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11 Thousa dynamic geometry for a non-extensive ideal σ $\frac{1}{12}$ Thermodynamic geometry for a non-extensive ideal gas $\frac{1}{78}$

13 79 14 80 J.L. López ^a*,*b, O. Obregón b, J. Torres-Arenas ^b

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a Departamento de Matemáticas, Facultad de Ciencias, Universidad Autónoma de Baja California, A.P. 1880, C.P. 22860, Ensenada, Baja California, Mexico antigornia, Mexico antigornia, Mexico antigornia, Mexico antigornia, $\frac{16}{2}$ b División de Geneiros Investigates Computed de Cuanquate A D E 142 C D 27150 León Cuanquate Meyica Mayica Mayor 2001 ¹⁰ b División de Ciencias e Ingenierías Campus León, Universidad de Guanajuato, A.P. E-143, C.P. 37150, León, Guanajuato, Mexico
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¹⁹ ARTICLE INFO ABSTRACT ⁸⁵

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28 metatration et al. 2001 et al. 2002 et al. 2003 et al. 2004 et 29 Pluctuation theory 95 30 Thermodynamic geometry Superstatistics Fluctuation theory

²¹ Article history: **Action 2018** A generalized entropy arising in the context of superstatistics is applied to an ideal gas. The curvature ⁸⁷ 22 Received 16 November 2017 scalar associated to the thermodynamic space generated by this modified entropy is calculated using 88 23 Received in revised formally 2018 two formalisms of the geometric approach to thermodynamics. By means of the curvature/interaction 89 24 Auch to the local and the state of the geometric approach to thermodynamic geometry it is found that as a consequence of 90 25 Communicated by CR Doering a generalized statistics, an effective interaction arises but the interaction is not enough to 91
25 Communicated by CR Doering 26 92 generate a phase transition. This generalized entropy seems to be relevant in confinement or in systems ₂₇ *Keywords:* entity result to the so many degrees of freedom, so it could be interesting to use such entropies to characterize the ₉₃ thermodynamics of small systems.

> are invariant with respect to Legendre transformations resembling the fact that the thermodynamic information does not depend on what fundamental relation (thermodynamic potential) is used. In this formalism, the representation invariance of the metric and its corresponding curvature scalar has been proved for simple thermodynamic systems [\[14\]](#page--1-0). On the other hand, inspired in fluctuation theory, a distance between points in a thermodynamic space can also be defined $[11,12,16]$, and we can associate to this space a thermodynamic metric, a corresponding Riemann tensor and consequently a curvature scalar *R*. Both approaches coincide in the physical interpretation of the curvature scalar as a manifestation of the existence of intermolecular interactions. When the curvature associated to the corespondent thermodynamic metric is non-zero, an interaction of some nature is present $[12,13]$, this is known as the curvature/interaction hypothesis. Other physical aspects that the scalar reveals, which has been proven for several thermodynamic systems, is the existence of first order phase transitions. The curvature scalar diverges at some point if a phase transition exists. For some systems, the point where the scalar diverges happens to be the critical point where the phase transition occurs [\[16,](#page--1-0)

> of the curvature scalar also provides additional information. For some systems it is clear that the sign of *R* represents the kind

> the bosonic or fermionic nature of the thermodynamic system, the

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\bullet 100 the context of the so called geometrothermodynamics [\[15,14\]](#page--1-0), the \bullet 100 the context of the so called geometrothermodynamics [15,14], the 61 127 for a positive scalar [\[17\]](#page--1-0). The sign of *R* can also be associated to

33 99 by means of Riemannian geometry [\[10–12,15,13,14\]](#page--1-0). Particularly in **1. Introduction**

