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Torsion balances with fibres of zero length

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ABSTRACT

Torsion balances have good immunity to tilt and low rotational stiffness. However precise control of the position of the suspended torsion 'bob' is difficult in the presence of ground vibrations and tilt and this is a limiting factor in applications where Casimir forces or putative non-Newtonian short-range forces are being measured. We describe how the desirable characteristics of torsion balances can be reproduced in a rigid body that is suspended using applied forces rather than a torsion fibre. The suspension system can then provide a more precise control of the degrees of freedom of the suspended body. We apply these ideas to a superconducting levitated torsion balance, developed by the authors, and a generic electrostatic suspension. We present results of preliminary experiments that provide support for our analyses.

1. Introduction

The torsion balance has been the work-horse of many areas of physics and engineering since the time of Cavendish [3,5]. The advantages associated with a state of the art Cavendish torsion balance are well understood and appreciated: measurements of torques can be made without the influence of Earth's gravity; fibres can be easily manufactured that give very small torsional stiffness and this minimises the problem of noise associated with the sensor that detects the rotational motion; finally torsion balances can be made so that, to a good approximation, horizontal acceleration or tilt of the laboratory does not couple to their rotation. As a result of these attributes the torsion balance has been used with great success to test the principle of weak equivalence and the inverse square law of gravitation [15]. However the classic torsion balance does have its drawbacks: its dynamics are complex as the suspended object (the bob) is essentially suspended as a simple pendulum that can swing in the presence of ground vibrations that accelerate the attachment point of the fibre. This makes the precise control of the linear displacements of the bob difficult. Ground tilt in a typical laboratory is of the order of a few µrad and the displacement of the bob attached to the end of a fibre of a few 10's of cm in length can make measurements of forces whose range is less than a few µm problematic [10]. There is residual tilt coupling due to the asymmetry of the fibre and, therefore, most torsion balances convert ground tilt into rotation about the torsion balance fibre axis [13,1]. The question arises as to whether there could be devices that can equal the performance of the Cavendish balance in terms of their signal to noise for torque measurement, have low sensitivity to ground tilt, but have more controllable dynamics. Many attempts have been made at dispensing with the standard torsion fibre. For example there have been: superconducting gradiometers [4] and torsion balances [6]; room temperature magnetic suspensions [9]; fluid suspensions [8] and electrostatic suspensions [16]. Nevertheless our knowledge of weak forces with ranges larger than about 50 µm is still dominated by a device that was devised more than 200 years ago.

The goal of the work described here picks up from the instrument developments of reference [6] and [16] (see also [7]) where the fibre is absent. We refer to these devices as torsion balances with 'fibres of zero length'. We aim to realise post-Cavendish torsion balances that have simple dynamics such that surfaces, that provide the source and test bodies for short range forces, can be accurately maintained in close proximity but still have the desirable properties of the Cavendish torsion balance. Such a development would potentially allow more accurate measurement of forces of shorter range than 50 µm and provide a more sensitive device than atomic force microscopes (see [11]) that are currently commonly used in this regime.

We present a scheme for tuning the dynamics of a suspended object in order to decouple its rotational motion from translational accelerations and also to tune one or more of its rotational modes to give a low stiffness. We will consider a general case of a mechanically rigid object (i.e. with no internal degrees of freedom) suspended by some combination of actuators relative to some rigid structure. The actuators provide forces and stiffnesses and could be magnetic, electrostatic, air pressure actuators, mechanical springs, or some combination thereof. The forces provided by the actuators

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could individually be attractive or repulsive and their equivalent springs could produce a stability or instability in the suspended object. However, the forces and their stiffnesses have to be tunable. Clearly, if the force provided by an actuator is either attractive or repulsive, it will not by itself produce an equilibrium position, either stable or unstable. Two such actuators acting in opposition can produce a stable equilibrium if the second derivative of the potential energy with respect to a particular degree of freedom is positive for both springs, and an unstable one if the second derivatives are both negative. We will specifically address the cases of superconducting diamagnetic and electrostatic suspensions. Diamagnetic superconducting suspensions can be used to suspend an object in a stable configuration. In this case, the equilibrium position will represent a minimum of the body's potential energy in the six-dimensional space of translations and rotations. The motion, for sufficiently small deviations from the equilibrium position, should be well described by a combination of up to six harmonic oscillator modes. By Earnshaw's theorem it is not possible to stably electrostatically suspend an object in three dimensions, but it may be in a stable configuration if it is mechanically constrained in one or more dimensions, and it may also be servo-controlled such that it is effectively stable below some frequency range that is characteristic of the servo system.

In the following article we analyse the general case of a levitated rigid body that is constrained by some combination of actuators relative to some rigid external structure (attached to the Earth). We refer to the levitated object as the 'float', the actuators as 'springs' and the rigid structure as the 'bearing'. We present a model for the potential energy in terms of generalised stiffnesses which constrain the float's motion relative to the bearing. We show how the stiffnesses can be defined in order to allow one of the float's rotational degrees of freedom to be decoupled from translational vibrations of the bearing. We also show, using the specific examples of an electrostatic suspension and a superconducting torsion balance, how the stiffnesses can be modified to achieve a rotational degree of freedom of low stiffness.

