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Characterizing quantum phase transition by teleportation

Meng-He Wu [a,b], Yi Ling [b,c], Fu-Wen Shu [a], Wen-Cong Gan [a]

[a] *Center for Relativistic Astrophysics and High Energy Physics, Department of Physics, School of Sciences, Nanchang University, Nanchang 330031, China*
[b] *Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China*
[c] *School of Physics, University of Chinese Academy of Sciences, Beijing 100049, China*

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ABSTRACT

In this paper we provide a novel way to explore the relation between quantum teleportation and quantum phase transition. We construct a quantum channel with a mixed state which is made from one dimensional quantum Ising chain with infinite length, and then consider the teleportation with the use of entangled Werner states as input qubits. The fidelity as a figure of merit to measure how well the quantum state is transferred is studied numerically. Remarkably we find the first-order derivative of the fidelity with respect to the parameter in quantum Ising chain exhibits a logarithmic divergence at the quantum critical point. The implications of this phenomenon and possible applications are also briefly discussed.

1. Introduction

Quantum phase transition (QPT) is a prominent phenomenon caused by quantum fluctuations in a many-body system, reflecting the degeneracy of the ground states [1]. Unlike thermal phase transition which is caused by thermal fluctuations and can always be characterized by some order parameters due to the symmetry breaking, quantum phase transition is very hard to be diagnosed when the system is lack of classical order parameters or manifest symmetry breaking cannot be found. In this circumstance it has been suggested that the entanglement may play a key role in characterizing quantum phase transition. Therefore, the critical behavior of some typical quantities which can be used to measure the degree of entanglement has been extensively investigated in literature, including the quantum concurrence [2], entanglement entropy [3] as well as the fidelity. In particular, the fidelity as a very crucial notion in quantum information science, which measures the quality of information transformation, has been widely used to investigate the occurrence of quantum phase transition [4–10]. Nevertheless, as far as we know, in all previous literature the fidelity used in this context is the Hilbert–Schmidt fidelity which is defined as the overlap between two pure quantum states $F(\lambda, \lambda+\delta\lambda) = |\langle\varphi(\lambda)|\varphi(\lambda+\delta\lambda)\rangle|$, where $|\varphi(\lambda)\rangle$ is a ground state of a many-body Hamiltonian $\hat{H}(\lambda)$, and λ is an external field parameter. Roughly speaking, in those papers the fidelity just measure the difference between two ground states when the system parameter is shifted from λ to $\lambda+\delta\lambda$, while the concept of information loss during the transmission as described in information science is absent in this context. Without surprise, in this setup one can find that the fidelity itself would exhibit extremal behavior at the critical point since two ground states at the critical point are orthogonal to each other in the thermodynamic limit, which is also known as the Anderson orthogonality catastrophe [4,11].

In this paper we intend to provide a novel way to diagnose the occurrence of quantum phase transition by quantum teleportation. In contrast to the strategy as mentioned above, we will construct a specific quantum channel by picking up two qubits in a one dimensional quantum Ising chain with infinite length, and then consider the fidelity when a specific quantum state is teleported through this channel. Now, the fidelity becomes a figure of merit [12] to characterize the quality of transmission indeed. Usually the quantum channel is described by a mixed state with density matrix ρ_c and the fidelity is given by $F(\rho_{in}, \rho_{out}) = Tr\left(\sqrt{\sqrt{\rho_{in}}\rho_{out}\sqrt{\rho_{in}}}\right)$, where ρ_{in} denotes the density matrix of input mixed state while ρ_{out} corresponds to the density matrix of output.

