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Mutual-friction induced instability of normal-fluid vortex tubes in superfluid helium-4

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ABSTRACT

It is shown that, as a result of its interactions with superfluid vorticity, a normal-fluid vortex tube in helium-4 becomes unstable and disintegrates. The superfluid vorticity acquires only a small (few percents of normal-fluid tube strength) polarization, whilst expanding in a front-like manner in the intervortex space of the normal-fluid, forming a dense, unstructured tangle in the process. The accompanied energy spectra scalings offer a *structural* explanation of analogous scalings in fully developed finite-temperature superfluid turbulence. A macroscopic mutual-friction model incorporating these findings is proposed.

1. Prologue

At temperatures smaller than 2.17 K (the lambda point), the quantum field that describes helium-4 becomes Bose–Einstein condensed, giving rise to a non-zero ground state that corresponds to an inviscid fluid (*"superfluid"*). The superfluid coexists with the (classical-like) *"normal-fluid"* of the Bogoliubov quasiparticles that comprise the thermalized quantum fluctuations. Turbulence in such systems, *"finite-temperature superfluid turbulence"* or FTST for short, has a unique characteristic [1,2]: it is the only known type of turbulence, such that two different types of fluid-vortices interact with each other. These are the topological defects in the superfluid (linear vortices of *quantized* circulation), and the classical vortices in the normal-fluid. In FTST, superfluid and normal-fluid vortices interact via "mutual friction" forces [3,4]. Like any other problem in statistical physics, FTST can be studied in either of the Liouville/Hamiltonian representations, that are analogs of the Schroedinger/Heisenberg representations of quantum mechanics, and in *dissipative*, normal-fluid turbulence context consist of the familiar Hopf/Navier–Stokes formulations. Notably, the "realization" ($\mathcal{R}$) formulations (Hamilton/Heisenberg/Navier–Stokes) can only be interpreted stochastically, i.e., subject to random initial conditions, they provide the means to generate sample paths of the random fields (e.g., velocity and pressure in turbulence) whose ensemble averages are the key objectives of the theory, and the only quantities of empirical value. In turbulence research, an ergodic hypothesis allows the inference of ensemble averages via spatial averaging. On the other hand, the "probabilistic" ($\mathcal{P}$) formulations (Liouville/Schroedinger/Hopf) are closed, but in the context of analytically intractable problems like turbulence, they face difficulties in providing closed statistical moment equations that can accurately capture the effects of vortical coherent structures and their interactions (the *"turbulence problem"*). For this reason, the present research employs the $\mathcal{R}$ formulation of FTST, and, by directly calculating the interactions between vortical structures in both fluids, draws conclusions about its statistical structure. This approach has a long tradition in classical turbulence theory [5–7]. For example, [8] indicated that a system of viscous, reconnecting vortex tubes reproduces the Kolmogorov $k^{-5/3}$ scaling of inertial range turbulence, and [9] showed that systems of vortex elements give rise to Levy rather than Gaussian distributions for turbulent flow velocity. Similar connections between vortices and spectra [10–12], as well as, vortices and velocity distributions [13,14] were also obtained in quantum fluids. In FTST, [12] showed that interactions between one normal-fluid and many superfluid vortex rings generate a tendency towards energy-level matching in the wavenumber intervals of the velocity spectra which correspond to the normal-fluid ring diameter scale. Remarkably, there is no accompanying vorticity matching, because, for typical normal-fluid Reynolds numbers, the inertia of the normal-fluid ring is much stronger than mutual-friction effects on superfluid vortices, hence, the latter cannot be coaxed into aligning with the former within the time-scales of normal-fluid ring motion. Fig. 1 (left) shows the isosurfaces of enstrophy in homogeneous, isotropic Navier–Stokes turbulence of Taylor Reynolds number $Re_\lambda \approx 100$. The results have been produced with a projection-type, incompressible Navier–Stokes solver with periodic boundary conditions. The results of Fig. 1 indicate

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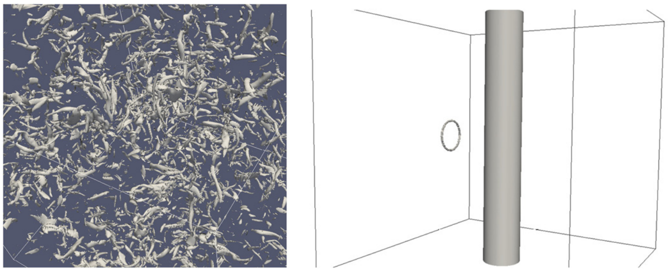

