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Nonlinear stochastic interacting dynamics and complexity of financial gasket fractal-like lattice percolation

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ABSTRACT

A novel nonlinear stochastic interacting price dynamics is proposed and investigated by the bond percolation on Sierpinski gasket fractal-like lattice, aim to make a new approach to reproduce and study the complexity dynamics of real security markets. Fractal-like lattices correspond to finite graphs with vertices and edges, which are similar to fractals, and Sierpinski gasket is a well-known example of fractals. Fractional ordinal array entropy and fractional ordinal array complexity are introduced to analyze the complexity behaviors of financial signals. To deeper comprehend the fluctuation characteristics of the stochastic price evolution, the complexity analysis of random logarithmic returns and volatility are preformed, including power-law distribution, fractional sample entropy and fractional ordinal array complexity. For further verifying the rationality and validity of the developed stochastic price evolution, the actual security market dataset are also studied with the same statistical methods for comparison. The empirical results show that this stochastic price dynamics can reconstruct complexity behaviors of the actual security markets to some extent.

1. Introduction

It is well known that the security market is a representative nonlinear complex system, and its fluctuations often exhibit strong nonlinear characteristics, nonstationary features and complexity behaviors [1–9]. Recently, modeling and investigating nonlinear behaviors of financial signals have been a central topic for a further understanding the mechanisms of market dynamics due to its vital roles in theoretical research and practical application in financial fields, such as risk management, derivatives pricing, hedging, physical assets valuation, forecasting [2,9–13]. In recent years, a wealth of stochastic agent-based financial price models have been introduced, based on statistical physics models [14,15], to reproduce the stylized facts observed in a great number of empirical works, for example, fat-tailed distribution, power-law, correlation, scaling, volatility clustering, multifractality and complexity [9,16–27]. For instance, the percolation system and oriented bond percolation are employed to model financial market price evolution, where the flowing of fluids through porous materials is utilized to imitate the local financial attitude' dissemination, and the traders sharing the same investment attitudes or trading strategy toward the security market are defined by a group of occupied adjacent lattices [17,18]. A financial price evolving model is introduced by the bond percolation on Sierpinski carpet fractal-like lattice to study the fluctuation dynamics of financial signals, where the spread of investment strategies or trading attitudes in the security market is mimicked by the bond percolation system [19]. A stochastic financial price evolving process is presented by the exclusion system to survey the complexity dynamics of actual financial market price returns, where the trading attitudes interaction among investment agents in the market is mimicked by the stochastic exclusion interacting laws [21]. A large number of nonlinear analyses based on permutation entropy (PermEn) and sample entropy (SampleEn) are proposed to measure the complexity behaviors of time series in various fields [28–39]. Fractional sample entropy (FSE) [31] developed from sample entropy method to explore fractional order dynamics in a system, is applied to discuss the complexity dynamics of price return series in the paper.

In the present paper, inspired by the work [17–19], according to the bond percolation on Sierpinski gasket fractal-like lattice, we present a new microscope financial price dynamics to reconstruct and study the price evolution of stock markets. Fractional ordinal array entropy (FOAE) and fractional ordinal array complexity (FOAC) are developed based on Refs. [31,32] to study the complexity behaviors of the returns. Statistical characters, power-law distributions, fractional sample entropy (FSE), FOAE and FOAC are utilized to study the fluctuation properties, the fat-tailed phenomenon and the complexity dynamics of stock price logarithmic

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return series and volatility (means absolute price logarithmic return in the paper) series. Moreover, through the comparative analyses between the real security market dataset (consists of Shanghai Stock Exchange (SSE) Composite Index and Standard & Poor's 500 Index (S&P500)) and the simulation dataset with different parameters, the empirical research confirms that the introduced nonlinear stochastic price evolving model can reconstruct primary complexity dynamics of actual price logarithmic returns to some extent.

2. Description of interacting price dynamics model

2.1. Description of bond percolation on gasket fractal-like lattice

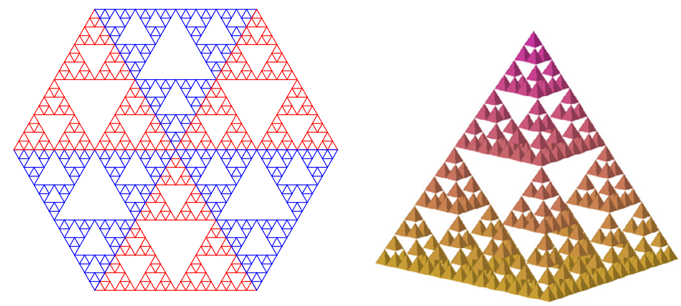
Sierpinski gasket and Sierpinski carpet on $\mathbb{Z}^d$ are two kinds of most well-known fractals which have self-similarity [40,41]. The former is a finitely ramified fractal, while the latter is an infinitely ramified fractal (i.e., by removing a finite number of vertices and edges the former can be disconnected but the latter is still connected). A fractal-like lattice (or a lattice fractal) is a graph with edges and edges which corresponds to a fractal, and all of lattice fractals have self-similarity, while some of them have no properties such as translation invariance, for details see Refs. [42–45]. For example, Chen [42] shows that in any dimension ($d \geq 2$) the Ising model on Sierpinski carpet lattice exhibits the phase transition, but on Sierpinski gasket lattice has a completely opposite conclusion (for the nature of the finitely ramified fractal). Shinoda [44,45] also establish the almost same conclusions about phase transitions property for the bond percolation system on Sierpinski carpet and Sierpinski gasket lattice. Fractals have strong link to financial markets and mechanical properties of percolating [41,43].

