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Discussion

Dimension-dependent stimulated radiative interaction of a single electron quantum wavepacket

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A R T I C L E I N F O

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A B S T R A C T

In the foundation of quantum mechanics, the spatial dimensions of electron wavepacket are understood only in terms of an expectation value – the probability distribution of the particle location. One can still inquire how the quantum electron wavepacket size affects a physical process. Here we address the fundamental physics problem of particle–wave duality and the measurability of a free electron quantum wavepacket. Our analysis of stimulated radiative interaction of an electron wavepacket, accompanied by numerical computations, reveals two limits. In the quantum regime of long wavepacket size relative to radiation wavelength, one obtains only quantum-recoil multiphoton sidebands in the electron energy spectrum. In the opposite regime, the wavepacket interaction approaches the limit of classical point-particle acceleration. The wavepacket features can be revealed in experiments carried out in the intermediate regime of wavepacket size commensurate with the radiation wavelength.

1. Introduction

When interacting with a radiation wave under the influence of an external force, free electrons can emit radiation spontaneously, or be stimulated to emit/absorb radiation and get decelerated/accelerated. Such an interaction can also be facilitated without an external force when the electron passes through polarizable medium. Numerous spontaneous radiative emission schemes of both kinds are well known: Synchrotron radiation, Undulator radiation, Compton Scattering, Cherenkov radiation, Smith–Purcell radiation, transition radiation [1–6]. Some of these schemes were demonstrated to operate as coherent stimulated radiative emission sources, such as Free Electron Lasers (FEL) [7–9], as well as accelerating (stimulated absorption) devices, such as Dielectric Laser Accelerator (DLA) and Inverse Smith–Purcell effect [10–12].

All of these spontaneous and stimulated radiation schemes have been analyzed in the classical limit – where they are modeled by point particles, and in the quantum limit – where they are normally modeled as plane waves [13–16]. Semi-classical wavepacket analysis of Kapitza–Dirac scattering was presented in [17]. However, a comprehensive quantum analysis of stimulated radiative interaction of a *free electron wavepacket* (radiative emission/absorption or equivalently acceleration/deceleration) in a finite interaction length is not available yet. It is required for bridging the classical "point particle" theory of accelerators and free electron radiators (FEL, DLA) with the quantum plane-wave limit theories of such devices, and of related important effects as multiphoton emission/absorption quantum-recoil spectrum in Photon-Induced Near-field Electron Microscopy (PINEM) [18–22].

The interpretation and the essence of the electron quantum wavepacket and its electromagnetic interactions have been a subject of debate since the early conception of quantum mechanics [43]. Modern QED theory and experiments indicate that spontaneous emission by a free electron is independent of its wavepacket dimensions [23–30]. However, in the present paper we focus on the stimulated emission process, and show that in this case the wavepacket dimensions do affect the interaction in a certain range of operation that we define.

In the following we analyze the stimulated radiative interaction of a single-electron wavepacket of arbitrary size, establishing first the consistency of our analysis with previous theory and experimental measurements of PINEM, FEL and DLA. We then present the main result: derivation of a new "*phase-dependent*" stimulated radiative interaction regime of an electron wavepacket in which the physical significance of the wavepacket size and the history of its generation and transport to the interaction region are exhibited. We demonstrate then how the quantum wavepacket theory evolves to the classical point-particle interaction limit in this regime.