Fig. 1. Left: Vorticity magnitude isosurfaces at level equal to 0.35 times its maximum value, for a homogeneous, isotropic, *pure* normal-fluid turbulence at Taylor Reynolds number $Re_\lambda \approx 100$. Although sheet-like structures are present, we predominantly see linear vortices. Right: The initial configuration consists of a straight normal-fluid vortex tube of circulation strength $\Gamma = 20 \times 10^3\ \nu$, and a superfluid ring of circulation strength $\kappa \approx \Gamma/46,747$.

that the normal-fluid vorticity within the inertial range of turbulence has a predominantly linear (rather than ring-like) structure, and although the vortices are curved and expected to move, the normal-fluid vortex motion effect is not as vigorous as in [12]. Perhaps then, a *straight* tube could be a helpful (albeit approximate) model for the normal-fluid vortex structures shown in Fig. 1, in the sense, that, since a straight tube does not move, superfluid vorticity polarization effects would be present in the highest degree possible. In other words, one would reasonably anticipate the phenomenology of actual FTST to lie in between the ring [12] and straight tube cases.

Notably, straight tube/superfluid vorticity interactions have been studied before [10,15], yet these studies are incomplete from the physics point of view, since they employ a prescribed normal-flow, and ignore the effects of mutual-friction on the latter. In this work, we account for the full physics, since we employ the mesoscopic model of superfluid dynamics [3], that describes the interactions between (turbulent) vortex structures and *individual* topological defects. We shall indicate that the combination of present and previous findings of the mesoscopic model lead to a novel formulation of the macroscopic (i.e., assuming a continuous superfluid vorticity field) equations of superfluid dynamics. In particular [16,17], there are presently two macroscopic prescriptions for the mutual-friction force per unit volume $\mathbf{f}_{\mathrm{MF}}$ (as it appears in the equation for the normal-fluid): (a) the Gorter and Mellink (GM) formula $\mathbf{f}_{\mathrm{GM}} = -\rho_s \rho_n A V_{ns}^2 \mathbf{V}_{ns}$, where ρ_s and ρ_n are (correspondingly) the superfluid and normal-fluid mass densities, $\mathbf{V}_{ns} = \mathbf{V}_n - \mathbf{V}_s$, where $\mathbf{V}_n$, $\mathbf{V}_s$ are (correspondingly) the normal-fluid and superfluid velocities, V_{ns} is the magnitude of $\mathbf{V}_{ns}$, and A is a function of the temperature T and V_{ns}. The GM formula is consistent with a chaotic, isotropic, superfluid vortex tangle, hence, a tangle whose organization does not mimic the (inertial range) vortex structure of normal-fluid turbulence shown in Fig. 1, (b) the Hall–Vinen–Bekharevich–Khalatnikov (HVBK) formula $\mathbf{f}_{\mathrm{HVBK}} = \frac{B\rho_s\rho_n}{\rho\omega_s}\boldsymbol{\omega}_s \times (\boldsymbol{\omega}_s \times \mathbf{V}_{ns}) + \frac{B'\rho_s\rho_n}{\rho}\boldsymbol{\omega}_s \times \mathbf{V}_{ns}$, where B, B' are macroscopic mutual-friction parameters that depend on temperature, second sound frequency, and flow velocity, $\rho = \rho_s + \rho_n$, $\boldsymbol{\omega}_s$ is the continuous superfluid vorticity field, and ω_s its magnitude. Although the HVBK formula has also been employed in homogeneous, isotropic turbulence situations, it is, in principle, a model of rotating superfluid turbulence, since it assumes a highly organized superfluid vorticity state, in the form of superfluid vortex bundles that mimic normal-fluid vorticity structures. Although the organization of flow vorticity into columnar structures of large-eddies that are parallel to the rotation axis is typical of rotating turbulent flows [5], we shall see here that, in the absence of rotation, they do not follow from more microscopic formulations of superfluid dynamics. The aforementioned *presuppositions* carry over to the vortex dynamics of the HVBK equations as formulated and solved in the pioneering contributions of Schwarz [18]. It is important to note here that, since HVBK vortex dynamics refers to the vortex lines in the *continuous* superfluid vorticity field, its comparison with experiments that measure the vortex line density of *discrete* topological defects tangles (that, moreover, do not obey organization assumptions embodied in the HVBK equations), is not methodologically sound. The usefulness of HVBK vortex dynamics (as a means for understanding the structure of superfluid turbulent flows) becomes even more questionable when we notice that, in previously published papers following this approach, the normal-fluid is kinematically prescribed, instead of been dynamically resolved via the HVBK equation for the normal-fluid. The latter shortcoming (which is on top of the aforementioned HVBK presuppositions) is a very important one, since, as we shall demonstrate here, the effects of superfluid back-reaction on normal-fluid vortices are simply too important to be ignored [3,4,12]. The mesoscopic model on the other hand, describes individual topological defects whose dynamics are coupled with the Navier–Stokes equations and *not* with the HVBK equation for the normal-fluid. Because of these characteristics, the mesoscopic model has *predicted* quantities in direct correspondence and impressive agreement with experiments. These are the prediction of tracer particle velocities in thermal counterflow turbulence [19,20], and the temporal scaling for the superfluid vortex line density in grid turbulence decay [4] experiments. As we shall see, a combination of current and previous mesoscopic model results indicate that neither GM or HVBK formulas are directly applicable to homogeneous, isotropic superfluid turbulence. Instead, we propose here a new formula for macroscopic mutual-friction effects that takes into account topological defect curvature and superfluid vorticity intensity factors. An important goal is to explain the phenomenology of, recently performed, fully resolved superfluid turbulence calculations [3,4] by analysing in great detail the physics of key elementary vortex processes in superfluid turbulence.

