Physics Letters A 382 (2018) 1058–1064

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Quasi-local action of curl-less vector potential on vortex dynamics in superconductors

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ARTICLE INFO

Article history:
Received 8 August 2017
Received in revised form 21 December 2017
Accepted 23 December 2017
Communicated by L. Ghivelder

Keywords:
TDGL finite element modeling
Phase slip centers
Abrikosov vortex motion
Aharonov–Bohm effect
Curl-less potential observability

ABSTRACT

Studies of the Abrikosov vortex motion in superconductors based on time-dependent Ginzburg–Landau equations reveal an opportunity to detect the values of the Aharonov–Bohm type curl-less vector potentials without closed-loop electron trajectories encompassing the magnetic flux.

1. Introduction

Anyone who has dealt with Maxwell's equations is familiar with the vector potential $\boldsymbol{A}$. Originally, it was introduced by Maxwell as a result of the influence of Faraday's concept of the "electrotonic state" (see, *e.g.*, Ref. [1]). Later, however, Heaviside and Hertz eliminated the need for $\boldsymbol{A}$ in these equations [2]. Indeed, because of gauge invariance, the addition of the gradient of an arbitrary function to $\boldsymbol{A}$ does not affect the predicted electric $\boldsymbol{E}$ and magnetic $\boldsymbol{H}$ fields as the solution of Maxwell's equations. Thus, it was believed that $\boldsymbol{A}$ did not fit the definition of a real field and was rather an auxiliary, mathematical tool for solving equations for $\boldsymbol{E}$ and $\boldsymbol{H}$, the "real" fields.

This point of view, however, has been altered with the appearance of quantum mechanics. The phase of the wave function of charged quantum objects couples with the vector potential in a way that makes gauge transformations themselves, not the vector potential, merely mathematical tools. Moreover, there are subtle effects in the quantum world, such as the Aharonov–Bohm (AB) effect [3], in which electrons interfere in the presence of the curl-less $\boldsymbol{A}$-field. Such a field can be created, for example, by a very long solenoid. In the area of electron propagation, both the local $\boldsymbol{E}$ and $\boldsymbol{H}$ are zero, but electron motion is affected by the $\boldsymbol{A}$-field [4]. In his classical text [5], Feynman defined the real field as a "mathematical device for avoiding the idea of action at a distance." Commenting on the AB-effect, he came to the conclusion that "the classical electromagnetic field acting locally on a particle is not sufficient to predict its quantum-mechanical behavior". It follows from this statement that either $\boldsymbol{H}$ should yield its role to $\boldsymbol{A}$, which then acquires a status of a real field, or, alternatively, the magnetic field inside the solenoid (*i.e.*, in its core) acts non-locally on electronic motion outside of the solenoid. Such a "*spooky action at a distance*", as Einstein would have said [6], follows from Stokes' theorem:

$$\oint_C \boldsymbol{A}\cdot d\boldsymbol{l} = \int_S curl\,\boldsymbol{A}\cdot d\boldsymbol{s} = \int_S \boldsymbol{H}\cdot d\boldsymbol{s} = \Phi \tag{1}$$

where S is a surface strained on the contour C and Φ is the magnetic flux inside of the solenoid (see, *e.g.*, [7]). The contour C follows a closed trajectory of electronic wave propagation. In all the experiments (see, *e.g.*, Ref. [8] and citations therein), the detection becomes possible only if the electron trajectories fully encircle the source of the curl-less vector potential.

In this article, a new effect is uncovered theoretically. It can be used for the detection of the curl-less $\boldsymbol{A}$. In principle, the detection of the curl-less $\boldsymbol{A}$ occurs in the AB-effect. However, in the case of the AB-effect, the presence of a "quantum ring" around the source of magnetic flux is mandatory. In our case, the vector-potential is being detected quasi-locally in space, without encir-

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https://doi.org/10.1016/j.physleta.2017.12.046
0375-9601/

