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Experimental verification of a new Bell-type inequality

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A B S T R A C T

Arpan Das et al. proposed a set of new Bell inequalities (Das et al., 2017 [16]) for a three-qubit system and claimed that each inequality within this set is violated by all generalized Greenberger–Horne–Zeilinger (GGHZ) states. We investigate experimentally the new inequalities in the three-photon GGHZ class states. Since the inequalities are symmetric under the identical particles system, we chose one Bell-type inequality from the set arbitrarily. The experimental data well verified the theoretical prediction. Moreover, the experimental results show that the amount of violation of the new Bell inequality against locality realism increases monotonically following the increase of the tangle of the GGHZ state. The most profound physical essence revealed by the results is that the nonlocality of GGHZ state correlate with three tangles directly.

1. Introduction

Quantum entanglement and nonlocality are the two most controversial characteristics of quantum theory [1,2]. From the beginning of quantum mechanics, there was a constant debate. Einstein especially was strongly opposed to existence of the entanglement and nonlocality. Einstein–Podolsky–Rosen (EPR) proposed a hypothetical experiment to against the completeness of quantum mechanics [3]. They believed that the reason some observations are not accurately predicted in quantum mechanics is the non-deterministic value of this implicit variable. These theories of introducing new variables or complementary conditions, as required, are the theory of implicit variables. Generally, if a system cannot be described as a simple product of its constituent subsystems, we call it an entangled system [4]. In 1964, Bell published an article and proposed a famous measurable Bell inequality [5]. It is not only proven theoretically that the theory of hidden variables and quantum mechanics are contradictory, but it also gives a physical method for experimental measurement of nonlocality. At the same time, the completeness of quantum mechanics and the quantum entanglement theory are well proven. In order to simplify the experimental measurement complexity of Bell type inequality, Clauser–Horne–Shimony–Holt (CHSH) advanced an optimizational inequality based on the two-particle system [6]. The CHSH inequality plays an important role in proving the theory of quantum entanglement and the local hidden variable (LHV) model [7]. According to the quantum theory, the nonlocal nature appears clearly in the case that entangled states allow for violation of Bell-type inequalities. Furthermore, for multi-particle systems, several inequalities have been proposed to characterize the nonlocality of the systems, such as the Svetlichny inequality [8]. Because the novel theoretical and experimental applications of multiqubit entanglement and nonlocality in quantum communication and quantum computation [9,10]. These inequalities against the local realistic model and the hidden variable theory are important, not only on theoretical research, but also on engineering applications for quantum information processing, such as estimating the security of quantum key distribution [11,12] and improving the fidelity of quantum teleportation [13]. Nonlocality of three or more photons may also play an important role in property description in many-body systems [14,15]. Arpan D. introduced a set of Bell inequalities for a three-qubit system [16] and proved that the more entangled a generalized GHZ state is, the greater the violation will be. So we can accurately describe the relation between nonlocality and entanglement for this three-qubit system.

In this letter, we have prepared GGHZ state experimentally using parametric down conversion of nonlinear crystals and characterized the state by extracting its density matrix by the quantum state tomography [17]. The fidelity of generalized GHZ states are about 0.84. Based on the generalized GHZ state, we tested the new inequality in Ref. [16]. A systematical investigation on tripartite entanglement and nonlocality has been carried out. The results show that the experimental measurements and theoretical predictions are in agreement, and they also show the concordant relationship between tripartite entanglement and tripartite nonlocality in the generalized GHZ states.

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2. Theoretical description of new Bell inequality

For the two qubits entangled state with spin-1/2 particles, assuming that Alice and Bob share the two particles. Alice can measure the particle with the operator A and A', and Bob measure the particle with the operator B and B'. The eigenvalue of the operator A and B is α and β respectively. The joint probability $p(A, B)$ can be obtained when Alice and Bob measure two-photon of the entangled system at the same time.

$$p(A, B) = \sum p_\lambda p(A, \lambda) p(B, \lambda) \tag{1}$$

For the prepared two-photon states ρ, the probability of joint operation $\langle AB\rangle$, $\langle AB'\rangle$, $\langle A'B\rangle$, and $\langle A'B'\rangle$ can be measured separately. According to the result of local realism, the Bell inequality satisfies the following relation

$$\begin{aligned} S = {} & p(\alpha, \beta|A, B) + p(\alpha', \beta|A', B) + p(\alpha, \beta'|A, B') \\ & - p(\alpha', \beta'|A', B') \le 2 \end{aligned} \tag{2}$$

On the other hand, in accordance with the theory of quantum mechanics, the particles measured jointly by Alice and Bob have non-local correlation [5,6,12]. The maximum allowed expectation value of the Bell operator for this two-photon system is $2\sqrt{2}$. The measurement results between $2 \sim 2\sqrt{2}$ are the violation of quantum nonlocality against local realism. However, it is important to point out that the violation of a Bell inequality is only sufficient criteria for certifying entanglement, but not a necessary one.