³⁵ Because of the existence of anomalous systems which do not geometrical relevant quantities, like the thermodynamic metric, ¹⁰¹ ³⁶ seem to obey the rules of common statistics generally associated are invariant with respect to Legendre transformations resembling 102 37 to non-equilibrium processes, a more general statistics has been the fact that the thermodynamic information does not depend on 103 ³⁸ proposed [\[1\]](#page--1-0) based on superstatistics [\[3,2\]](#page--1-0) which considers large a what fundamental relation (thermodynamic potential) is used. In 104 ³⁹ fluctuations of intensive quantities [\[2\]](#page--1-0). We review here this formu-
this formalism, the representation invariance of the metric and its ¹⁰⁵ ⁴⁰ lation in which the intensive fluctuating quantity is the tempera-
corresponding curvature scalar has been proved for simple thermo-⁴¹ ture. This fluctuation gives rise to a certain probability distribution dynamic systems [14]. On the other hand, inspired in fluctuation 107 ⁴² characterized by a generalized Boltzmann factor. We can, in prin-
theory, a distance performance performant is thermal to same performance between points in a thermodynamic space can ⁴³ ciple, associate an entropy for every probability distribution and also be defined [11.12.16], and we can associate to this space a 108 ⁴⁴ in this context the Boltzmann–Gibbs entropy corresponds to the thermodynamic metric a corresponding Riemann tensor and con-⁴⁵ usual Boltzmann factor. It is, however, possible to obtain other **sequently a curvature scalar R** Both annoaches coincide in the ¹¹¹ ⁴⁶ generalized expressions for entropies associated to different prob-
hysical interpretation of the curvature scalar as a manifestation ⁴⁷ ability distributions [\[4\]](#page--1-0) depending on one or several parameters $\frac{p_{\text{H}}}{p_{\text{H}}}$ he existence of intermedecular interactions. When the curvature 48 [\[3,5\]](#page--1-0). In [\[1,5\]](#page--1-0), it was shown how to generate an entropy depend-
associated to the corespondent thermodynamic metric is pop-zero 49 ing only on the probability. The particular entropy considered here $\frac{320 \text{ interaction of some nature in present [12.13]}}{20 \text{ interaction of some nature in present [12.13]}}$ this is known as 50 arises from a generalized gamma distribution depending only on $\frac{1}{16}$ expressive distribution by potential process (ther physical aspects that $\frac{1}{16}$ ⁵¹ the probability p_l . This entropy has several interesting features and the scalar reveals which has been proven for several thermody- 52 it seems to be relevant for particular thermodynamic systems like $\frac{1}{2}$ is the systems is the systems of first order phase transitions $\frac{53}{119}$ confined systems [\[6\]](#page--1-0) and in this context we find an interesting and $\frac{1}{10}$ and $\frac{1}{10}$ curvature solar diverges at some point if a phase transition 54 necessary application of such generalized entropies. The quantum $\frac{120}{120}$ is the pay of the paint was the paint was the paint was the paint of the subsequence in the state of the state of the paint of the state 55 version of this entropy which is a generalization of the Von Neu-
 $\frac{253335}{8000}$ to the gritical suit where these transition expection of the Von Neu- 56 mann entropy arises by means of a natural generalization of the $\frac{121}{10}$ be the construction point where the fluctuation to put the fluctuation of the $\frac{10}{10}$ $\frac{14}{2}$. In the thermodynamic geometry of fluctuation theory, the sign $\frac{123}{2}$ Because of the existence of anomalous systems which do not replica trick [\[7–9\]](#page--1-0).

 58 In the formalism of the geometric approach to thermodynamics, 58 III curvature scalar also provides alternational information. For the state is not the state of Ω 59 a geometric structure is given for usual thermodynamic systems some systems it is clear that the sign of R represents the kind 125 60 126 of interaction, being attractive for a negative scalar and repulsive

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⁶² 128 *E-mail addresses:* jlopez87@uabc.edu.mx (J.L. López), octavio@fisica.ugto.mx 63 (O. Obregón), jtorres@fisica.ugto.mx (J. Torres-Arenas). Bose and Fermi ideal gases are a clear example of this, we have 129 (O. Obregón), jtorres@fisica.ugto.mx (J. Torres-Arenas).

