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Energetic and dynamical instability of spin–orbit coupled Bose–Einstein condensate in a deep optical lattice

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ABSTRACT

We investigate the energetic and dynamical instability of spin–orbit coupled Bose–Einstein condensate in a deep optical lattice via a tight-binding model. The stability phase diagram is completely revealed in full parameter space, while the dependence of superfluidity on the dispersion relation is illustrated explicitly. In the absence of spin–orbit coupling, the superfluidity only exists in the center of the Brillouin zone. However, the combination of spin–orbit coupling, Zeeman field, nonlinearity and optical lattice potential can modify the dispersion relation of the system, and change the position of Brillouin zone for generating the superfluidity. Thus, the superfluidity can appear in either the center or the other position of the Brillouin zone. Namely, in the center of the Brillouin zone, the system is either superfluid or Landau unstable, which depends on the momentum of the lowest energy. Therefore, the superfluidity can occur at optional position of the Brillouin zone by elaborating spin–orbit coupling, Zeeman splitting, nonlinearity and optical lattice potential. For the linear case, the system is always dynamically stable, however, the nonlinearity can induce the dynamical instability, and also expand the superfluid region. These predicted results can provide a theoretical evidence for exploring the superfluidity of the system experimentally.

1. Introduction

The optical lattice created by interference of two laser beams has provided an ideal platform for the manipulation of quantum atomic gases owing to its high controllability [1–3]. It has induced to an explosion of research both in theories [4–11] and experiments [12–14], where some interesting phenomena have been discovered, such as Bloch oscillations [6], Landau–Zener tunneling [7,8], Josephson effect [12], energetic and dynamical instability [9,13], and superfluidity [10,11,14], etc. The Bloch state and the corresponding quasienergy have provided the basic concept and language for understanding the periodic systems [15,16]. In the optical lattice, the superfluidity of Bose–Einstein condensate (BEC) can be regarded as a Bloch wave [17,18]. As well known, the energetic criterion [19] results in a critical speed, beyond which the superfluidity becomes unstable against perturbation, then the Landau instability occurs. In a linear system, the BEC in the optical lattice is always dynamically stable, nevertheless the nonlinearity can generate the dynamical instability, and also expand the superfluid region. The nonlinearity can also modify the dispersion relation, and induce a loop structure in the energy band [17]. The investigation of nonlinear phenomena [20,21] in BEC has attracted more and more attentions in theory and experiment, which is also closely related to the stability of system.

Recently, the realization of BEC with spin–orbit coupling (SOC) in the optical lattice [22] has opened a completely new avenue for exploring superfluidity, and motivated the interesting of investigating the physics of the SOC in the optical lattice [23–26]. In absence of SOC, there is a minimum of the lowest energy band at the center of the Brillouin zone and a maximum at the edge [17,18]. However, the SOC can modify the dispersion relation, and generate a center peak of the lowest energy band at the center of the Brillouin zone, thus an isolated flat band can be presented in a certain parameter region [27], which has been observed experimentally [28]. The combination of SOC and optical lattice has also resulted in the predictions and discoveries of topological insulators, superfluidity and dynamical instability. However, the exploring for the energetic and dynamical instability of BEC with the combination of SOC and nonlinearity in a deep optical lattice is still missing. The tight-binding model [29,30] including the SOC and nonlinearity has made it possible to explore the physics of SOC in the deep optical lattice.

In this paper, the dependence of energetic and dynamical instability on the combination of SOC, Zeeman splitting and nonlinear-

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ity is revealed in a deep optical lattice, while the complete stability phase diagram is presented in full parameter space. Furthermore, the mechanism for generating the superfluidity is explicitly illustrated, which depends on the dispersion relation of the system. It is well known that the superfluidity occurs when the momentum of condensate is not beyond the critical value, i.e., the center of the Brillouin zone, otherwise the system is energetically unstable, i.e., Landau instability arises. However, the coupling effects of SOC, Zeeman splitting, nonlinearity and optical lattice potential can modify the dispersion relation of the system, and generate a transition between zero momentum state and non-zero momentum state, which results in the superfluidity appearing in the region either around or deviated from the center of the Brillouin zone. Namely, these coupling effects can make the superfluidity occur at optional position of the Brillouin zone dependent on the momentum of the minimum energy. In a linear system, the BEC is always dynamically stable. However, the nonlinearity can induce the dynamical instability, and also enhance the superfluid region. This can be used to manipulate the energetic and dynamical instability in experiment.