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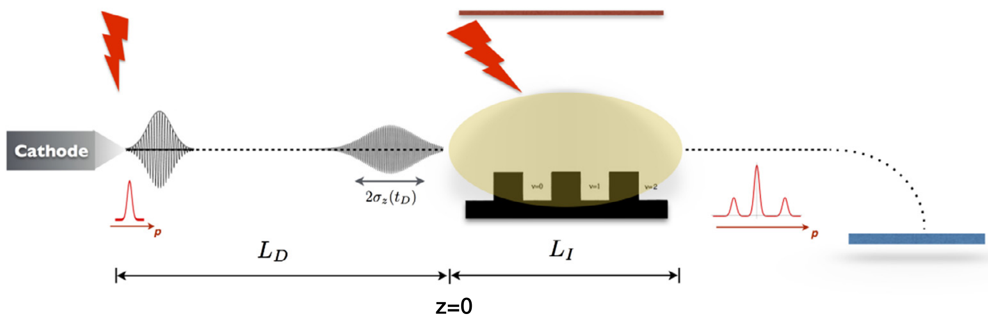

Fig. 1. The experiment setup. Single electron wavepackets are photo-emitted from a cathode driven by a ftSec laser. After a free propagation length L_D, the expanded wavepacket passes next to the surface of a grating, and interacts with the near-field radiation, that is excited by an IR wavelength laser, phase locked to the photo-emitting laser. The momentum distribution of the modulated wavepacket is measured with an electron energy spectrometer.

2. Modeling and methods

First order perturbation analysis. Our one-dimensional interaction model is based on the first order perturbation solution of the relativistic "modified Schrödinger equation", derived from Klein–Gordon equation for the case when the spin effect is negligible [13] (see Supplementary 1):

$$i\hbar\frac{\partial\psi(z,t)}{\partial t}=\left(H_0+H_I(t)\right)\psi(z,t)\,, \tag{1}$$

where $H_0=\varepsilon_0+v_0(-i\hbar\nabla-\mathbf{p}_0)+\frac{1}{2m^*}(-i\hbar\nabla-\mathbf{p}_0)^2$ is the free space Hamiltonian, $m^*=\gamma_0^3 m$, and the interaction part is:

$$H_I(t)=-\frac{e\left(\mathbf{A}\cdot(-i\hbar\nabla)+(-i\hbar\nabla)\cdot\mathbf{A}\right)}{2\gamma_0 m} \tag{2}$$

This model is fitting for description of the variety of interaction schemes mentioned, some of them operating with a relativistic beam. In the present one-dimensional model of electron interaction in a "slow-wave" structure we use a longitudinal vector potential $\mathbf{A}=-\frac{1}{2i\omega}\left(\tilde{\mathbf{E}}(z)\,e^{-i(\omega t+\phi_0)}-\tilde{\mathbf{E}}^*(z)\,e^{i(\omega t+\phi_0)}\right)$, where $\tilde{\mathbf{E}}(z)=E_0e^{iq_z z}\hat{\mathbf{e}}_\mathbf{z}$ represents the dominant slow component of the radiation wave, and transverse field components, as well as transverse variation of the field are neglected. We exemplify our modeling here for a case of Smith–Purcell radiation (see Fig. 1), for which the radiation wave is a Floquet mode: $\tilde{\mathbf{E}}(z)=\sum_m\tilde{\mathbf{E}}_m e^{iq_{zm}z}$ with $q_{zm}=q_{z0}+m2\pi/\lambda_G$, λ_G is the grating period, $q_{z0}=q\cos\Theta$, $q=\omega/c$ and Θ is the incidence angle of the radiation wave relative to the axial interaction dimension. The radiation wave number $q_z=q_{zm}$ represents one of the space harmonics m that satisfies synchronism condition with the electron [6]: $v_0\cong\omega/q_{zm}$. We note that the analysis would be similar for the Cherenkov interaction scheme with $q_z=n(\omega)\cos\Theta$, and $n(\omega)$ the index of refraction of the medium. Furthermore, the analysis can be extended to the case of FEL and other interaction schemes [13,14].