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¹ R < 0 in the first case and R > 0 in the second case [\[18,22\]](#page--1-0). For that leads to the following generalized entropy [1] ⁶⁷ ² some other systems, it appears a change of sign in the curvature 68 ³ scalar, from negative to positive or the other way around. These $\frac{3}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{69}{2}$ 4 cases generally appear where a different statistics, other than that $\Delta = K - (1 - p_I)$. (4) (4) 70 5 of Boltzmann's, is considered [\[17,22\]](#page--1-0). Following the vast literature $l=1$ and the vast literature $l=1$ ⁶ of the subject related to the thermodynamic geometry of the two $\frac{1}{1}$ thas been observed that when the fluctuations are small the $\frac{72}{1}$

11 In this work we particularly consider the curvature of a non-
 $\{2, 1, 2, 3, 4, 5\}$ and this is the case for the curvature of a non-12 extensive ideal gas characterized by a generalized non-extensive $\frac{11}{12}$ Cq. (4), we will identify consider small includedness and we will be ¹³ entropy. The particular entropy we use depends only on the prob-
13 entropy. The particular entropy we use depends only on the prob-¹⁴ ability distribution and arises in the realm of superstatistics $[1,6]$. generalized entropy there is a generalized H function, ¹⁵ In order to get a better insight of the physics behind the curva-¹⁶ ture scalar of our thermodynamic system, we calculate the curva- $H = d^3p e^{f \ln f} - 1$, (5) ⁸² ¹⁷ ture scalar using the two formalisms mentioned earlier, we will all the state of the scalar using the two formalisms mentioned earlier, we will all the state of the ¹⁸ call these two scalars, the geometrothermodynamic scalar for the it can be shown that it satisfies a generalized H-theorem [19], Us- 84 ¹⁹ scalar calculated following the formalism in [\[13\]](#page--1-0) and the fluctua-
ing a Maxwell distribution to calculate this new *H* metrion, keep-²⁰ tion theory scalar, to the scalar calculated following [\[17\]](#page--1-0). We will ing only the first order correction and the relation $H = -S/kV$ it ⁸⁶ ²¹ find that the particular entropy (statistics) we propose $[1,6]$ mod- $\frac{60}{100}$ follows 22 ifies the geometric structure of the generalized thermodynamic 88 ²³ space considered, namely a generalized ideal gas, giving rise to the $S_{\text{cc}} = -kN \ln(n^2 - 3) = \frac{3}{2}$ (6) ⁸⁹ 24 appearance of an effective interaction. $\begin{array}{c} 2 \end{array}$ $\begin{array}{c} 2 \end{array}$ $\begin{array}{c} 2 \end{array}$

²⁵ The paper is organized as follows: First in section 2 we ex-
 ν_{Vn^2} 3 3 3 3 15 ⁹¹ ²⁶ plain how our modified entropy, and its associated Boltzmann $-\frac{\ln 2}{\ln 2} \ln^2 (n\lambda^3) - \frac{1}{2} \ln (n\lambda^3) + \frac{1}{n^2}$, ²⁷ factor, arises by assuming a particular probability distribution. In $2^{2/2}$ $2^{2/2}$ $2^{2/2}$ $2^{2/2}$ 10 ²⁸ section [3](#page--1-0) we briefly introduce first the formalism of geometrother-
⁹⁴ section 3 we briefly introduce first the formalism of geometrother-
 h can be identified with the man thermal wave 29 modynamics developed by H. Quevedo [\[13\]](#page--1-0) and describe how the $\sqrt{2\pi mkT}$ can be defined with the mean thermal with $\sqrt{2\pi mkT}$ 30 thermodynamic metric is calculated. In this same section we also length, k is the boltzmann constant, V is the volume and T is the 31 introduce the thermodynamic metric in the formalism of G. Rup-
 31 absolute temperature. The authors in [19] studied the thermody- 32 peiner [\[12\]](#page--1-0). We calculate the curvature scalar in both formalisms and properties of the corresponding generalized ideal gas. in this α 33 to further analyze and compare the thermodynamic information context, analysis of response functions shows a first correction hav-34 contained in the scalars using the interpretation of both for- ing a universal form, that is, the same functional correction to all 100 35 malisms. In section [4](#page--1-0) we discuss the interpretation of both scalars the enhancement quantities derived from the generalized equations 101 ³⁶ and in section [5](#page--1-0) we conclude and present the main results of our ^{or state.} 37 *Nork* 2022 **In order to obtain a thermodynamic potential, we assume that 203** work.

