Physics Letters A 382 (2018) 1020–1023

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Langmuir wave phase-mixing in warm electron-positron-dusty plasmas

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ARTICLE INFO

Article history:
Received 22 November 2017
Accepted 14 February 2018
Available online 15 February 2018
Communicated by F. Porcelli

Keywords:
Electron-positron-dusty plasma
Electrostatic Langmuir wave
Phase-mixing
Wave-breaking

ABSTRACT

An analytical study on nonlinear evolution of Langmuir waves in warm electron-positron-dusty plasmas is presented. The massive dust grains of either positively or negatively charged are assumed to form a fixed charge neutralizing background. A perturbative analysis of the fluid-Maxwell's equations confirms that the excited Langmuir waves phase-mix and eventually break, even at arbitrarily low amplitudes. It is shown that the nature of the dust-charge as well as the amount of dust grains can significantly influence the Langmuir wave phase-mixing process. The phase-mixing time is also found to increase with the temperature.

Dusty plasma, as its name suggests, generally carries a small amount of charged dust grains of micrometre or sub-micrometre size along with the electrons, ions and other probable components [1,2], and they are quite ubiquitous in astrophysical environments [3,4]. Charged dust particles can also naturally coexist in electron-positron plasmas maintaining overall charge neutrality in equilibrium [5–7]. Such pair-dusty plasmas are believed to exist in supernova and pulsar environments [8] and have been created in laboratory experiments [9]. In recent decades, a considerable attention has been paid on the role the dust particles playing in the collective effects and strong electromagnetic interaction between the charged particles [10–16]. In this Letter, we investigate what kind of physical effect the presence of dust grains brings in on the phase-mixing of excited electrostatic Langmuir waves in warm electron-positron plasmas. We show here that the process of Langmuir wave phase-mixing can be strongly influenced by the nature and amount of the background dust charge grains.

It is well known that phase-mixing is a novel physical phenomenon which can cause an excited wave or oscillation to break at arbitrary amplitudes, even if the perturbation amplitude is kept well below the threshold value [17–22]. Over the years, breaking of waves/oscillations in plasmas has been discussed as a fundamental topic of research owing of its potential applications like ion and electron heating [23,24], particle acceleration to high energies [25,26], etc. An excited wave or oscillation is called phase-mixed when its characteristic frequency becomes space dependent due to some physical reasons like inhomogeneity [17,27,28], relativistic effects [29,30], etc. Physically, space dependent frequency causes different plasma species to oscillate with different local frequencies, leading to crossing of their trajectories in a finite time (wave-breaking). The particle-bunching and spiky density profile are common signatures of phase-mixing/wave-breaking [30–34].

For our analysis, we assume a system of warm electron-positron plasma containing a small amount of dust grains which are either positively or negatively charged. Massive dusts are considered to form a uniform background with density N_{0d}. In equilibrium, the plasma-species maintain an overall charge neutrality, i.e., $n_{0d} + n_{0p} = n_{0e}$, where n_{0p} and n_{0e}, respectively, denote the unperturbed densities of the positrons and electrons. And, $n_{0d} = sZ_d N_{0d}$, where Z_d quantifies the amount of dust charge on dust grain with $s = +1$ or $s = -1$ for positively or negatively charged dust, respectively. In one space dimension, the space-time evolution of Langmuir waves in electron-positron-dusty (EPD) plasmas can be fairly described by the following electron-positron fluid and Maxwell's equations:

$$\partial_t n_j + \partial_x (n_j v_j) = 0, \tag{1}$$

$$m_j n_j \left(\partial_t + v_j \partial_x\right) v_j = q_j n_j E - \partial_x p_j, \tag{2}$$

$$\partial_x E = 4\pi e n_{0d} + 4\pi \sum_j q_j n_j, \tag{3}$$

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https://doi.org/10.1016/j.physleta.2018.02.020
0375-9601/

where the index j equals 'p' for positrons and 'e' for electrons. And, n_j, v_j, m_j, p_j, and q_j represent the densities, velocities, masses, pressures, and charges of either electrons or positrons, respectively, with $q_j = e$ for the positrons and $q_j = -e$ for electrons. For equal mass species $m_p = m_e = m$. The pressure is assumed to be isotropic: $p_j = \gamma n_j T_j$ [35], where γ is the ratio of the specific heats and T_j denotes the temperature of the j-th species in Boltzmann unit. For simplicity in analysis, we further assume $T_p \simeq T_e = T$. Now linearizing Eqs. (1)–(3) we find the dispersion relation of Langmuir waves in warm EPD plasmas: $\omega^2 = \omega_{pe}^2 + \omega_{pp}^2 + k^2 v_{th}^2$, where $\omega_{pe} = \sqrt{4\pi n_{0e} e^2/m}$ and $\omega_{pp} = \sqrt{4\pi n_{0p} e^2/m}$ denote the electron and positron plasma frequencies, respectively, with $v_{th} = \sqrt{\gamma T/m}$ denoting the thermal velocity of either electrons or positrons.