Quantum teleportation was originally proposed by C.H. Bennett et al. in 1993 [13]. An unknown quantum state can successfully be transferred through a quantum channel which is made of a pure but entangled state, given that a classical information channel could also instruct local observers taking appropriate operations. Next applying arbitrary mixed state as the quantum channel associated with the standard teleportation protocol has been demonstrated in [14]. Later on a more specific scheme was proposed to teleport entangled Werner state [15] via thermally entangled states of two-qubit Heisenberg XX chain in [16]. Inspired by this scheme

E-mail addresses: menghewu.physik@gmail.com (M.-H. Wu), lingy@ihep.ac.cn (Y. Ling), shufuwen@ncu.edu.cn (F.-W. Shu), ganwencong@gmail.com (W.-C. Gan).

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we will provide a novel way to construct the quantum channel with a quantum mixed state which is made from the ground state of the quantum Ising chain. It is this key point that makes it plausible to link quantum teleportation to quantum critical phenomenon in our paper. Thanks to the tensor network techniques recently developed in [17–21], we will numerically find the ground states of quantum Ising chain in terms of matrix product states (MPS), then construct the quantum channel by picking up two qubits which could be nearest neighboring or next-nearest neighboring to each other in the quantum Ising chain. By tracing out all the other qubits in the chain the quantum channel will be a mixed state described by a reduced-density matrix.

Our paper is organized as follows. In next section we will present the setup for the construction of quantum channel with MPS. Then in section 3 we will numerically calculate the entanglement entropy and the fidelity of the quantum channel when a Werner state is transferred. More importantly, we will demonstrate that the first order derivative of the fidelity to the system parameter will display a logarithmic divergence at the critical point. We conclude this paper with some discussion on the implications and possible applications of this phenomenon.

2. Basic setup

2.1. The ground states of quantum Ising chain in terms of MPS

In this subsection we will present the setup for the quantum channel of teleportation. We start with the one-dimensional Ising model composed of an infinite spin chain, which is one of the simplest models in many-body physics and exactly solvable [22]. The Hamiltonian of the quantum Ising chain considered in our paper is given by

$$\hat{H} = \sum_{j=1}^{\infty} \sigma_1^j \sigma_1^{j+1} + \lambda \sum_{j=1}^{\infty} \sigma_3^j, \tag{1}$$

which only involves the neighboring interactions of spins and $\sigma_1 = \sigma_x$, $\sigma_3 = \sigma_z$ are ordinary Pauli matrices.

The ground states of above quantum Ising chain with infinite length can be described by matrix product states (MPS) very efficiently [23]. For an MPS with infinite qubits, we will employ infinite time evolving block decimation (iTEBD) algorithm to simulate the ground states of quantum Ising chain [17,24]. This algorithm tells us that starting from any random MPS and performing an imaginary time evolution by acting the Hamiltonian operators on MPS, one could finally reach the ground state of the system provided that the time lasts long enough.

Next we demonstrate the algorithm of iTEBD in our paper briefly, closely following the logic presented in [17]. First, we construct the MPS with infinite length. Because the quantum Ising chain in Eq. (1) has $\mathcal{Z}_2$ symmetry, the infinite chain of MPS is only composed of two distinct pairs of tensors $\{\Gamma_A, \lambda_A, \Gamma_B, \lambda_B\}$ which could be viewed as the unit cells of the system, where λ_A, λ_B are diagonal matrices with non-negative diagonal elements, as shown in Fig. 1. Second, we build the unitary time evolution operator $U = e^{-\hat{H}\delta\tau}$ with the use of the Hamiltonian in Eq. (1), where $\delta\tau$ is a tiny time step. Third, we perform the unitary operation by acting U on the infinite MPS and then contract them into a new tensor Θ, as illustrated in Fig. 2. Fourth, singular value decomposition (SVD) is used to decompose Θ into individual tensors X and Y, as shown in Fig. 2. Lastly, we contract X and Y using matrix λ_B^{-1} and obtain updated Γ_A and Γ_B, as shown in Fig. 2. So far, we have finished the process of updating the unit cells of MPS except for λ_B. By exchanging λ_A and λ_B, we repeatedly perform above process until the ground state is reached within a precision setting.