41 **2. Generalized entropies depending only on the probability distribution**

 10 crostate associated with a local cell of average temperature $1/\beta$
 110
 110
 110
 110
 111 is given by

$$
B(E) = f(\beta)e^{-\beta E}d\beta
$$
 (1)
where $u = \frac{U}{2} - \frac{V}{2}$ and $h = \frac{3h^2}{2}$. The first terms correspond to

tors. Following the procedure stated in [\[2,4\]](#page--1-0) it is possible, in principle, to associate a modified entropy to every Boltzmann factor. As the form

$$
f_{p_l}(\beta) = \frac{1}{\beta_0 p_l \Gamma \frac{1}{p_l}} \frac{\beta}{\beta_0} \frac{1}{p_l} \frac{\frac{1-p_l}{p_l}}{e^{-\beta/\beta_0 p_l}},
$$
(2)

to the Boltzmann factor

$$
B_{p_l}(E) = (1 + p_l \beta_0 E)^{-\frac{1}{p_l}},
$$
\n(3)

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that leads to the following generalized entropy [\[1\]](#page--1-0)

$$
S = k \qquad (1 - p_l^{p_l}). \tag{4}
$$

$$
l=1
$$

⁷³ approaches considered here, we find that the sign interpretation β polymann factor (3) can be approximated as an infinite sum ⁸ is more clear in the formalism of G. Ruppeiner [\[17\]](#page--1-0) but even so, where the first term is the wavel Beltzmann factor and the first 74 9 it is not at all clear that this interpretation could be valid for all $\frac{1}{2}$ is not all $\frac{1}{2}$ if $\frac{1}{2}$ 10 thermodynamic systems. The contract of the contract of the state of the stat It has been observed that when the fluctuations are small, the Boltzmann factor (3) can be approximated as an infinite sum where the first term is the usual Boltzmann factor and the first correction term seems to be the same even for different statistics [\[2\]](#page--1-0). It was shown in $[21,19]$ that this is the case for the entropy in Eq. (4). We will further consider small fluctuations and we will take only the first correction term in entropy. Associated to this generalized entropy there is a generalized *H* function,

$$
H = d^3 p \cdot e^{\int \ln f} - 1 \quad , \tag{5}
$$

it can be shown that it satisfies a generalized H-theorem [\[19\]](#page--1-0). Using a Maxwell distribution to calculate this new *H* function, keeping only the first order correction and the relation $H = -S/kV$, it follows

$$
S_{eff} = -kN \ln(n\lambda^3) - \frac{3}{2}
$$
 (6)

$$
-\frac{kV n^2 \lambda^3}{2^{5/2}} \ \ \ln^2(n\lambda^3) - \frac{3}{2} \ln(n\lambda^3) + \frac{15}{16} \ ,
$$

where $\lambda = \frac{h}{\sqrt{2\pi m kT}}$ can be identified with the mean thermal wavelenght, *k* is the Boltzmann constant, *V* is the volume and *T* is the absolute temperature. The authors in [\[19\]](#page--1-0) studied the thermodynamic properties of the corresponding generalized ideal gas. In this context, analysis of response functions shows a first correction having a universal form, that is, the same functional correction to all thermodynamic quantities derived from the generalized equations of state.

