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# Testing local-realism and macro-realism under generalized dichotomic measurements

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ABSTRACT

Generalized quantum measurements with two outcomes are fully characterized by two real parameters, dubbed as sharpness parameter and biasedness parameter and they can be linked with different aspects of the experimental setup. It is known that sharpness parameter characterizes precision of the measurements and decreasing sharpness parameter of the measurements reduces the possibility of probing quantum features like quantum mechanical (QM) violation of local-realism (LR) or macro-realism (MR). Here we investigate the effect of biasedness together with that of sharpness of measurements and find a trade-off between those two parameters in the context of probing QM violations of LR and MR. Interestingly, we also find that the above mentioned trade-off is more robust in the latter case.



## 1. Introduction

Nonclassical features of quantum mechanics, for example, quantum mechanical (QM) violations of local-realism (LR) [1,2] or macro-realism (MR) [3] are probed through performing incompatible measurements on systems. Ideal measurements, also known as projective measurements, are described by a set of projectors acting on the system's Hilbert space. This is also known as Von Neumann measurement after his seminal work formalising measurement scheme in QM [4]. Later this concept is extended to positive operator valued measurement (POVM) and presently it describes the most general kind of quantum measurements. POVM has operational advantages in many tasks over projective measurements, for example, distinguishing nonorthogonal states [5], demonstrating hidden nonlocality [6,7].

In POVM formalism two observables can be measured jointly even when they do not commute. It is well known [8] that the observables which can be measured jointly do not lead to the violations of Bell–CHSH (Bell–Clauser–Horne–Shimony–Holt) inequalities [1,2]. Moreover for two dichotomic measurements, POVMs are not better than projective measurements in the context of QM violations of Bell–CHSH inequalities [9]. It was shown by considering the implication of quantum entanglement to nonlocal game, which is a kind of cooperative game of incomplete information. Nonlocal game can be described as follows: a referee, who determines the game, randomly chooses questions, drawn from finite sets according to some fixed probability distribution and send them to two players (say, Alice and Bob) at distant locations. Alice and Bob respond to the referee with an answer without communicating themselves. The referee then evaluates some predicate based on the questions asked and their answers, to determine whether they win or lose. Alice and Bob can gain advantage in winning if they share quantum correlations instead of classical correlations [9].

Condition for joint measurability of two noncommuting observables was derived [10] and for two dichotomic observables it is fully characterized [11–13]. It is also shown that for particular two-level observables the border of joint measurability coincides with the one for the violation of the Bell–CHSH inequality [14]. In [15] it has been shown that, for two non-jointly measurable observables at one side, there always exists state and projective measurements for the other side such that the violation of the CHSH inequality is enabled. This result was shown in [15] by casting joint measurability, considering its implicit characterization, as a problem of semi-definite programme (SDP) [16].

Moving to the practical origin of POVM, it is known that they occur in quantum measurement formalism mainly due to two reasons [17]. Firstly, POVM may account for the ever-present imperfections of any measurement and secondly, there are measurement

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situations for which there exists no ideal background observable represented by a projective valued measurement (PVM). Examples of the second reason include genuine phase space observables, as the individual measurement outcomes are fuzzy phase space points in accordance with the Heisenberg uncertainty relation. Regarding the first reason it is known that in Von Neumann measurement scheme there is a cut between classical and quantum domain where quantum systems are measured by classical apparatus. System variable to be measured becomes entangled with classical probe in the process of measurement interaction. By sharply distinguishing different probe states one can achieve PVM. In reality, measurements are usually not PVM reflecting non-zero overlap between the probe states. For detail study one can see [17].

Dichotomic POVM are characterised by two real parameters dubbed as sharpness and biasedness [11,13]. These two parameters can be linked with different types of nonidealness, with respect to ideal projective measurement, involved in the real experimental scenario. Sharpness characterizes measurement precision which is related to the overlap between non-orthogonal probe states [19]. On the other hand, biasedness can be linked with the error in alignment of Stern–Gerlach (SG) apparatus or the deviation from the Gaussian nature of spatial wave-packet of incident spin-$\frac{1}{2}$ particles which is recently shown by some of us [20]. Any physical system is described with respect to some reference frame. For example, spin direction is defined with respect to some gyroscope in the laboratory instead of any purported absolute Newtonian space. Setting up SG apparatus requires a attached reference frame with respect to which the direction of inhomogeneous magnetic field, incident particle beam, position of screen are defined. Incident particle beam may not pass through the center of SG apparatus due to some alignment problem which reflects in the biasedness of POVM measured with such non-ideal device. In information processing tasks communication between different observers without shared reference frame is an interesting area of research and for detailed study one can see [18]. These two quantities, therefore, have well defined physical interpretations beyond mathematical constructions.