2. Model

We consider a BEC with equal contributions of Rashba and Dresselhaus SOC loaded into a deep one-dimensional optical lattice, which has been realized in experiment [22] recently. For sufficiently deep lattice, the system can be regarded as a chain of separately trapped BEC that are weakly linked. Thus the tight-binding approximation is appropriate and applicable. By using the tight-binding approximation, with the mean-field frame, the stability of BEC with SOC in optical lattice can be described by the following dimensionless equations [22,27,29–32]

$$\mathrm{i}\frac{d\psi_{\sigma n}}{dt} = -\Gamma(\psi_{\sigma n+1}+\psi_{\sigma n-1}) + \mathrm{i}\frac{k_L}{2}(\psi_{\bar\sigma n+1}-\psi_{\bar\sigma n-1}) \pm \delta\psi_{\sigma n} + (\gamma|\psi_{\sigma n}|^2+\beta|\psi_{\bar\sigma n}|^2)\psi_{\sigma n} \quad (1)$$

where $\psi_{\sigma n}$ ($\psi_{\bar\sigma n}$) is the normalized wave function of BEC with spin σ ($\bar\sigma$) in the nth site, and $\{\sigma,\bar\sigma\}=\{\uparrow,\downarrow\}$, i.e., $\sum_{\sigma n}|\psi_{\sigma n}|^2=1$. $\Gamma \equiv \Gamma_{n,n+1} = \int w^*(x-n)\partial^2 w(x-n-1)/\partial x^2 dx$ is the tunneling constant with Wannier wave function $w(x-n)$. $k_L \equiv k_{L(n,n+1)} = (4\kappa/k_{OL})\int w^*(x-n)\partial w(x-n-1)/\partial x dx$ denotes the dimensionless SOC strength with the optical lattice wave number k_{OL} and the corresponding equal Rashba and Dresselhaus SOC strength κ. $\delta \equiv \bar\delta/\omega_R$ refers to dimensionless Zeeman splitting frequency, where $\bar\delta$ is defined by the detuning or by the external Zeeman field, and $\omega_R \equiv E_R/\hbar \equiv \hbar k_{OL}^2/(2m)$ with the recoil energy E_R and the atomic mass m. $\gamma = (a_{\sigma\sigma}/a_0)\int|w(x-n)|^4dx$ and $\beta = (a_{\sigma\bar\sigma}/a_0)\int|w(x-n)|^4dx$ respectively represents dimensionless intra- and inter-species nonlinearity strength with the background scattering length a_0 [31], and the two-body scattering lengths between intra- and interspecies $a_{\sigma\sigma}$ and $a_{\sigma\bar\sigma}$. The physical variables are rescaled as $\psi_{\sigma n} \sim [\omega_R/(2\omega_\perp a_0)]^{1/2}\psi_{\sigma n}$, $x \sim k_{OL}^{-1}x$, and $t \sim \omega_R^{-1}t$ with the transversal frequency $\omega_\perp$ [29–32]. This tight-binding model has already been widely used to detect the physics of BEC with SOC in a deep optical lattice, where composite localized modes [29] and SOC-induced symmetry breaking of localized discrete matter waves [30] are discovered.

The stability analysis of BEC in the optical lattice can be performed by using the Bogoliubov theory. Thus, the tight-binding Eq. (1) has a Bloch wave solution [9,10,18]

$$\psi_{\sigma n} = [\psi_{\sigma 0} + u_\sigma(t)\mathrm{e}^{\mathrm{i}qn\pi} + v_\sigma^*(t)\mathrm{e}^{-\mathrm{i}qn\pi}]\mathrm{e}^{\mathrm{i}(kn\pi-\mu t)}, \quad (2)$$