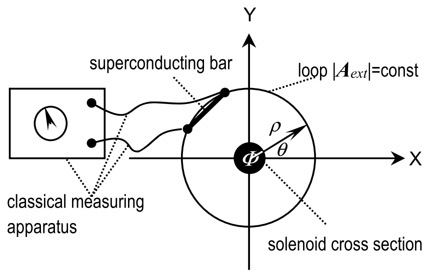

Fig. 1. Experimental setup. The (x, y)-plane is orthogonal to the axis of the solenoid. The vector $\boldsymbol{A}$ is tangential to the loop with an amplitude $A_\theta(\rho) \equiv A(\rho)$. $\Phi = H_0\pi R^2$ is the magnetic flux inside the solenoid.

cling the source of the magnetic field by a quantum ring. The curl-less static $\boldsymbol{A}$-field affects nonlinear quantum dynamics in a short, thin, current-carrying superconducting strip. The resulting features due to the $\boldsymbol{A}$-field are detectable by classical instruments.

2. Choice of model and basic equations

2.1. General physical setup

Let us consider an infinitely long solenoid and direct the z-axis of the coordinate system along it. Then, the curl-less vector potential, $\boldsymbol{curl}\,\boldsymbol{A}_{ext} = \boldsymbol{H}_{ext} = 0$, in cylindrical coordinates (z, ρ, θ), is:

$$\boldsymbol{A}_{ext}(z,\rho,\theta) = (A_z, A_\rho, A_\theta) = (0, 0, \alpha/\rho), \qquad \rho > R \tag{2}$$

where $\alpha = H_0R^2/2$, H_0 is the magnetic field amplitude inside the solenoid, and R is the radius of the solenoid (Fig. 1).

In this experimental layout, the quantum object, a superconducting bar, is affected by the vector potential, which is tangential to the circumference to which the bar is a chord. Obviously, if the bar is short enough, $\boldsymbol{A}_{ext}$ along its length may be regarded approximately as a constant at a given distance $\rho = \rho_0$ from the solenoid. Moreover, if the width of the bar is much smaller than the distance ρ_0, one can neglect the variation of $\boldsymbol{A}_{ext}$ across the cross section of the bar. Thus, we consider $\boldsymbol{A}_{ext}$ inside the bar to be directed along its length and constant in space and time. Two cases are still possible: $\boldsymbol{A}_{ext} = \boldsymbol{A}_0$ and $\boldsymbol{A}_{ext} = -\boldsymbol{A}_0$. Switching the current direction in the solenoid will change the direction of the vector potential: $\boldsymbol{A}_{ext} \to -\boldsymbol{A}_{ext}$. It is very important to notice that such an action should be performed with special care to have justifiable experimental results: any temporal variation of the $\boldsymbol{A}$-field creates an electric field $\boldsymbol{E} = -\partial\boldsymbol{A}/\partial t$ in the reference frame coupled with the superconducting bar (here and below, we use the units $\hbar = c = e = 1$), and the effects imposed upon the bar can be attributed to $\boldsymbol{E}$. To avoid this, one should start the experiment at temperatures above the critical temperature of the superconductor ($T > T_c$), mount the bar in the vicinity of the solenoid (as shown in Fig. 1), set up the value of Φ (*i.e.*, $\boldsymbol{A}_{ext}$), connect the bar to a classical measuring circuit, and then only cool down to $T < T_c$ in order for the bar to become superconductive. At the instant of superconducting transition, the coupling between the phase of the Cooper pairs' condensate wave function and the vector potential is setting up. After equilibrium is reached, one can switch on the external current through the bar and watch the classically observable dynamic quantum effects. The expected events will be modeled in this article by the time-dependent Ginzburg–Landau (TDGL) equations.

2.2. TDGL equations

The motion of current in the superconducting bar in Fig. 1 is affected by two fields: i) the field induced by a "classical measuring apparatus", which principally represents a current source and a voltmeter measuring the voltage across the bar; ii) the field of the solenoid described above. To describe the current flowing through this bar, we utilize a set of TDGL equations [9–15] in the simplest "gapless" limit [9]. This is not an oversimplification: the condensate of Cooper pairs fully reveals its quantum properties in this limit. Moreover, we also performed calculations in the more general "finite gap" case with a qualitatively similar outcome. The adopted restriction makes the consideration more transparent.