Similarly, the three-qubit states, Alice, Bob and Charlie share the three particles. They can operate three space-separated observers A, B and C. The joint probability can be written as follows

$$p(A, B, C) = \sum p_\lambda p(A, \lambda) p(B, \lambda) p(C, \gamma) \tag{3}$$

In the three qubits spin-1/2 system, the Bell inequality satisfies the following relation:

$$\begin{aligned} S = {} & p(\alpha, \beta, \gamma|A, B, C) + p(\alpha', \beta, \gamma|A', B, C) \\ & + p(\alpha, \beta'\gamma|A, B', C) - p(\alpha', \beta', \gamma|A', B')C \\ & + p(\alpha, \beta, \gamma'|A, B, C') - p(\alpha', \beta, \gamma'|A', B, C') \\ & - p(\alpha, \beta'\gamma'|A, B', C') - p(\alpha', \beta', \gamma'|A', B')C' \le 2 \end{aligned} \tag{4}$$

Arpan D. introduced a set of Bell inequalities [16]. Two of the parties will make two measurements, while the third party will make only one measurement. This third party can be either of Alice, Bob, and Charlie. Since the inequalities are symmetric under the identical particles, we can choose one Bell inequality from the set arbitrarily. We chose the new inequality

$$S_A = A_1(B_1 + B_2) + A_2(B_1 - B_2)C_1 \le 2 \tag{5}$$

where, $A_1 = \sigma_z$, $A_2 = \sigma_x$, $B_1 = cos\theta\sigma_x + sin\theta\sigma_z$, $B_2 = -cos\theta\sigma_x + sin\theta\sigma_z$, and $C = \sigma_x$. Similar to the Bell inequality, the maximum value of the new Bell operator S_A is $2\sqrt{2}$ in the framework of quantum theory. Let's consider the three-qubit generalized GHZ state

$$|\psi\rangle_{GGHZ} = sin\theta|000\rangle + cos\theta|111\rangle \tag{6}$$

The upper bound of S_A on the expectation value can be written as

$$\begin{aligned} S_A &= \langle\psi|_{GGHZ} A_1(B_1 + B_2) + A_2(B_1 - B_2)C_1|\psi\rangle_{GGHZ} \\ &= 2\sqrt{1 + 4sin^2\theta + 4cos^2\theta} \end{aligned} \tag{7}$$

We can plot the relationship between S_A and θ shown as the solid line in Fig. 3. Because a CHSH type operator for second and third qubits is embedded in this operator, the amount of violation will be exactly same as in the case of twoqubit entangled state and the CHSH operator. Not only this inequality, but there is another inequality within this set, which will also be violated in this case. Also, as all the two operators have the same form (the CHSH form) in second and third particle, the amount of maximal violations will be same in two cases. And the last important fact is that, no other states (except biseparable pure states) will have same kind of violations.

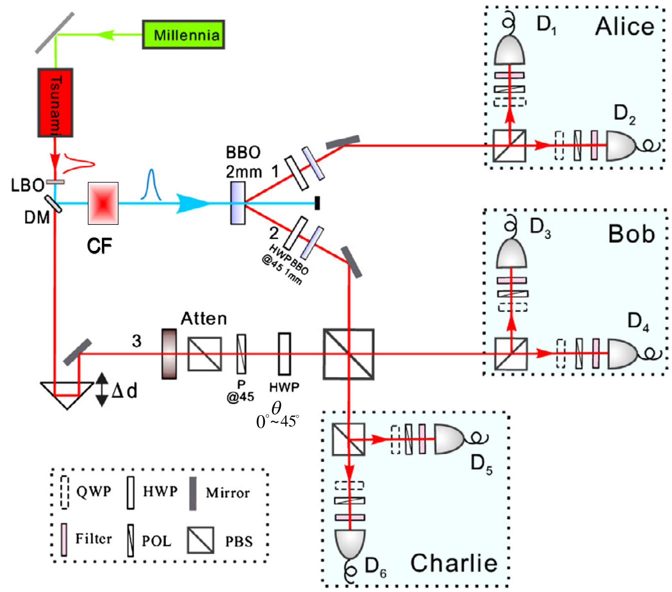

Fig. 1. (Color online.) Scheme of the experimental setup. CF is a combined filter used to filter the mixed IR. High-precision small-angle prisms are inserted for fine adjustments of the relative delay Δd of the two different paths. The angle θ can be adjusted by HWP. To achieve good temporal and spatial overlap at PBS_{12}, every output is spectrally filtered ($\Delta_{FWHM} = 3$ nm) and monitored by fiber coupled single-photon detectors. Throughout the experiment, the coincidence time-window is set to be 5 ns, which ensures that accidental coincidence is negligible.

3. Preparation and characterization of GGHZ states

The experimental set-up is shown as Fig. 1. The first step of the experiment is the preparation of the polarization entangled three-qubit GHZ state. A continuous laser of 532 nm is produced by a semiconductor laser (Millennia, Spectra-Physics). Continuous laser pumped Ti: sapphire mode locked femtosecond laser. The femtosecond laser generated an infrared (IR) pulse laser with a central wavelength of 780 nm, pulse a width of 100 nm and a repetition rate of 78 MHz. The infrared laser pulse focused on the LiB_3O_5 (LBO) crystal at a certain angle. The 780 nm IR pulse is converted to 390 nm near ultraviolet (UV) pulse in LBO crystal by parametric up-conversion. Then the UV light pulse passes through a combined filter (CF) which is used to filter the mixed IR. Behind the CF, the UV pulse is focused on a β-barium borate (BBO) crystal. When the phase is a good match, type II parametric down-conversion will occur with a certain probability. The polarization of the generated two photons is in an entangled state $(|H_1H_2\rangle + |V_1V_2\rangle)/\sqrt{2}$. $|H\rangle$, which indicates that the photon is horizontally polarized and $|V\rangle$ indicates that the photon is vertically polarized. While the transmitted IR light from dichroic beamsplitters (DM) is attenuated to a weak pseudo-single-photon source by a combined attenuator. The weak light source in path 3 is further prepared in the state $(|H_3\rangle + |V_3\rangle)/\sqrt{2}$ by the polarizer (P) with 45°. Then, photon 2 is superposed with photon 3 in a polarizing beam splitter (PBS). By finely adjusting the delay Δd between paths 2 and 3 to make sure photon 2 and photon 3 arrive at the PBS simultaneously, the three-