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Quantum-state transfer through long-range correlated disordered channels

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ABSTRACT

We study quantum-state transfer in XX spin-1/2 chains where both communicating spins are weakly coupled to a channel featuring disordered on-site magnetic fields. Fluctuations are modeled by long-range correlated sequences with self-similar profile obeying a power-law spectrum. We show that the channel is able to perform almost perfect quantum-state transmissions even in the presence of significant amounts of disorder provided the degree of those correlations is strong enough, with the cost of having long transfer times and unavoidable timing errors. Still, we show that the lack of mirror symmetry in the channel does not affect much the likelihood of having high-quality outcomes. Our results suggest that coexistence between localized and delocalized states can diminish effects of static perturbations in solid-state devices for quantum communication.

1. Introduction

Spin chains have been widely addressed as quantum channels for (especially short-distance) communication protocols since proposed in Ref. [1] that spin chains can be used for carrying out transfer of quantum information with minimal control, i.e., with no manipulation being required during the transmission process. Basically, Alice prepares and sends out an arbitrary qubit state through the channel and Bob only needs to make a measurement at some prescribed time. The evolution itself is given by the natural dynamics of the system.

Since then, several schemes for high-fidelity quantum-state transfer (QST) [1–19] and entanglement creation and distribution [20–33] in spin chains have been put forward (for reviews on the subject, see Refs. [34–36]). Perfect QST can be attained in *fully* modulated networks [2–4,37] (cf. [38,39] for proof-of-concept realizations). Other less-demanding (on the engineering side) approaches rely on optimization of the outer couplings of the chain [13] or setting *very weak* couplings between the communicating parties and the bulk of the chain [6–9,11,19,27–29] Similarly, one can also strategically apply local strong magnetic fields in order to establish resonances between the sender and receiver [16,17,23].

One factor that should be taken into account when dealing with the above protocols is disorder arising from, e.g. manufacturing errors, that could potentially damage the planned output [29,40–50]. It is known that the slightest amount of disorder is already capable of promoting Anderson localization effects [51] in 1D systems. That is not necessarily true, however, in the case of *correlated* disorder. The breakdown of Anderson localization has been reported when short- [52,53] or long-range correlations [33,54–60] are present in disordered 1D models. In particular, the latter case finds a set of extended states in the middle of the band with well detached mobility edges thereby signalling an Anderson-type metal-insulator transition [54,55]. This is also manifested in low-dimensional spin chains [33,57]. Long-range correlations with power-law spectrum can actually be found in various physical systems such as in, to name a few, DNA molecules [61], plasma fluctuations [62], patterns in surface growth [63], and graphene nanoribbons [64]. Other kinds of correlated defects have been considered in [40,42] in the context of QST.

Here, we consider a one-dimensional XX spin chain in which the local magnetic fields (on-site potentials) of the channel follow a long-range correlated disordered distribution with power-law spectrum $S(k) \propto 1/k^{\alpha}$, with k being the corresponding wave number and α being a characteristic exponent governing the degree of such correlations. We show that when perturbatively attaching two communicating (end) spins to the channel and setting their frequency to lie in the middle of the band, we are still able to perform nearly perfect QST rounds in the presence of correlated disorder, the major drawback being the requirement of long transfer times and loss of accuracy in the measurement time. Surprisingly, we find it happens even in the presence of considerable amounts of asymmetries in the channel. The reason for that is the appearance of extended states in the middle of the band which

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offers the necessary end-to-end *effective* symmetry thereby supporting the occurrence of Rabi-like oscillations between the sender and receiver spins. We further show that perfect mirror symmetry is not a crucial factor as long as there exists a proper set of delocalized eigenstates in the channel.

In the following, Sec. 2, we introduce the XX spin Hamiltonian with on-site long-range correlated disorder. In Sec. 3 we derive an effective two-site Hamiltonian that accounts for the way both communicating parties are coupled to the channel. In Sec. 4 we display the results for the QST fidelity and timing errors. Our final remarks are addressed in Sec. 5.

2. Spin-chain Hamiltonian

We consider a pair of spins (communicating parties) coupled to a one-dimensional quantum channel consisting altogether of spin-1/2 chain with open boundaries featuring XX-type exchange interactions described by Hamiltonian $\hat{H} = \hat{H}_{\text{ch}} + \hat{H}_{\text{int}}$ with ($\hbar = 1$)

$$\hat{H}_{\text{ch}} = \sum_{i=1}^{N} \frac{\omega_i}{2}(\hat{1} - \hat{\sigma}_i^z) - \sum_{\langle i,j \rangle} \frac{J_{i,j}}{2}(\hat{\sigma}_i^x\hat{\sigma}_j^x + \hat{\sigma}_i^y\hat{\sigma}_j^y), \quad (1)$$

where $\hat{\sigma}_i^{x,y,z}$ are the Pauli operators for the i-th spin, ω_i is the local (on-site) magnetic field, and $J_{i,j}$ is the exchange coupling strength between nearest-neighbor sites. Supposing the sender (s) and receiver (r) spins are connected to sites 1 and N from the channel at rates g_s and g_r, respectively, the interaction part reads

$$\hat{H}_{\text{int}} = \frac{\omega_s}{2}(\hat{1} - \hat{\sigma}_s^z) + \frac{\omega_r}{2}(\hat{1} - \hat{\sigma}_r^z) - \frac{g_s}{2}(\hat{\sigma}_s^x\hat{\sigma}_1^x + \hat{\sigma}_s^y\hat{\sigma}_1^y) - \frac{g_r}{2}(\hat{\sigma}_r^x\hat{\sigma}_N^x + \hat{\sigma}_r^y\hat{\sigma}_N^y). \quad (2)$$

Note that since $\hat{H}$ conserves the total magnetization of the system, i.e., $\left[\hat{H}, \sum_i \hat{\sigma}_i^z\right] = 0$, the Hamiltonian can be split into independent subspaces with fixed number of excitations. Here we focus on the single-excitation Hilbert space spanned by the computational basis $|i\rangle = \hat{\sigma}_i^+|\downarrow\downarrow \ldots \downarrow\rangle$ with $i = r, s, 1, \ldots, N$, that means every spin pointing down but the one located at the i-th position. In this case, we end up with a hopping-like matrix with $N+2$ dimensions.

