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Analysis, calculation and utilization of the *k*-balance attribute in interdependent networks

Zheng Liu [a], Qing Li [b], Dan Wang [c], Mingwei Xu [a,*]

[a] *Department of Computer Science and Technology, Tsinghua University, Beijing 100084, China*
[b] *Graduate School at Shenzhen, Tsinghua University, Shenzhen 518055, China*
[c] *Department of Computing, The Hong Kong Polytechnic University, Hong Kong*

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ABSTRACT

Interdependent networks, where two networks depend on each other, are becoming more and more significant in modern systems. From previous work, it can be concluded that interdependent networks are more vulnerable than a single network. The robustness in interdependent networks deserves special attention. In this paper, we propose a metric of robustness from a new perspective—the balance. First, we define the balance-coefficient of the interdependent system. Based on precise analysis and derivation, we prove some significant theories and provide an efficient algorithm to compute the balance-coefficient. Finally, we propose an optimal solution to reduce the balance-coefficient to enhance the robustness of the given system. Comprehensive experiments confirm the efficiency of our algorithms.

1. Introduction

Nowadays, the phenomenon that networks in nature display interdependency on each other is more and more common. The interdependency between two or more networks means that some losses of functionality in one network can exert effects on the others [1] [2] [3]. A particular example is the interdependency between the power grid and communication networks: a lack of power supply weakens the functionality of communication networks, which in turn may affect the power supply of the grid because of the losses of some control messages [4] [5]. Besides, examples can be found in several other domains: social networks (for examples, Facebook, Twitter) are interactional because they share the same users, some changes in the relational graph of Facebook can make the relational graph of Twitter change; multimodal transportation networks are composed of different layers (for examples, airplane, train) that share the same locations [6].

Robustness is one of the most required properties of a network. In interdependent networks, robustness deserves more attention since interdependent networks are more vulnerable than a single network [7] [8] [9] [10]. The failure of elements in one network generally causes the failure of dependent elements in the other network, and the new failure in turn influences the former network [11]. The failure process can cascade several times between two networks and result in catastrophic consequences. That is, small perturbations in one network can be amplified by the interaction between networks. The 2003 catastrophic blackout affecting 50 million people in Northeast America, is exactly the result of cascading failures, caused by the interdependency between the power grid and communication networks [12].

In 2010, Buldyrev et al. propose a model and an analytical framework to study the robustness of interdependent networks [7]. They investigate the remaining nodes after cascading failures caused by some initial invalid nodes, by utilizing techniques from the percolation theory. However, the theoretical approach is of little help in practical contexts since it applies only to the case of infinitely large system. In [12] [13] [14], the authors study the minimum number of node failures needed to cause the total failure of the power grid and communication networks. In [15] [16], the authors provide techniques to identify the κ most vulnerable nodes of interdependent networks using a new model. However, they study robustness from the perspective of the entire system instead of either network respectively. We use lethality to define the ability of either network to damage the entire system. The difference between lethality degree of two networks plays a vital role and it is crucial to robustness.

A new metric of robustness from the perspective of balance in interdependent networks is proposed. Given an interdependent system consisting of two networks, either network has its own

* Corresponding author.
E-mail address: xmw@cernet.edu.cn (M. Xu).

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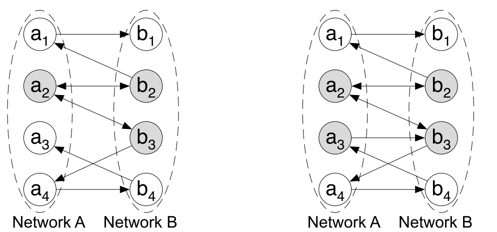

(a) Before adding $a_3 \to b_3$ (b) After adding $a_3 \to b_3$

Fig. 1. A cyber-physical system consisting of two interdependent networks.

lethality to the entire system since an initial failure (from natural disaster or malicious attack) in either network can possibly cause the entire system to fail ultimately. In practice, making the same degree of damage on the two networks respectively can evolve different consequences. In other words, when one network's lethality to the system is higher than the other's, attacking the network with higher lethality can get more benefits easily than the other in terms of attackers. According to the Cannikin Law, the robustness of the system depends on the network with the higher lethality to the system. Under the condition of limited overhead, reducing its lethality to balance the system can enhance the robustness. In order to achieve this goal, several links connecting the two networks can be added.

