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Analyses of a heterogeneous lattice hydrodynamic model with low and high-sensitivity vehicles

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A R T I C L E I N F O

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A B S T R A C T

Basic lattice model is extended to study the heterogeneous traffic by considering the optimal current difference effect on a unidirectional single lane highway. Heterogeneous traffic consisting of low- and high-sensitivity vehicles is modeled and their impact on stability of mixed traffic flow has been examined through linear stability analysis. The stability of flow is investigated in five distinct regions of the neutral stability diagram corresponding to the amount of higher sensitivity vehicles present on road. In order to investigate the propagating behavior of density waves non linear analysis is performed and near the critical point, the kink antikink soliton is obtained by driving mKdV equation. The effect of fraction parameter corresponding to high sensitivity vehicles is investigated and the results indicates that the stability rise up due to the fraction parameter. The theoretical findings are verified via direct numerical simulation.

1. Introduction

Traffic networks consisting of highways, streets, traffic control devices and various kind of vehicles provide convenient atmosphere to passengers. The problem of traffic jams, with respect to increasing strength of automobile, has become one of the challenging issue for scientists and researchers even with the development of technology and road infrastructure. To reduce jamming states of traffic, it is necessary to predict the complex mechanism and features of traffic congestion and investigate its properties. Therefore, a notable amount of traffic models ranging from micro to macro level [1–11] have been produced to study the complex traffic phenomena of self-driven non equilibrium system of interacting particles.

Among all these approaches, the lattice hydrodynamic (LH) model [7] is one of the simplest and well known traffic models which collaborates the merits of both macroscopic and microscopic models. On one hand it can analyze the transitions and the separation of the traffic phases while on other hand it can describe dynamical evolution of traffic jams in terms of kink–antikink and soliton traffic density waves.

Subsequently, the basic LH model proposed by Nagatani [7] was widely referred and extended by considering a series of realistic factors influencing traffic conditions for example backward looking effect [12], density difference effect [13], lateral effect of lane width [14], driver's behavior [15] etc. All the above cited models are developed to explain some traffic situations for a single lane road. In order to explain the real traffic phenomena mathematically, lattice approach is not only extended for two-lane road [16–21] but for two-dimensional traffic network also [22].

In reality, traffic system either of a whole city or a highway contains a mixture of variety of vehicles. The vehicle heterogeneity in traffic stream is a significant if not dominant factor in modeling highway traffic flow operations accurately. For example, high percentages of big vehicles may induce congestion at relatively lower volumes, and hence dissimilar network traffic conditions may result than with low percentages. Moreover, the proportion of small vehicles in the city traffic stream may vary considerably from highway traffic. To investigate the effect of heterogeneous traffic, some efforts have been made to extend the macro as well as micro models. Lighthill–Whitham–Richard's (LWR) and Rascle's model [23,24] was extended to discuss about flow of heterogeneous traffic, but the extended model fails to analyze the important non-equilibrium features of mixed traffic flow as the speed of each class cannot deviate from its equilibrium velocity. The micro models in which main focus is on car-following behavior [25,26] provide another way to include the heterogeneity of traffic and analyze its effect on traffic flow dynamics. Following microscopic approach, Mason and Woods [27] have performed stability analysis of the multi class optimal velocity model. A few attempts have also been made to generalize the higher order continuum models for heterogeneous traffic flow [28]. Because of its capability in describing the jam-

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ming transitions of traffic flow in the form of kink–antikink soliton density waves, lattice hydrodynamic approach is suitable to investigate the role of heterogeneity in flow of traffic. But the focus of previously discussed LH models was mainly on homogeneous traffic phenomena for single-lane road and/or multi lane highways. These models were unable to completely describe the features of heterogeneous traffic flow on roads, since they do not consider different heterogeneous factors of traffic for example heterogeneity of vehicle and/or driver's heterogeneity etc.

In this paper, we attempt to develop a simple lattice hydrodynamic model to incorporate a distribution of heterogeneous vehicles in section 2, and show by means of theoretical investigation that the resulting model, though simple in nature, is efficient to capture the influence of heterogeneous vehicles on the stability of flow in section 3 and 4. Simulation is performed in section 5, in order to confirm the analytical results and finally, in section 6 conclusion is provided.

2. Proposed model

Basic lattice hydrodynamic model was proposed by Nagatani [7] for unidirectional single lane road. The first equation is the continuity equation which relates the local traffic density with the local velocity via conservation law in the absence of any on/off ramp and given as:

Continuity eq.:

$$\partial_t \rho_j + \rho_0(\rho_j v_j - \rho_{j-1} v_{j-1}) = 0. \tag{1}$$

Further the idea of microscopic car following model is incorporated to describe the density waves in traffic flow. This provides the evolution equation in which the variation of traffic current at jth site is obtained by the difference between the optimal current at $(j+1)$th site and the actual current at site j as:

Evolution eq.:

$$\partial_t(\rho_j v_j) = a[\rho_0 V(\rho_{j+1}) - \rho_j v_j], \tag{2}$$

where $\rho_j(t)$ and $v_j(t)$ are the local density and velocity at jth site on the one dimensional lattice at time t, respectively. ρ_0 and a, respectively, are the average density and the driver's sensitivity. $V(.)$ represents the optimal velocity function (OVF) [29], having the property of monotonically decreasing function with an upper bound. We have adopted the optimal velocity function as given in [26] for the symmetry of density, which is same as given by Bando [29].