The equation for the order parameter $\Delta = |\Delta|\exp(i\theta)$ can be written in the form:

$$-\frac{\pi}{8T_c}\left(\frac{\partial}{\partial t} + 2i\varphi\right)\Delta + \frac{\pi}{8T_c}\left[D\left(\nabla - 2i\boldsymbol{A}\right)^2\right]\Delta + \left(\frac{T_c - T}{T_c} - 7\zeta(3)\frac{|\Delta|^2}{8(\pi T_c)^2}\right)\Delta = 0. \tag{3}$$

Here, $\boldsymbol{A}$ and φ are vector and scalar potentials of the electromagnetic field, T_c is the critical temperature of the superconductor, D is the electronic diffusion coefficient, and $\zeta(3)$ is the Riemann zeta function. Obviously, $\boldsymbol{A}$ should include both the contribution of the solenoid and the contribution of the current flowing through the superconductor. For the numerical modeling which follows, Eq. (3) should be rewritten in its dimensionless form [16–18]:

$$\left(\frac{\partial}{\partial\tau} + i\phi\right)\Psi = -\left(\frac{i}{k}\nabla + \mathbf{A}\right)^2\Psi + \left(1 - |\Psi|^2\right)\Psi. \tag{4}$$

Here, $\kappa = \lambda_L/\xi$ is the Ginzburg–Landau parameter, $\Psi = \Delta/\Delta_0$, $\tau = tD/\xi(T)^2$, $\phi = 2\varphi\xi(T)^2/D$, and $\mathbf{A} = 2\boldsymbol{A}\xi$, where $\xi^2(T) = \pi D/[8(T_c - T)]$ and $\Delta_0^2 = 8\pi^2T_c(T_c - T)/[7\zeta(3)]$. In the same notations, the equation for the current density can be represented as

$$\sigma\left(\frac{\partial\mathbf{A}}{\partial\tau} + \nabla\phi\right) = \frac{1}{2ik}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) - |\Psi|^2\mathbf{A} - \nabla\times\nabla\times\mathbf{A}. \tag{5}$$

Note that equations (4) and (5) are gauge-invariant, and we can use the most convenient gauge when solving them. In Eq. (5), one can recognize the current expression $\boldsymbol{j} = \boldsymbol{j}_n + \boldsymbol{j}_s$, where $\boldsymbol{j}_n = -\sigma(\dot{\mathbf{A}} + \nabla\phi)$ is the normal component of the current $\boldsymbol{j} = \nabla\times\nabla\times\mathbf{A}$, and the rest is the superconducting component $\boldsymbol{j}_s$. We chose the gauge $\phi = 0$. This choice is common when solving TDGL equations (see, *e.g.*, [16–18]). Other choices are also feasible, and their well-posedness is proven for both the finite element, finite difference, and finite volume settings (see, *e.g.* Ref. [19] and citations therein). For our particular task, the gauge $\phi = 0$ is convenient because the solenoidal potential is static and is not associated with any electric field: $\boldsymbol{E}_{sol} = -\partial\boldsymbol{A}_{ext}/\partial t + \nabla\phi \equiv 0$. This does not mean that electric fields are excluded from consideration: the $\boldsymbol{A}$-field inside the bar, as we will see, is time dependent and the $\boldsymbol{E}$-field plays an important role in the dynamics to be considered.

2.3. Equations in 2D- and 1D-cases

TDGL equations in the $\phi = 0$ gauge have been successfully used in various 1D and 2D problems. In particular, the dynamics of the penetration of single-flux quantum vortices into superconducting disks have been studied [16,17]. In these cases, a magnetic field is applied externally to a thin film of finite geometry. Alternatively, in the case of no applied external $\boldsymbol{H}$-field, one can consider a finite strip of a superconductor, in which the current enters through one facet and exits through the opposite facet (Fig. 2).

The flow of this current generates a magnetic field with the field lines encircling the strip and thus directed oppositely at the horizontal edges of the strip. This field also initiates single flux