Let us now make a few assumptions in regard to the channel described by Hamiltonian (1). Here we consider the spin-exchange coupling strengths to be uniform $J_{i,j} \to J$ and, in order to study the robustness of the channel against disorder we introduce correlated static fluctuations on the on-site magnetic field ω_n, $n = 1, \ldots, N$. A straightforward way to generate random sequences featuring internal long-range correlations is through the trace of the fractional Brownian motion with power-law spectrum $S(k) \propto 1/k^{\alpha}$ [54,59]

$$\omega_n = J \sum_{k=1}^{N/2} k^{-\alpha/2} \cos\left(\frac{2\pi nk}{N} + \phi_k\right), \quad (3)$$

where $k = 1/\lambda$, is the inverse modulation wavelength, $\{\phi_k\}$ are random phases distributed uniformly within $[0, 2\pi]$, and α controls the degree of correlations. This parameter is related to the so-called Hurst exponent [65], $H = (\alpha - 1)/2$, which characterizes the self-similar character of a given sequence. When $\alpha = 0$, we recover the case of uncorrelated disorder (white noise) and for $\alpha > 0$ underlying long-range correlations take place. The resulting long-range correlated sequence becomes nonstationary for $\alpha > 1$. Furthermore, according to the usual terminology, when $\alpha > 2$ ($\alpha < 2$) the series increments become persistent (anti-persistent). Interestingly, this brings about serious consequences on the spectrum profile of the system. As shown in [54,59], when $\alpha > 2$ there occurs the appearance of delocalized states in the middle of the one-particle spectrum band. In the QST scenario with weakly-coupled spins r and s, i.e. $g_s, g_r \ll J$, that promotes a strong enhancement in the likelihood of disorder realizations with very-high fidelities F, most of them yielding $F \approx 1$. This will be elucidated along the paper.

Hereafter we set the sequence generated by Eq. (3) to follow a normalized distribution, that is $\omega_n \to (\omega_n - \langle\omega_n\rangle)/\sqrt{\langle\omega_n^2\rangle - \langle\omega_n\rangle^2}$. We also stress that such a disordered distribution has no typical length scale which is a property of many natural stochastic series [66].

3. Effective two-site description

We now work out a perturbative approach to write down a proper representation of an effective Hamiltonian involving only the sender and receiver spins provided they are very weakly coupled to the channel. Intuitively, we expect they span their own subspace with renormalized parameters and thus QST takes place via effective Rabi oscillations between them [7,19]. Our goal here is to investigate the influence of disorder in such subspaces and evaluate their resilience to imperfections in the channel.

Following a second-order perturbation approach (for details, see Refs. [7,8] or Supplementary Material), we can obtain an effective Hamiltonian projected onto $\{|s\rangle, |r\rangle\}$ which reads

$$\hat{H}_{sr} = \begin{pmatrix} h_s & -J' \\ -J' & h_r \end{pmatrix}, \quad (4)$$

with

$$h_\nu = \omega_\nu - \epsilon^2 g_\nu^2 \sum_k \frac{|a_{\nu k}|^2}{E_k - \omega_\nu}, \quad (5)$$

$\nu \in \{s, r\}$, and

$$J' = \frac{\epsilon^2 g_s g_r}{2} \sum_k \left(\frac{a_{sk}a_{rk}}{E_k - \omega_s} + \frac{a_{sk}a_{rk}}{E_k - \omega_r}\right), \quad (6)$$

where ϵ being the perturbation parameter, $a_{sk} \equiv \langle 1|E_k\rangle$, $a_{rk} \equiv \langle N|E_k\rangle$, and $\{|E_k\rangle\}$ are the eigenstates of the channel [Eq. (1)] with corresponding (nondegenerate) frequencies $\{E_k\}$. Note that we are assuming all parameters to be real.

Hamiltonian (4) describes a two-level system which performs Rabi-like oscillations in a time scale set by the inverse of the gap between its normal frequencies. In order to have as perfect as possible QST one should guarantee that $h_s = h_r$. This is automatically fulfilled, given $\omega_s = \omega_r$ and $g_s = g_r = g$, for mirror-symmetric chains since $|a_{sk}| = |a_{rk}|$ for every k. In that case, for a noiseless uniform channel and in the limit of very weak outer couplings, which implies in the validity of Hamiltonian (4), an initial state prepared in $|s\rangle$ will evolve in time to $|r\rangle$ with nearly unit amplitude at times $\tau = n\pi/(2J') = n\pi J/(2\epsilon^2 g^2)$, with n being an odd integer [6,7]. Note that as N increases more eigenstates get in the middle of the spectrum and thus ϵg_ν must be adjusted accordingly (we shall drop out the perturbation parameter ϵ hereafter).

4. Quantum-state transfer protocol

4.1. General scheme

In the standard QST procedure [1], Alice is able to control the spin located at position s and wants to send an arbitrary qubit $|\phi\rangle_s = \alpha|\downarrow\rangle_s + \beta|\uparrow\rangle_s$ to Bob which has access to spin r. Now let us assume that the rest of the chain is initialized in the fully polarized spin-down state so that the whole state reads