The fundamental idea behind first lattice model [7] is that the optimal current regulate the traffic current with a delay time. Later, the above lattice model was extended to consider the optimal current difference effect which has been found to have an important influence on multi-lane traffic flow system [19]. In the improved lattice model by Peng [19], the continuity equation remains intact while the evolution equation is modified to include the optimal current difference effect (OCDE) between two forward sites and is given by:

$$\partial_t(\rho_j v_j) = a\rho_0 V(\rho_{j+1}(t)) - a\rho_j v_j + a\lambda\rho_0[V(\rho_{j+2}) - V(\rho_{j+1})], \tag{3}$$

here, the last term represents the OCDE on site $j+1$ at time t and λ is the reaction coefficient of optimal current difference.

The above model has been found to have greater stability as compared to Nagatani's model. But this model can not be used to analyze the characteristics of heterogeneous traffic as it considers the drives to be homogeneous.

In general, traffic stream comprises of many different types of vehicles starting from very small to large. In this paper, our purpose is to model the behavior characteristics of heterogeneous traffic involving two types of vehicles small and large. The distinguishing features of both type of vehicles refers to the sensitivities. First type (small) of vehicles are assumed to have high-sensitivity as compared to second type (large) of vehicles. By keeping this in view, new heterogeneous lattice hydrodynamic model is proposed for both types of vehicles. The traffic is considered as unidirectional because both type of vehicles will move in the same direction. Hence, the continuity equations remain intact for both type of vehicles at site j and are given by

$$\partial_t(\rho_{1,j}) + \rho_0(\rho_{1,j} v_{1,j} - \rho_{1,j-1} v_{1,j-1}) = 0, \tag{4}$$

$$\partial_t(\rho_{2,j}) + \rho_0(\rho_{2,j} v_{2,j} - \rho_{2,j-1} v_{2,j-1}) = 0, \tag{5}$$

where, $\rho_{1,j}$ $(\rho_{2,j})$ and $v_{1,j}$ $(v_{2,j})$ denotes respectively the local density and velocity of first type (second type) vehicles at jth site at any time t, respectively.

Further, the evolution equations for each type will be modified to include the effect of delay which is inversely proportional to the sensitivity. Assuming $a_1 = \frac{1}{\tau_1}$ and $a_2 = \frac{1}{\tau_2}$ be the sensitivities of small and large vehicles, respectively, we propose new evolution equations for each type in heterogeneous traffic stream as

$$\begin{aligned}\partial_t(\rho_{1,j} v_{1,j}) &= a_1\rho_0 c V(\rho_{j+1}(t)) - a_1\rho_{1,j} v_{1,j} \\ &\quad + a_1\rho_0 c[V(\rho_{j+2}) - V(\rho_{j+1})],\end{aligned} \tag{6}$$

$$\begin{aligned}\partial_t(\rho_{2,j} v_{2,j}) &= a_2\rho_0(1-c) V(\rho_{j+1}(t)) - a_2\rho_{2,j} v_{2,j} \\ &\quad + a_2\rho_0(1-c)\lambda[V(\rho_{j+2}) - V(\rho_{j+1})],\end{aligned} \tag{7}$$

here, $0 \le c \le 1$ denotes the fraction of small vehicles while $(1-c)$ represents the fraction of large vehicles in the heterogeneous traffic stream. Note that both types of vehicles will interact with each other and the presence of one will affect the other. This has been incorporated via optimal velocity function which is the function of total density $(\rho_j(t))$ in the evolution equation for both types of vehicles. Combining (4) with (6), the density evolution equation for first type of vehicle is obtained as

$$\begin{aligned}&\rho_{1,j}(t+2\tau_1) - \rho_{1,j}(t+\tau_1) + \tau_1 c\rho_0^2[V(\rho_{j+1}(t)) - V(\rho_j(t))] \\ &\quad + \tau_1\lambda c\rho_0^2[V(\rho_{j+2}(t)) - 2V(\rho_{j+1}(t)) + V(\rho_j(t))] = 0.\end{aligned} \tag{8}$$

Similarly, the density equation for second type of vehicles is given as

$$\begin{aligned}&\rho_{2,j}(t+2\tau_2) - \rho_{2,j}(t+\tau_2) \\ &\quad + \tau_2(1-c)\rho_0^2[V(\rho_{j+1}(t)) - V(\rho_j(t))] \\ &\quad + \tau_2\lambda(1-c)\rho_0^2[V(\rho_{j+2}(t)) - 2V(\rho_{j+1}(t)) + V(\rho_j(t))] = 0.\end{aligned} \tag{9}$$

As the major purpose of the present model is to describe the characteristics of heterogeneous flow as a whole instead of describing it for different vehicles. In order to fulfill the purpose we need to get density equation for combined traffic. So, we are choosing $\tau_2 = K\tau_1$ where K is very small number, because the time delay corresponding to bigger vehicles is more than that of small vehicles. Then by adding eqs. (8) and (9) we obtain the following eq.

$$\begin{aligned}&[\rho_{1,j}(t+2\tau_1) + \rho_{2,j}((t+2\tau_1) + (K-1)2\tau_1)] \\ &\quad - [\rho_{1,j}(t+\tau_1) + \rho_{2,j}((t+\tau_1) + (K-1)\tau_1)] \\ &\quad + {\rho_0}^2(\tau_1 c + (1-c)\tau_2)[V(\rho_{j+1}) - V(\rho_j)] \\ &\quad + \lambda{\rho_0}^2(\tau_1 c + (1-c)\tau_2)[V(\rho_{j+2}) - 2V(\rho_{j+1}) + V(\rho_j)] = 0.\end{aligned} \tag{10}$$