Next, we perform nonlinear analysis of Eqs. (1)–(3). For the purpose, introducing new variables:

$$\delta n_s = \delta n_p + \delta n_e, \; \delta n_d = \delta n_p - \delta n_e,$$

$$V = v_p + v_e, \text{ and } v = v_p - v_e,$$

where $\delta n_p = n_p - n_{0p}$ and $\delta n_e = n_e - n_{0e}$, followed by normalizing the relevant variables as $t \to \omega_{pe} t$, $x \to kx$, $n_j \to n_j/n_{0e}$, $v_j \to k v_j/\omega_{pe}$, $E \to ekE/m\omega_{pe}^2$, one can simply arrive at the following equations:

$$\partial_t \delta n_d + \partial_x \left[(1-\alpha_d) v_p + (V\delta n_d + v\delta n_s)/2\right] = (\alpha_d/2)\, \partial_x (V - v), \tag{4}$$

$$\partial_t \delta n_s + \partial_x \left[(1-\alpha_d) V + (V\delta n_s + v\delta n_d)/2\right] = -(\alpha_d/2)\, \partial_x (V - v), \tag{5}$$

$$\partial_t V + \partial_x [(v^2 + V^2)/4] = -v_t^2 \partial_x \left(\ln n_p + \ln n_e\right), \tag{6}$$

$$\partial_t v + \partial_x (vV/2) = 2E - v_t^2 \partial_x (\ln n_p - \ln n_e), \tag{7}$$

$$\partial_x E = \delta n_d, \tag{8}$$

where $\alpha_d = n_{0d}/n_{0e}$, denoting the ratio of equilibrium dust density to equilibrium electron density, and v_t is the normalized thermal velocity of either electrons or positrons. We now solve Eqs. (4)–(8) by employing a straightforward perturbation expansion technique subjected to the initial conditions: $\delta n_s(x,0) = \delta \cos x$, $\delta n_d(x,0) = -\delta \cos x$, $V(x,0) = v(x,0) = 0$, where δ is the amplitude of perturbation. Treating δ as a small expansion parameter, we expand all the field variables as $f(x,t) = \sum_{i=1}^{\infty} f^{(i)}(x,t)$ [36]. Inserting the field expansion series into the set of Eqs. (4)–(8) followed by solving the first order equations, we find the following solutions:

$$\delta n_d^{(1)} = \delta \cos x \,[A_1(1 - \cos \bar{\omega} t) - 1],$$
$$\delta n_s^{(1)} = -\delta \cos x \,[B_1(1 - \cos \bar{\omega} t) + B_2(1 - \cos v_t t) - 1],$$
$$V^{(1)} = \delta \sin x \,(v_t \sin v_t t),$$
$$v^{(1)} = \delta \sin x \,[C_1 \sin v_t t - C_2 \sin \bar{\omega} t],$$

and

$$E^{(1)} = \delta \sin x \,[A_1(1 - \cos \bar{\omega} t) - 1], \tag{9}$$

where the coefficients are given by

$$A_1 = \left[\frac{\alpha_d v_t^2 + (2-\alpha_d)(2+v_t^2)}{2\bar{\omega}^2}\right],$$

$$B_1 = \frac{1}{2}\left[\frac{\alpha_d^2 v_t^2}{\bar{\omega}^2(2-\alpha_d)} + \frac{\alpha_d(2+v_t^2)}{\bar{\omega}^2}\right],$$

$$B_2 = \frac{1}{2}\left[(2-\alpha_d) - \frac{\alpha_d^2}{(2-\alpha_d)}\right],$$

$$C_1 = \frac{\alpha_d v_t}{(2-\alpha_d)}, \text{ and } C_2 = \left[\frac{(2+v_t^2)}{\bar{\omega}} + \frac{\alpha_d v_t^2}{\bar{\omega}(2-\alpha_d)}\right],$$

with $\bar{\omega} = \sqrt{(2-\alpha_d) + v_t^2}$, denoting the normalized frequency of the Langmuir waves. The first order solutions given in Eq. (9) shows the presence of Langmuir modes, acoustic modes, and zero frequency modes. The zero frequency modes bear the dominant significance in the context of phase-mixing, because the fast time scale average of $E^{(1)}$ or $\delta n_s^{(1)}$ results in nonzero DC terms, indicating an onset of phase-mixing in the excited Langmuir waves [21].