2. Mathematical model and solution methods

As mentioned above, the $\mathcal{R}$ formulation of superfluid turbulence follows here the incompressible, mesoscopic model of refs. [3,4]. In this formulation, discrete (albeit coarse-grained) topological defects (vortices) in the condensate, interact with a normal-fluid continuum. The motion of point $\mathbf{X}_v(t)$ belonging to the superfluid vortex tangle $\mathcal{L}$ is governed by the zero sum of (from start to end) Magnus, Hall–Vinen, Iordanskii, and reconnection forces

$$\rho_s\kappa\mathbf{X}'_v \times (\mathbf{V}_s - \dot{\mathbf{X}}_v) + D_0\mathbf{X}'_v \times [\mathbf{X}'_v \times (\mathbf{V}_n - \dot{\mathbf{X}}_v)] +$$
$$\rho_n\kappa\mathbf{X}'_v \times (\mathbf{V}_n - \dot{\mathbf{X}}_v) - \int_{\mathcal{L}} d|\mathbf{X}_{\mathcal{L}}|\,\mu_v\ddot{\mathbf{R}}\,\delta(|\mathbf{X}_v - \mathbf{X}_{\mathcal{L}}|) = 0.$$

Here, μ_v is the vortex mass per unit length, $\mathbf{X}'_v$ the unit tangent to the line vortices, ρ_s the superfluid mass density, κ the quantum of circulation, ρ_n the normal-fluid mass density, D_0 the coefficient of the Hall–Vinen force, $\mathbf{V}_s$ the Biot–Savart velocity, $\mathbf{V}_s(\mathbf{X}_v) = \frac{\kappa}{4\pi}\int_{\mathcal{L}} \frac{(\mathbf{x}-\mathbf{X}_v)\times d\mathbf{x}}{|\mathbf{x}-\mathbf{X}_v|^3}$, and $\mathbf{R}$ a deterministic, *pointwise-exact*, reconnection-jump process that models the *topological* (i.e., cut and glue) transition from one *smooth* superfluid tangle configuration to another [3,21]. It is important to note here that, although this equation does not include vortex inertia/acceleration, it includes the vortex mass per unit length μ_v in the formal term depicting reconnections. This is not an inconsistency, since in the reconnection term, μ_v multiplies $\ddot{\mathbf{R}}$ which is an (instantaneous) *jump* process, that is *not resolved* at the mesoscopic range of scales of interest here, hence, it does not contribute to the vortex acceleration. In other words, since the reconnection process is written formally as a jump process, it is not dynamically resolved (and the actual