2. Decoupling of translational forces from the rotational mode

The low flexural rigidity of the fibre suspension of a Cavendish torsion balance is such that the centre of mass of the suspended torsion bob lies to a high accuracy directly below the axis of rotation of the balance. This means that a transverse acceleration of the laboratory (e.g. from seismic noise) acting on the suspended mass (bob) through its centre of mass (COM) will not produce a torque on the balance. In general, for an electromagnetically suspended object, there will be some offset between the COM and the point through which the suspension forces act. This leads to a coupling between translational and rotational modes. This situation is illustrated in Fig. 1.

In Fig. 1 an object, represented by the outer rectangle (but that can in principle be of an arbitrary shape), is shown in an arbitrary orientation being supported by an array of forces, with their associated stiffnesses acting at various points on the surface of the body. We can assume, for the sake of ease of conceptual understanding, that the figure is a plan view and that local gravity acts in a perpendicular direction to the page. A steady acceleration of the bearing in the plane of the page will produce a force, $\vec{F}_c$, acting at the centre of mass of the float and a reaction force $\overrightarrow{F_b}$ at a position that we will define as the centre of buoyancy of the float/bearing system. We have from Newton's third law that

$$\vec{F}_c + \vec{F}_b = 0, \tag{1}$$

and the net torque acting on the system is

$$\Gamma = \vec{r_b} \times \vec{F}_b + \vec{r_c} \times \vec{F}_c = \left(\vec{r_c} - \vec{r_b}\right) \times \vec{F}_c. \tag{2}$$

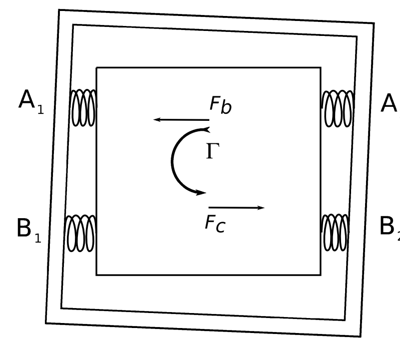

Fig. 1. A schematic diagram in two dimensions of a float (outer tilted rectangular object) that is suspended from a bearing using a combination of forces that are represented by springs in compression. A linear acceleration of the bearing creates forces F_c and F_b that act at the centre of mass of the float and centre of buoyancy of the bearing, respectively. If the centre of buoyancy is not located at the centre of mass of the float, a torque Γ is also produced. The labels for the coils are used in a discussion in Section 4.

We can equate the force acting on the float with the product of its mass, m_f, and the acceleration, $\ddot{\vec{r_0}}$, of the bearing. The coordinates of the centre of buoyancy, $\vec{r_b}$, and the centre of mass of the float, $\vec{r_c}$, can be defined with respect to a coordinate system centred on the bearing. We notice that the centre of buoyancy is the point at which a force can be applied such that it only produces a displacement of the float and not a rotation. It follows that, if the centre of mass of the float coincided with the centre of buoyancy, the float would displace but not rotate. Tuning of the centre of mass position can be achieved by adjustment of the distribution of mass, as is the case in standard mechanical beam balances where an appropriate period of oscillation can be thus achieved, for example. However here we explore the possibility of adjustment of the stiffnesses.

For a general positional configuration of the float, we can calculate the potential energy, E, stored in the ensemble of suspension springs in terms of the linear and angular displacements of the float, Δ, with respect to the bearing as

$$E = \frac{1}{2}\underline{\Delta} \cdot \underline{\underline{K}} \cdot \underline{\Delta}, \tag{3}$$

where $\underline{\underline{K}}$ is the symmetric stiffness matrix,

$$\underline{\underline{K}} = \begin{pmatrix} K_{\xi\xi} & K_{\xi\eta} & K_{\xi\varsigma} & K_{\xi\theta} & K_{\xi\phi} & K_{\xi\psi} \\ & K_{\eta\eta} & K_{\eta\varsigma} & K_{\eta\theta} & K_{\eta\phi} & K_{\eta\psi} \\ & & K_{\varsigma\varsigma} & K_{\varsigma\theta} & K_{\varsigma\phi} & K_{\varsigma\psi} \\ & & & K_{\theta\theta} & K_{\theta\phi} & K_{\theta\psi} \\ & & & & K_{\phi\phi} & K_{\phi\psi} \\ & & & & & K_{\psi\psi} \end{pmatrix}, \tag{4}$$

with $\Delta = (\xi, \eta, \varsigma, \theta, \phi, \psi)$ which contains, respectively, the float's spatial and angular displacements with respect to the bearing in a cartesian coordinate system. The components of the forces and torques, $\underline{F}$, can be calculated using virtual work arguments in the usual way,

$$\underline{F} = -\underline{\underline{K}}\,\underline{\Delta}. \tag{5}$$

Initially we will concentrate on the equations of motion in a plane perpendicular to Earth's gravity. With reference to equation (2), we can take moments about the centre of buoyancy and use subscripts to denote the cartesian components of torque and force,

$$\Gamma_z = \Delta x F_y - \Delta y F_x, \tag{6}$$

where $(\Delta x, \Delta y)$ is the location of the centre of mass with respect to the centre of buoyancy.