1 → aa1 | 3 | $a^2u$ | $\frac{1+2g}{3}$ | 0 | $\frac{2-2g}{3}$ | 0 |
| 19 | aab | aab → aab | 3 | $a^2b$ | $\frac{2g-f}{3}$ | 0 | $\frac{2-2g}{3}$ | $\frac{1+f}{3}$ |
| 20 | aaa | aaa → aaa | 1 | $a^3$ | $g$ | 0 | $1-g$ | 0 |

The paper is organized as follows. We introduce the dynamical systems model of Galam opinion dynamics in the second section. In the third section, we present the results of numerical calculations. In the fourth section, we analyze the fixed points of the dynamical system, and discuss their stability with the linearized map analysis. The existence of phase transition-like behavior is also pointed out. In the fifth section, the analysis of oscillation period is presented. The paper is concluded with some discussions in the last, sixth section.

## 2. The dynamical extension of Galam model

We construct a dynamical extension of the Galam model, that enables describing the temporal variation of the number of inflexibles, thereby establishing a model for the secular changes of political majority. Our basic observations on the role of extremism in the political process are:

- It is a committed few who often drive political change by tirelessly pushing their cause.
- Extremists thrive in hostile environment, but lose their edge after success.

To model this tendency, we assume that, after each update in Galam model, the number of inflexible agents increase with a probability $f$ if the local majority goes against them, while it decreases with a probability $g$. We assume that the probabilities are proportional to the number of existing inflexibles. We also include the appearance of inflexibles inside the group which has no inflexibles, which we represent by the probability $h$. Here $f$, $g$ ad $h$ are positive numbers between 0 and 1.

Starting with $N$ agents capable of independently taking two values 1 and 0, signifying the support for party $A$ and $B$, respectively, we repeat the following process. All agents are randomly divided into groups of $r$ agents, and within each group the values of agents are updated to conform to the initial local majority within the group, with the following exceptions:

- There are agents called *inflexibles*, who do not follow the group-majoritarian update rule simply keeping their own fixed value. There are both $A$-inflexibles and $B$-inflexibles whose respective preset values are 1 and 0. Agents who do not belong to inflexibles are called *floaters*.
- Within a group whose majority has gone to party $A$, after the update the number of $A$-inflexibles decreases probabilistically by the factor $1 - g$, and the number of $B$-inflexibles increases probabilistically by factor the $1 + f$. When there are no $B$-inflexible, one appears anew in the group with probability $h$.
- Within a group whose majority has gone to party $B$, the number of $B$-inflexibles decreases probabilistically by the factor $1 - g$, and the number of $A$-inflexibles increases probabilistically by the factor $1 + f$. When there are no $A$-inflexible, one appears anew in the group with probability $h$.

Here we assume $N$ to be a multiple of $r$. At time step $t$, we denote the ratio of agents supporting the party $A$ by $p_t$, and resultantly the ratio of agents supporting the party $B$ by $1 - p_t$. It includes both floaters and inflexibles. The ratio of $A$-inflexibles and $B$-inflexibles at the time step $t$ are denoted by $a_t$ and $b_t$ respectively.

The evolution of $\{p_t, a_t, b_t\}$ can be calculated by tabulating all possible agent configurations of a group (indexed by $k$), their multiplicity ($m_k$), the probability of their occurrence ($P_k$), their contributions to the appearance of each agent type and values ($K_k^{f1}$ and $K_k^{f0}$ for floaters supporting $A$ and $B$ parties respectively, and $K_k^a$ and $K_k^b$ for $A$ and $B$ inflexibles), taking their products and summing by $k$ in the form,

$$p_{t+1} = \sum_k m_k P_k(p_t, a_t, b_t)(K_k^{f1} + K_k^a) \quad (1)$$

$$a_{t+1} = \sum_k m_k P_k(p_t, a_t, b_t) K_k^a \quad (2)$$

$$b_{t+1} = \sum_k m_k P_k(p_t, a_t, b_t) K_k^b \quad (3)$$

For $r = 3$, we have tabulated these quantities explicitly in Table 1, from which, we obtain the formulae for the evolution of probabil-