The Sierpinski gasket is also referred to as the Sierpinski triangle or the Sierpinski sieve. The Sierpinski gasket may be constructed from an equilateral triangle by recursively removing triangular subsets, i.e., starting with an equilateral triangle, equally dividing it into four smaller congruent equilateral triangles by connecting three midpoints of the three edges and remove the central one (but remain the corresponding edges), then infinitely repeating the same procedure with each of remaining smaller triangles. In this work, we consider the Sierpinski gasket lattice fractal which corresponds to the Sierpinski gasket. A financial price evolution process based on the percolation on Sierpinski gasket lattice is established, only the vertices and edges are required in the construction. We now simply describe two-dimensional Sierpinski gasket lattice, for details see [40–42,44]. Suppose that there is an equilateral triangle on $\mathbb{Z}^2$ with three vertices $O=(0,0)$, $a_0=(1/2,\sqrt{3}/2)$ and $b_0=(1,0)$, F_0 be the graph made up of the three vertices mentioned above and three edges connect the three vertices with length 1. Let $\{F_n,\ n=0,1,\cdots\}$ be the sequence of graphs given by iteration as

$$F_{n+1}=F_n\cup(F_n+2^n a_0)\cup(F_n+2^n b_0) \tag{1}$$

where n is the recursive number, $A+a=\{x+a|\ x\in A\}$ and $kA=\{kx|\ x\in A\}$. Let V_n be the set of vertices, E_n be the set of edges with length 1 in graph F_n. Then the Sierpinski gasket fractal-like lattice is defined as $F=\cup_{n=0}^{\infty}F_n$, and the corresponding Sierpinski gasket is $\widetilde{F}=\overline{\cup_{n=0}^{\infty}2^{-n}F}$. Let V be the vertex set of F, and E be the corresponding edge set. Fig. 1 depicts two diagrams of two- and three-dimensional Sierpinski gasket lattice fractals, respectively. Fig. 1(a) shows six two-dimensional Sierpinski gasket lattices put together in a hexagonal shape. Fig. 1(b) displays three-dimensional Sierpinski gasket lattice.

We combine the Bernoulli bond percolation with the bounded graph F_n. Each edge in E_n is open with probability p $(0\leq p\leq 1)$ and closed with probability $1-p$ respectively, and all edges are



(a) (b)

Fig. 1. Sierpinski gasket lattice fractal: (a) Sierpinski gasket lattices put together in a hexagonal shape on $\mathbb{Z}^2$. (b) Three-dimensional Sierpinski gasket lattice.

open or close independently. Let P_p represent its joint probability distribution, and we consider investors can consult their trading attitudes along the open bond. Denote $x\leftrightarrow y$ if there is an open path (a cluster of connected open edges) from vertex x to vertex y. Let the random open cluster containing vertex x be $C(x)=\{y\in V,\ \ x\leftrightarrow y\}$, and $C(0)$ be the random open cluster containing the origin. The percolation probability is denoted as $\theta(p)=P_p(|C(0)|=\infty)$, where $|C|$ represents the cardinal number of the set C, and the critical probability is $p_c=\inf\{p:\ \theta(p)>0\}$. Then $p_c=1$ for Sierpinski gasket lattice because it is finitely ramified.

2.2. Formation of financial price dynamics

In the section, a nonlinear financial price evolving model is constructed by the bond percolation on two-dimensional Sierpinski gasket lattice. Let six Sierpinski gasket lattice F_n put together in a hexagonal shape (as shown in Fig. 1(a)) denoted as G_n, and $C_t(0)$ be a stochastic open cluster containing the origin in G_n. We construct a financial price evolving model of auctions for a stock in a security market. Suppose there is an investment agent at every vertex in G_n, and each investment agent can trade the stock several times at each trading day $t\in\{1,2,\cdots,T\}$, but at most one at each time. At the beginning in each trading day, we assume that only the investment agent at the origin can obtain some random investment attitude according to economic environment. Let a random variable ξ_t represent the random trading attitude, such that the investment agent takes buying ($\xi_t=1$), selling ($\xi_t=-1$), or neutral position ($\xi_t=0$) with probability q_1, q_{-1} or $1-q_1-q_{-1}$ $(0\leq q_1,\ \ q_{-1},\ \ q_1+q_{-1}\leq 1)$, respectively. Then this investment agent send bullish, bearish or neutral trading position to its nearest investors, and the sending process keeps going by its neighbors. Based on the bond percolation on G_n, investment agents can consult their nearest agents mutually or the investment positions can be disseminated, which is considered to be the main incentive of price fluctuations. For a fixed $t\in\{1,2,\cdots,T\}$, the aggregate demand is defined as

$$\mathcal{B}_t=\frac{\xi_t\cdot|C_t(0)|}{|G_n|} \tag{2}$$

where $|A|$ represents the cardinal number of set A. Based on the above several definitions and Refs. [9,16,17], we define the stock price returns proportional to the aggregate demands $\mathcal{B}_t$ as

$$r_t=\beta_t\,\mathcal{B}_t \tag{3}$$

where β_t (>0) denotes the market depth parameter which measures the price fluctuation's sensitivity to the aggregate demand. Let $\mathcal{P}_t$ be the closing price of t-th trading day, and hence the stock