which results in a small perturbation above the ground state, where μ is the chemical potential of the system, $\psi_{\sigma 0}$ is the ground state wave function, $u_\sigma(t)$ and $v_\sigma^*(t)$ are the Bogoliubov quasiparticle amplitudes, while k and q is the quasimomentum of the condensate and the quasiparticle excitation, respectively. The perturbation depends on the quasimomentum of the quasiparticle excitation and the site of the lattice, and it is a periodic perturbation. In the experiment, the perturbation can be generated by a sudden displacement of the magnetic potential along the lattice axis [14]. Substituting Eq. (2) into Eq. (1), one can obtain Bogoliubov–de Gennes equation

$$\mathrm{i}\frac{d}{dt}\begin{pmatrix} u_\uparrow \\ u_\downarrow \\ v_\uparrow \\ v_\downarrow \end{pmatrix} = \hat\sigma\hat A \begin{pmatrix} u_\uparrow \\ u_\downarrow \\ v_\uparrow \\ v_\downarrow \end{pmatrix}, \quad (3)$$

where $\hat\sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ is the Pauli matrix and the matrix

$$\hat A = \begin{pmatrix} U(k+q) & W_{\uparrow\downarrow} \\ W_{\uparrow\downarrow}^* & U(k-q) \end{pmatrix}, \quad (4)$$

with

$$U(k) = \begin{pmatrix} L_\uparrow(k) & M_\uparrow(k) \\ M_\downarrow(k) & L_\downarrow(k) \end{pmatrix}, \quad (5)$$

and

$$W_{\sigma\bar\sigma} = \begin{pmatrix} \gamma\psi_{\sigma 0}^2 & \beta\psi_{\sigma 0}\psi_{\bar\sigma 0} \\ \beta\psi_{\bar\sigma 0}\psi_{\sigma 0} & \gamma\psi_{\bar\sigma 0}^2 \end{pmatrix}. \quad (6)$$

Here $L_\sigma(k) = -2\Gamma\cos(k\pi) \pm \delta + 2\gamma|\psi_{\sigma 0}|^2 + \beta|\psi_{\bar\sigma 0}|^2 - \mu$ and $M_\sigma(k) = -k_L\sin(k\pi) + \beta\psi_{\sigma 0}\psi_{\bar\sigma 0}^*$. The stability of the system can be obtained by examining the eigenvalues of the matrix $\hat A$ and $\hat\sigma\hat A$. If the matrix $\hat A$ is positive definite, the Bloch wave is an energy local minimum which represents a superfluidity, then the system can be energetically stable. Otherwise, an energy saddle point arises when matrix $\hat A$ exists one or more negative eigenvalues, which results in Landau instability, i.e., energetic instability. Thus, the boundary of energetic instability is defined by $\min(\varepsilon)=0$, where ε is the eigenvalues of matrix $\hat A$. If the eigenvalues of the matrix $\hat\sigma\hat A$ are all real numbers, the Bloch wave is dynamically stable, otherwise the dynamical instability appears for existing one or more imaginary eigenvalues, where the corresponding mode will grow exponentially in time, which can result in period doubling and other forms of spontaneous breaking of the periodicity of the system [9,10,14]. Namely, the border of dynamical instability is defined by $\max\{\mathrm{abs}[\mathrm{Im}(\lambda)]\}=0$, where λ is the eigenvalues of matrix $\hat\sigma\hat A$. Further, $\hat A$ and $\hat\sigma\hat A$ are both 4×4 matrix, and their eigenvalues can be directly obtained by the diagonalization of the matrix.

3. Dispersion relation

The dispersion relation of BEC in the lattice becomes complex owing to the coupling effects of SOC, Zeeman splitting, nonlinearity and optical lattice potential, thus it can not be obtained analytically, but can be described numerically. However, the dispersion relation can be acquired analytically for a linear system or in absence of the Zeeman splitting.

For linear case, i.e., $\gamma=\beta=0$, the dispersion relation for the lowest energy band is depicted as

$$\mu = -2\Gamma\cos(k\pi) - \sqrt{\delta^2 + k_L^2\sin^2(k\pi)}, \quad (7)$$

with k varying in the first Brillouin zone $k \in [-1,1]$. There is a critical Zeeman splitting value $\delta_c = k_L^2/(2\Gamma)$ that for $\delta < \delta_c$,