It is known that decreasing the sharpness of measurements reduces the possibility of obtaining QM violations of Bell–CHSH inequality [21,22] or Leggett–Garg inequality (LGI) [23–25] and below a certain value of the sharpness parameter, violations of these inequalities are not obtained. In the most general formulation of dichotomic measurements, as we have just mentioned, there is another parameter, apart from sharpness parameter, which is known as biasedness parameter.

In this paper we explore the effect of biasedness of measurements on probing quantumness in the context of QM violation of CHSH inequality as well as in the context of QM violations of three inequivalent necessary conditions for MR, namely LGI [3], Wigner's form of the Leggett–Garg inequality (WLGI) [24] and the condition of no-signalling in time (NSIT) [26]. Inequivalence of these necessary conditions of MR has been studied [27] and it was recently shown [28] that for a particular biased unsharp measurement there exists a state of two level system for which all these necessary conditions of MR are violated for any non-zero value of the sharpness parameter. In another work [29] the effect of biasedness over that of unsharpness of multi-outcome spin measurements has been explored for multilevel spin systems considering a particular measurement scheme.

In case of spatial correlations we find out the effect of the biasedness parameter on the minimum value of the sharpness parameter over which the QM violation of CHSH inequality persists. Furthermore, we derive the necessary and sufficient condition for the violation of CHSH inequality with biased unsharp measurements at one side and projective measurements at another side. As a corollary of this derivation we find out that the violation of the CHSH inequality cannot be enabled with dichotomic POVMs at one side if that is not enabled with projective measurements on both sides. This result is consistent with previous findings [9]. In case of temporal correlations we find out the effect of the biasedness parameter on the minimum values of the sharpness parameter over which QM violations of different necessary conditions of MR persist. Thus it is shown that there is a trade-off between the sharpness parameter and the biasedness parameter characterizing arbitrary dichotomic POVM in the context of probing QM violations of local-realist (LR) and macro-realist (MR) inequalities. It is also observed that the above mentioned trade-off is more robust in the latter case which means that the effect of biasedness parameter counters the effect of unsharpness of measurements more in the latter case.

We organize this paper in the following way. We briefly discuss the characterization of the most general dichotomic POVM in Section 2. In Section 3, we consider the QM violation of the CHSH inequality with most general dichotomic POVM at one side. Then in Section 4 we show the trade-off between sharpness and biasedness parameter in probing the QM violations of three inequivalent necessary conditions of MR, i.e., LGI, WLGI, NSIT. Section 5 contains discussion and concluding remarks.

## 2. Generalized dichotomic measurements

Projective valued measurement (PVM) is a set of projectors that add to identity, i.e., $A \equiv \{P_i | \sum P_i = \mathbb{I}, P_i^2 = P_i\}$ (where $P_i$s are projectors). The probability of getting the $i$-th outcome is given by, $\text{Tr}[\rho P_i]$ for the state $\rho$.

On the other hand, positive operator valued measurement (POVM) is a set of positive operators that add to identity, i.e., $E \equiv \{E_i | \sum E_i = \mathbb{I}, 0 < E_i \leq \mathbb{I}\}$. The probability of getting the $i$-th outcome is $\text{Tr}[\rho E_i]$. Effects ($E_i$s) represent quantum events that may occur as outcomes of a measurement.

In case of dichotomic measurements, the most general POVM is characterized by two parameters – sharpness parameter ($\lambda$) and biasedness parameter ($\gamma$) and the corresponding effect operators are given by,

$$E^{\pm} = \lambda P^{\pm} + \frac{1 \pm \gamma - \lambda}{2}\mathbb{I}, \tag{1}$$

where $P^+$ and $P^-$ are sharp projectors corresponding to the two outcomes $+$ and $-$ respectively. For $E^{\pm}$ being valid effect operators, the positivity ($E^{\pm} \geq 0$) and normalisation condition ($E^+ + E^- = \mathbb{I}$) have to be satisfied. From these conditions it is followed that $|\lambda| + |\gamma| \leq 1$. Sharpness parameter ($\lambda$) characterizes the measurement precision which is related to the overlap between non-orthogonal probe states [19]. We consider $\lambda$ to be positive as negative value of the sharpness parameter has no physical meaning. Eq. (1) with $\gamma = 0$ gives unbiased unsharp measurement, which is also a dichotomic POVM, but not the most general one [17]. For unbiased unsharp measurement $(1 - \lambda)$ characterizes the amount of unsharpness associated with the measurement.

## 3. QM violation of local-realism with most generalized dichotomic measurements

Quantum mechanical predictions are incompatible with local realist theory, which is probed through QM violation of Bell–CHSH inequality. Let us consider two spatially separated parties, say Alice and Bob. Alice performs two dichotomic observables $A$ and $A'$; Bob performs two dichotomic observables $B$ and $B'$. In this scenario the CHSH inequality [2] is given by

$$\langle AB \rangle + \langle AB' \rangle + \langle A'B \rangle - \langle A'B' \rangle \leq 2. \tag{2}$$