An Example. Using a concrete example in Fig. 1, we illustrate that balancing an interdependent system can enhance its robustness under the condition of limited overhead. Considering a cyber-physical system consisting of two interdependent networks each with 4 nodes, i.e., network A and network B. The dependence edges connecting nodes in different networks are denoted by directed edges. For example, as shown in Fig. 1(a), the edge with a one-way arrow $a_1 \to b_1$ means the functioning of node b_1 depends on node a_1, and b_1 will lose efficacy due to the failure of a_1. Similarly, the edge with a two-way arrow $a_2 \leftrightarrow b_2$ means the functioning of the two nodes depend on each other. a_2 will fail after b_2 lose efficacy, and b_2 will fail after a_2 lose efficacy as well. What calls for special attention is that the functioning of a node in one network can depend on the ability of more than one node in the other network. In this case, the node can function if at least one of its supporters still work. For example, as shown in Fig. 1(a), the node a_2 depends on both b_2 and b_3, so a_2 will lose efficacy only if both b_2 and b_3 lose efficacy. Note that only interdependency links between two networks are considered in our assumption. The reason will be provided in Section 2.1.1. Due to cascading failure, the result can be drawn that the initial failure of $\{a_2\}$ in network A will cause all nodes to fail by the cascading failure process, while the initial failure of $\{b_2, b_3\}$ will cause all nodes to fail. Thus, the lethality of network A to the whole system is higher than the lethality of network B. We evaluate the system robustness in the case of random node failure. Suppose 3 random nodes losing efficacy as initial failure in two networks respectively. Consequently, the probability that the whole system ceases to function for network A is 75%, and 50% for network B. To reduce the lethality of network A, an edge $a_3 \to b_3$ is added into the system in Fig. 1(b). As a consequence, removal of two nodes is necessary for the complete fragmentation for either network, and the probabilities of complete fragmentation caused by initial failure of 3 random nodes become the same, i.e., 50%. We can draw the conclusion that the robustness of the interdependent system is improved by reducing the lethality of network A to balance the system at the cost of adding only one edge.

In this paper, we define balance-coefficient of the interdependent system as a metric of balance. To the best of our knowledge, there is no prior work attempting to model the interdependency based on balancing interdependent networks. Given two interdependent networks, we can balance the system by reducing the balance-coefficient to zero to enhance its robustness. Firstly, we prove some significant theories and provide an efficient algorithm to compute the balance-coefficient. Further, we provide an algorithm to optimize the balance-coefficient by adding some dependence links to enhance the robustness of the system. We evaluate the impact of balance-coefficient and the performance of our algorithm by comprehensive simulations. The results show that balance-coefficient is an effective metric to robustness, and the error rate of our algorithm is quite low.

The contributions of this paper are summarized as follows:

- The balance attribute is proposed to evaluate the robustness from the aspect of difference between two interdependent networks' lethality in an interdependent system.
- Several theorems to decide two interdependent networks are balanced or not are proven and an algorithm is provided to compute the balance-coefficient as a metric of balance attribute.
- The balance-coefficient is optimized to balance interdependent networks and enhance the robustness of the system by adding dependence links between two networks.

The rest of this paper is organized as follows. Section 2 introduces the basic model and the problem formulation. In Section 3, we classify all the interdependent systems into four categories and solve the decision problem of balance attribute. In Section 4, the problem of computing the balance-coefficient is formulated and proved to be NP-complete. In Section 5, the balance-coefficient is optimized by adding some specific links. We conduct abundant experiments and show the experiment results in Section 6 and introduce related works in Section 7. Finally, this paper concludes in Section 8.

2. Balance in interdependent networks

2.1. *Interdependency model*

2.1.1. *Graph model*

Considering an interdependent system consisting of two interacting networks, i.e., network A and network B. We use graph $G = (V^I, E^I)$ to formulate either network. For simplicity and without loss of generality, both networks are assumed to have N nodes. In our work, we ignore the edges connecting nodes within both networks and focus on the *dependence edges* connecting two networks, i.e., set $E^I = \phi$. One node can function *iff* it depends on at least one functioning node in the other network. Given G_A and G_B, an interdependent system is defined as $G = (U, V, E)$. U and V represent the two sets of nodes in G_A and G_B respectively, and E is the set of dependence edges. The dependence edges are directional, such as the edge $u_i \to v_j$, meaning v_j is supported by u_i. The model of interdependent networks has a bipartite topology. We use $|U|$, $|V|$ and $|E|$ to denote the number of nodes in two networks and the number of dependence edges, respectively.

The assumptions in our interdependency model are summarized as follows:

- For simplicity and without loss of generality, both interdependent networks have N nodes, i.e., $|U| = |V| = N$.
- The internal edges within singles network are ignored since they do not affect interdependency relationship, i.e., $E^I = \phi$.
- A node in one network can function if it depends on at least one functioning node in the other network.

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