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Nondiffracting wave beams in non-Hermitian Glauber–Fock lattice

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ABSTRACT

We theoretically study non-Hermitian Glauber–Fock lattice with nonuniform hopping. We show how to engineer this lattice to get nondiffracting wave beams and find an exact analytical solution to nondiffracting localized waves. The exceptional points in the energy spectrum are also analyzed.

Diffraction is a phenomenon linked to the wave nature of light and occurs when a wave encounters an obstacle. The wave may be altered in amplitude and/or phase when it passes through an obstacle and diffraction takes place. In 1987, the term "nondiffracting beams" was introduced by Durnin [1]. The nondiffracting beam was comprehended as the monochromatic optical field whose transverse intensity remains unchanged in free-space propagation. Durnin examined the beams as exact solutions to the homogeneous Helmhotz equation. The transverse amplitude of such beams can be described by the Bessel functions. Berry and Balazs theoretically showed another nonspreading solution was available for the Schrodinger equation describing a free particle [2]. They showed that for a wave function in the form of an Airy function, the probability density propagates in free space without distortion and with constant acceleration. Other kinds of nondiffacing beams and their properties were studied in [3,4].

In optics, the nondiffracting beam can be obtained in the convenient media such as waveguides or nonlinear materials [5–11]. Diffraction in an array of waveguides is governed by hopping light from site to site by optical tunneling. By using the diffraction properties of waveguide arrays, the discrete diffraction can be controlled. Glauber Fock lattices refer to a special class of asymmetric and semi-infinite tight-binding lattices with inhomogeneous hopping rates. This kind of lattice has been introduced in [12,13]. The light propagation in a Glauber–Fock waveguide lattice is equivalent for the displacement of Fock states in phase space. The square-root distribution of the coupling parameter in such lattices displays a new family of intriguing quantum correlations not encountered in uniform arrays. The propagation of classical light waves in Glauber–Fock photonic lattices was observed experimentally [13]. In these photonic lattices, refractive indices and second-neighbor couplings define the mass and frequency of the analog quantum oscillator. The quantum model conserves the Ermakov Lewis invariant [14]. Altough the photonic array is effectively semi-infinite and the waveguide coupling is not uniform, Bloch-like revivals are observed in these optical structures [15]. Dynamic localization in Glauber Fock lattices has been studied in several papers [16–18].

In 1998, Bender et al. theoretically showed that a class of non-Hermitian Hamiltonians, the so called $\mathcal{PT}$ symmetric Hamiltonians, could have a real spectrum [19]. Since then, several examples of this kind of Hamiltonians with real eigenvalues was introduced in optics [20–27]. A coupled mode theory suitable for coupled optical $\mathcal{PT}$ symmetric systems has been developed in [20]. Constant intensity supermodes in non-Hermitian lattices have been considered in the framework of coupled mode theory [21]. Two dimensional photonic crystal possessing periodic $\mathcal{PT}$ symmetry has been theoretically analyzed and found that optical modes exhibits thresholdless spontaneous $\mathcal{PT}$ symmetry breaking near the Brillouin zone boundary [22]. Beam dynamics in $\mathcal{PT}$ symmetric periodic potentials has been examined for one and two dimensional lattice geometries [23].

One of the intriguing aspect associated with non-Hermitian Hamiltonian systems is the symmetry breaking at exceptional points (EPs) which was introduced in [28]. The symmetry breaking takes the system from a regime of real energy eigenvalues to a partial complex spectrum with conjugate pairs of eigenvalues. At these points, two or more eigenvalues and their corresponding

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eigenvectors become identical. The appearance of EPs and spectral singularities modify the dynamical and topological properties of optical lattices [29–32]. EPs in non-Hermitian tight binding lattice with unidirectional hopping have been theoretically investigated for three geometrical settings including open linear chain, finite linear chain and ring lattice [33]. Physical justification of non-Hermitian lattices with unidirectional hopping have been suggested in [34,35]. Experimental observation of EPs has been made in microwave cavities [36]. Absence of EPs are another interesting research area and it was investigated in square wave guide arrays with diagonally balanced gain-loss distribution [37].