In the next order, the obtained solutions of a few relevant field variables $V^{(2)}$ and $v^{(2)}$ are given as

$$\begin{aligned} V^{(2)} &= \delta^2 \sin 2x\,[a_0 \sin 2v_t t + a_1 \sin v_t t - a_2\, t \cos 2v_t t \\ &\quad - a_3 \sin \bar{\omega} t - a_4 \sin 2\bar{\omega} t - a_5 \sin(\bar{\omega} + v_t) t \\ &\quad - a_6 \sin(\bar{\omega} - v_t) t], \\ v^{(2)} &= \delta^2 \sin 2x\,[b_0 \sin \omega^* t + b_1 \sin v_t t + b_2 \sin 2v_t t \\ &\quad + b_3 \sin \bar{\omega} t - b_4 \sin 2\bar{\omega} t - b_5 \sin(\bar{\omega} + v_t) t \\ &\quad + b_6 \sin(\bar{\omega} - v_t) t], \end{aligned} \tag{10}$$

showing generation of higher harmonics of Langmuir and acoustic modes, as expected. And, in Eq. (10), the various coefficients are as follows:

$$a_0 = -\frac{a_1}{2} + \frac{a_2}{2v_t} + \frac{a_3 \bar{\omega}}{2v_t} + \frac{a_4 \bar{\omega}}{v_t} + \frac{a_5(\bar{\omega}+v_t)}{2v_t\{(\bar{\omega}+v_t)^2 - 4v_t^2\}} + \frac{a_6(\bar{\omega}-v_t)}{2v_t\{(\bar{\omega}-v_t)^2 - 4v_t^2\}},$$

with

$$a_1 = -\frac{1}{3}\left[(1 - B_1 - B_2)(\lambda_1 v_t + \lambda_2 C_1) + (A_1 - 1)(\lambda_1 C_1 + \lambda_2 v_t)\right],$$

$$a_2 = -\frac{1}{32}[2(v_t^2 + C_1^2) + 4v_t B_2(\lambda_1 v_t + \lambda_2 C_1) - v_t^2 B_2^2(1 + 4\lambda_2^2 \alpha_d^{-2})],$$

$$a_3 = \frac{v_t^2}{4(\bar{\omega}^2 - 4v_t^2)}\left[4C_2\{\lambda_1(A_1 - 1) + \lambda_2(1 - B_1 - B_2)\} + \bar{\omega}(B_1 - A_1)\left\{\left(1 + 4\lambda_2^2 \alpha_d^{-2}\right)(A_1 - B_1 - B_2) + 2(1 - A_1)\right\}\right],$$

$$a_4 = -\frac{1}{32(2-\alpha_d)}\left[2\bar{\omega} C_2^2 + 4v_t^2 C_2(\lambda_1 A_1 - \lambda_2 B_1) - \bar{\omega} v_t^2 \left\{4\lambda_2^2 \alpha_d^{-2}(A_1 - B_1)^2 + (A_1 + B_1)^2\right\}\right],$$

$$a_5 = -\frac{v_t^2}{2\{(\bar{\omega}+v_t)^2 - 4v_t^2\}}\left[\lambda_1(v_t B_1 - A_1 C_1) + \lambda_2(B_1 C_1 - B_2 C_2 - v_t A_1)\right],$$

$$a_6 = \frac{v_t^2}{2\{(\bar{\omega}-v_t)^2 - 4v_t^2\}}\left[\lambda_1(v_t B_1 - A_1 C_1) - \lambda_2(v_t A_1 - B_1 C_1 - B_2 C_2)\right],$$

and

$$b_0 = \frac{1}{\omega^*}[-v_t(b_1 + 2b_2) - \bar{\omega}(b_3 - 2b_4) + (\bar{\omega}+v_t) b_5 - (\bar{\omega} - v_t) b_6],$$