The solution of Schrödinger equation to zero order (i.e. free-space propagation) is well known. Assuming that the initial wavepacket, which is emitted at some point $z=-L_D$ near the cathode face (or any other electron source) at time $t=-t_D$, is a Gaussian at its waist, then:

$$\begin{aligned}\psi^{(0)}(z,t)&=\int dp c_p^{(0)}e^{-iE_pt/\hbar}\left|p(z)\right\rangle\\&=\left(2\pi\sigma_{p_0}^2\right)^{-\frac{1}{4}}\int\frac{dp}{\sqrt{2\pi\hbar}}\exp\left(-\frac{(p-p_0)^2}{4\sigma_{p_0}^2}\right)\\&\quad\times e^{ip(z+L_D)/\hbar}e^{-iE_p(t+t_D)/\hbar},\end{aligned} \tag{3}$$

where $|p(z)\rangle\equiv e^{ipz/\hbar}/\sqrt{2\pi\hbar}\,|p\rangle$, $L_D, t_D=L_D/v_0$ are the "effective" drift length and drift time of the wavepacket center from a "virtual cathode". Expanding the energy dispersion relation to second order $E_p=c\sqrt{m^2c^2+p^2}\approx\varepsilon_0+v_0(p-p_0)+\frac{(p-p_0)^2}{2m^*}$, the wavepacket development in momentum space is then given by (see supplementary 2):

$$c_p^{(0)}=\left(2\pi\sigma_{p_0}^2\right)^{-\frac{1}{4}}\exp\left(-\frac{(p-p_0)^2}{4\tilde{\sigma}_p^2(t_D)}\right)e^{i(p_0L_D-E_0t_D)/\hbar}, \tag{4}$$

with $\tilde{\sigma}_p^2(t_D)=\sigma_{p_0}^2\left(1+it_D/t_{R_\parallel}\right)^{-1}$, $\sigma_{p_0}=\hbar/2\sigma_{z_0}$, $t_{R_\parallel}=\frac{m^*\hbar}{2\sigma_{p_0}^2}=4\pi\frac{\sigma_{z_0}^2}{\lambda_c^*c}$, and we defined $\lambda_c^*=\lambda_c/\gamma^3$ with $\lambda_c=h/mc$ the Compton wavelength.

Note that the "virtual cathode" is not necessarily the physical face of the electron beam source. We define it as the point where the electron wavepacket is at its minimal length (at its longitudinal waist). Recent work showed that this position can be optically controlled by streaking techniques [37].

We now solve Eq. (1) in the interaction region $0<z<L_I$ using the first order perturbation theory in momentum space (see Supplementary 2)

$$\begin{aligned}\psi(z,t)&=\psi^{(0)}(z,t)+\psi^{(1)}(z,t)\\&=\int dp\left(c_p^{(0)}+c_p^{(1)}\right)e^{-iE_pt/\hbar}\left|p(z)\right\rangle,\end{aligned} \tag{5}$$

and then calculate the electron momentum density distribution after interaction:

$$\begin{aligned}\rho(p')&=\rho^{(0)}(p')+\rho^{(1)}(p')+\rho^{(2)}(p')\\&=\frac{\left|c^{(0)}(p')\right|^2+2\,\mathrm{Re}\left\{c^{(1)*}(p')\,c^{(0)}(p')\right\}+\left|c^{(1)}(p')\right|^2}{\int dp'\left(\left|c^{(0)}(p')\right|^2+\left|c^{(1)}(p')\right|^2\right)},\end{aligned} \tag{6}$$

where

$$\rho^{(0)}(p')=\left|c^{(0)}(p')\right|^2=\left(2\pi\sigma_{p_0}^2\right)^{-\frac{1}{2}}\exp\left(-\frac{(p'-p_0)^2}{2\sigma_{p_0}^2}\right) \tag{7}$$

is the initial Gaussian momentum density distribution.

3. Results

Phase-independent momentum distribution – FEL gain. First we draw attention to the second order density distribution (third term in Eq. (6)), as derived in Supplementary 2:

$$\rho^{(2)}(p')=\Upsilon^2\left[\left(\frac{p'+p_{rec}^e/2}{p_0}\right)^2\rho^{(0)}(p'+p_{rec}^e)\right.$$