In this paper, we consider non-Hermitian Glauber Fock lattice and investigate the EPs in its spectrum. In the next section, we give the model describing the non-Hermitian system we investigate. We discuss how we can engineer the system to get nondiffracting solution and the reality of the energy spectrum. At the end of the section, we sum up the results we get in the paper.

1. Model

We consider a semi-infinite and asymmetric Glauber–Fock lattice which consists of evanescently coupled waveguides. The tunneling amplitude varies with the square root of the site number n. Here, we assume the most general z-dependent tunneling and z-dependent refractive index, where z is the normalized propagation distance. We consider a variant of the Glauber Fock lattice in such a way that the system is non-Hermitian. The non-Hermiticity of the system is defined by the parameter δ. The complex field amplitude at the n-th waveguide $c_n(z)$ satisfies the following equation

$$i\partial_z c_n + Fnc_n + J(\sqrt{n+1}c_{n+1} + (1-\delta)\sqrt{n}c_{n-1}) = 0 \tag{1}$$

where $J = J(z)$ is the z-dependent first order tunneling amplitude through which particles are transferred from site to site, $F = F(z)$ is the z-dependent effective propagation constant proportional to the effective refractive index and $c_n(z)$ is the field amplitude at the n-th waveguide. For $n < 0$ $c_n(z) = 0$. The parameter δ is a real valued constant and the system becomes Hermitian if $\delta = 0$. The system described above is non-$\mathcal{PT}$ symmetric since the waveguides are no longer equidistant. There exist no inversion symmetric point in the system.

We follow the method introduced in [14,38] to get the analytical solution. We can define the state vector via the field amplitudes as $|\psi_n>=\sum_{n=0}^{\infty} c_n(z)|n>$, where the state $|n>$ is the Fock state and $n = 0, 1, 2, \ldots$ [14]. Substituting this solution into the equation (1) gives the Schrodinger-like equation $H\psi = i\dot{\psi}$ ($\hbar = 1$) with the Hamiltonian

$$H = -\left(F(z)\hat{n} + J(z)\left((1-\delta)\hat{a} + \hat{a}^{\dagger}\right)\right) \tag{2}$$

Here bosonic creation and annihilation operators satisfy

$$\hat{a}^{\dagger}|n>=\sqrt{n+1}|n+1> \quad \text{and} \quad \hat{a}|n>=\sqrt{n}|n-1>.$$

The number operator satisfies $\hat{n}|n>=n|n>$. At this point we can rewrite the Hamiltonian using $\hat{a} = \frac{q+ip}{\sqrt{2}}$ and $\hat{a}^{\dagger} = \frac{q-ip}{\sqrt{2}}$, where q and p normalized position and momentum operators, respectively. Then the Hamiltonian becomes

$$H = -\left(\frac{p^2}{2m} + \frac{m}{2}\omega^2 q^2 + \sqrt{2}Jq - \frac{J\delta}{\sqrt{2}}(q+ip) - \frac{F}{2}\right) \tag{3}$$

where the z-dependent mass is $m = 1/F(z)$ and the z-dependent frequency is $\omega^2 = F(z)^2$.