38 104 the conventional linear relation between internal energy and tem-³⁹ 2. **Generalized entropies depending only on the probability** experature holds. Within this approximation we obtain the following 105 ⁴⁰ **distribution thermodynamic fundamental relation thermodynamic fundamental relation 106**

$$
^{42}_{43}
$$
 The Boltzmann factor depending on the energy *E* of a mi-
\n
$$
S = kN \ln v + \frac{3kN}{2} \ln \frac{u}{b} + \frac{3kN}{2}
$$
 (7)

45 111 [−] *kN* 25*/*² *b*3*/*² *u*3*/*² *v* ln² *^b*3*/*² *vu*3*/*² [−] ³ 2 ln *^b*3*/*² *vu*3*/*² ⁺ 15 ¹⁶ *,*

 $\mathbf{u} = \frac{U}{N}$, $\mathbf{v} = \frac{V}{N}$ and $\mathbf{b} = \frac{3h^2}{4\pi m}$. The first terms correspond to ¹¹³
48 49 and different distributions $f(\beta)$ lead to different Boltzmann fac-
49 and different distributions $f(\beta)$ lead to different Boltzmann fac-50 tors. Following the procedure stated in $[2,4]$ it is possible, in prin-
 $\frac{1}{2}$ in the case of 51 ciple, to associate a modified entropy to every Boltzmann factor. As the local gas $3 - k \sqrt{m} v + \frac{1}{2} m \frac{1}{b} + \frac{1}{2}$ can be recovered by an $\frac{117}{2}$ 52 an example we have that for the distribution $f(\beta) = \delta(\beta - \beta_0)$ the second to the second to ϵ by the discussion and the second to ϵ and ϵ and ϵ the discussion ϵ of the discussion ϵ of the discussion $\frac{53}{15}$ usual Boltzmann factor is recovered and from this, the Boltzmann–
 $\frac{1}{15}$ usual this decay of the latter and this curve with the further and this with intervalsed in the latter of the second this in- 54 Gibbs entropy follows directly [\[4\]](#page--1-0). In [\[1\]](#page--1-0) a Gamma distribution of $\frac{1}{100}$ we have derivatives determines we have not to the contract term does not $\frac{35}{21}$ the form of the entropy is the entropy, and this constant term does not $\frac{121}{121}$ 56 122 affect the final result. At this point we have to clarify that the lin- $\frac{1}{123}$ $\frac{1-p_l}{1-2l}$ $\frac{1-p_l}{1-2l}$ ear relation between internal energy and temperature that we have $\frac{1}{23}$ $f_{n}(B) = \frac{1}{\sqrt{1-\frac{1}{2}}} \sum_{\mu=1}^{\mu} \frac{e^{-\beta/\beta_0 p_l}}{e^{-\beta/\beta_0 p_l}}$ (2) assumed makes our calculations to be more accurate for low den- $\beta_0 \frac{\partial f}{\partial x} = \frac{\partial f}{\partial y} \frac{\partial f}{\partial y}$ by $\beta_0 \frac{\partial f}{\partial z} = \frac{\partial f}{\partial x} \frac{\partial f}{\partial y}$ sities or high temperatures and we will take this into account to β_1 $\frac{p_1}{p_1}$ expansive the interpretation of the behavior of the curvature scalars. the usual entropy of the ideal gas, $S = kN \ln v + \frac{3kN}{2} \ln \frac{u}{b} + \frac{3kN}{2}$. We notice that the Sackur–Tetrode expression for the entropy of the ideal gas $S = kN \ln v + \frac{3kN}{2} \ln \frac{u}{b} + \frac{5kN}{2}$ can be recovered by an ad-hoc fixing term as it was originally proposed by Gibbs. It is not possible to recover the 5*kN/*2 term from the classical calculation we made but this does not affect the further analysis which in-

 61 was proposed where, by maximizing the appropriate information $\hskip1cm$ A previous thermodynamic analysis was made corresponding to $\hskip1cm$ $_{127}$ 62 measure, the parameter p_l can be identified with the probability a system characterized by a particular interaction. In [19] a sys-
 63 and β₀ is the average inverse temperature. This distribution yields tem was considered of gas particles exposed to square-well and 129 64 130 Lennard–Jonnes potentials. These potentials are well defined and 65 **131 1** $\mathcal{B}_{B_1}(E) = (1 + p_1 \beta_0 E)^{-p_1}$, $\qquad \qquad$ (3) to these systems. A further analysis with Monte Carlo simulations \qquad^{132} A previous thermodynamic analysis was made corresponding to a system characterized by a particular interaction. In [\[19\]](#page--1-0) a system was considered of gas particles exposed to square-well and

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