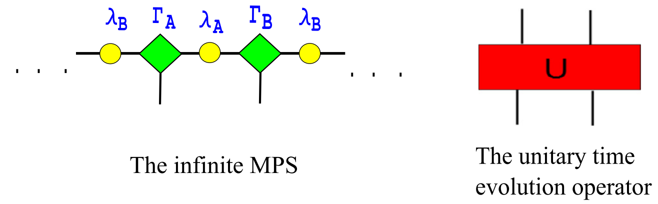

Fig. 1. The left is the infinite MPS, and the right is unitary time evolution operator $U = e^{-\hat{H}\delta\tau}$.

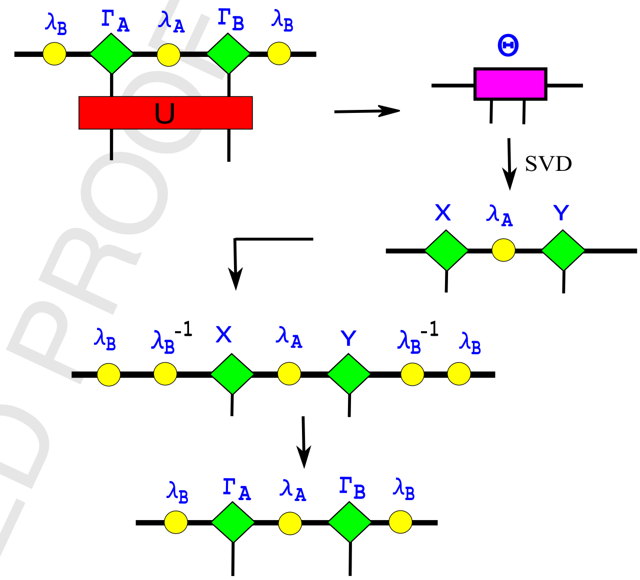

Fig. 2. The process of iTEBD algorithm.

2.2. Teleportation via mixed entangled states

In this subsection we will outline our strategy to construct a quantum channel for teleportation with ground states of quantum Ising chain. Our main purpose is to teleport a specific mixed state with a quantum channel which is also made from a mixed but entangled state. The standard teleportation protocol has previously been appeared in [14,16,25] and we will briefly review their setup as follows. In [14], it is originally shown that the standard teleportation with an arbitrary entangled mixed state χ_{AB} as quantum channel is equivalent to a generalized depolarizing channel $\Lambda(\chi_{AB})$ with probabilities given by the maximally entangled components of the quantum channel χ_{AB}, i.e.

$$\Lambda(\chi_{AB})\rho = \sum_i Tr[P_i \chi_{AB}] \sigma_i \rho \sigma_i, \tag{2}$$

where $P_i = \sigma_i P_0 \sigma_i$ $(i = 0, 1, 2, 3)$ with $P_0 = |\Phi^+\rangle\langle\Phi^+|$ and $|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$. σ_0 is the identity matrix and $\sigma_1 = \sigma_x$, $\sigma_2 = \sigma_y$, $\sigma_3 = \sigma_z$ are Pauli matrices. ρ is the single qubit that we wish to teleport.

Now in this paper, we intend to teleport entangled Werner states with two qubits $\rho_W = \frac{1}{4}(\sigma_0 \otimes \sigma_0 - \frac{2\gamma+1}{3} \sum_{i=1}^{3} \sigma_i \otimes \sigma_i)$, where $0 < \gamma \leq 1$. In paper [16], thermally entangled states of two-qubit Heisenberg XX chain are employed to construct the quantum channel. Given the Hamiltonian of two-qubit Heisenberg XX chain $\hat{H}$, one can write down the density matrix of the thermal entangled state as $\rho_c = \frac{1}{Z} e^{-\hat{H}/kT}$, where $Z = Tr(e^{-\hat{H}/kT})$ is the partition function, while T is the equilibrium temperature and k is Boltzmann constant. Now, taking entangled Werner state as the input and using two copies of the above thermal states as quan-

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