The exact analytical solution of this Hamiltonian is available if we make the variable change $z \to -z$. This is the Hamiltonian of a quantum harmonic oscillator which mass, frequency, and external driving force are dependent on z. To get the solution, let us transform the coordinate according to $q' = \frac{q-q_c}{L}$, where z dependent function $q_c(z)$ describes translation and $L(z)$ is a z dependent dimensionless scale factor. The center of the wave packet moves in accordance with $q_c(z)$ and the width of the packet changes in accordance with $L(z)$. Under this coordinate transformation, the z-derivative operator transforms as $\partial z \to \partial z - L^{-1}(\dot{L}q' + q_c')\partial q'$. We will look for the solution of the form

$$\psi_n(q', z) = \exp\left(\Gamma - \frac{mJ\delta}{\sqrt{2}}Lq' + i\Lambda\right)\frac{\phi_n(q')}{\sqrt{L}} \tag{4}$$

where $\dot{\Gamma} = -\frac{mJ\delta\dot{q}_c}{\sqrt{2}}$, the position dependent phase is

$$\Lambda(q', z) = m\left(\alpha q' + \frac{\beta}{2}q'^2 + S\right)$$

and α, β, and S are z-dependent functions. Substituting this ansatz into the corresponding Schrodinger-like equation results in an equation including harmonic and linear potential terms. Therefore we choose $\alpha(z) = L\dot{q}_c$, $\beta(z) = L\dot{L}$ and $\dot{S}(z) + \frac{\dot{m}}{m}S = \frac{1}{2}\dot{q_c}^2 - \frac{\omega^2}{2}q_c^2 - \frac{J^2\delta^2}{4} + \frac{J\delta q_c}{\sqrt{2m}} - \frac{\sqrt{2}Jq_c}{m} + \frac{F}{2m}$. The resulting equation is given by

$$-\frac{1}{2mL^2}\frac{\partial^2\phi}{\partial(q')^2} + \left(\frac{m}{2}\Omega^2(q')^2 + Uq'\right)\phi = i\frac{\partial\phi}{\partial z} \tag{5}$$

where $\Omega^2 = L(\ddot{L} + \frac{\dot{m}}{m}\dot{L} + \omega^2 L)$ and $U = mL(\ddot{q_c} + \frac{\dot{m}}{m}\dot{q_c} + \omega^2 q_c + \sqrt{2}\frac{J}{m} - \frac{J\delta}{\sqrt{2m}} + i\frac{\dot{m}}{m}\frac{J\delta}{\sqrt{2}} + i\frac{\dot{J}\delta}{\sqrt{2}})$. q_c and L are satisfied by the following equations

$$\ddot{q_c} + \frac{\dot{m}}{m}\dot{q_c} + \omega^2 q_c + \left(\sqrt{2} - \frac{\delta}{\sqrt{2}}\right)\frac{J}{m} = -i\frac{\delta}{\sqrt{2m}}\frac{d(Jm)}{dz} \tag{6}$$

$$\ddot{L} + \frac{\dot{m}}{m}\dot{L} + \omega^2 L = \frac{1}{m^2L^3} \tag{7}$$

The equation (7), known as the Ermakov equation, is easy to solve for the initial condition $\dot{L}(z=0) = 0$. Its solution is $L(z) = 1$. The solution of equation (6) will be discussed below. With the help of equations (6) and (7), the linear potential is eliminated from equation (5). The solution for $\phi_n(q')$ is found as

$$\phi_n(q') = N_n \exp\left(i\int\frac{E_n}{mL^2}dz - \frac{q'^2}{2}\right)H_n(q') \tag{8}$$

where $E_n = (n+\frac{1}{2})\omega$ and H_n and N_n are the Hermite polynomials and the normalization constant respectively.

We can find the exact solution inserting equation (8) into equation (4). It is well known that eigenfunctions of non-Hermitian Hamiltonians are not generally orthogonal to each other. This fact can be seen here, $\int_{-\infty}^{\infty}\psi_n^{\star}(q,0)\psi_{n'}(q,0)dq \neq \delta_{n,n'}$ and we emphasize that this integral changes during propagation. Complex amplitude in the n-th waveguide at the position z is given by $c_n(z) = <\psi(q,z)|n>$.

Having established the exact analytical wave packet solution, we can now discuss initial wave packet. Suppose that $\dot{q_c}(0) = \dot{\Gamma}(0) = 0$. The initial value of the total power P_0 is given by the