



A scheme of quantum state discrimination over specified states via weak-value measurement



Xi Chen ^{a,*}, Hong-Yi Dai ^{b,c}, Bo-Yang Liu ^a, Ming Zhang ^{a,*}

^a College of Artificial Intelligence, National University of Defense Technology, Changsha, 410073, China

^b Department of Physics, National University of Defense Technology, Changsha, 410073, China

^c Interdisciplinary Center for Quantum Information, National University of Defense Technology, Changsha, 410073, China

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ABSTRACT

The commonly adopted projective measurements are invalid in the specified task of quantum state discrimination when the discriminated states are superposition of planar-position basis states whose complex-number probability amplitudes have the same magnitude but different phases. Therefore we propose a corresponding scheme via weak-value measurement and examine the feasibility of this scheme. Furthermore, the role of the weak-value measurement in quantum state discrimination is analyzed and compared with one in quantum state tomography in this Letter.

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1. Introduction

Quantum state estimation (QSE) from experimental data is of importance in the quantum information processing and the quantum control, since a quantum state possesses all information about a quantum system. The QSE can be categorized by different task goals: it is called a quantum state tomography (QST) when a totally unknown state is determined based on practical measurement outcomes and the quantum state discrimination (QSD) refers to selecting a state closest to the practical measurement outcome from a fixed quantum state set with finite elements. To a certain degree, QST is a QSD with a set of infinite elements.

One of applications for the QSD is the transmission of classical information through quantum channels, *i.e.* distinguishing assigned quantum states with assigned probabilities via quantum measurements [1]. During such transmission the legal sender and receiver both know the *a priori* information about the target state, which illegal users do not know in principle. The following value range can be considered as the *a priori* information: the value range of each measurement outcome under different operators corresponding to every state in the set. Such *a priori* information is a necessary condition in the QSD, because the QSD must work against a fixed state set. On the contrary, the QST requires no *a priori* information.

* Corresponding authors.

E-mail addresses: chenxi_nudt@163.com (X. Chen), zhangming@nudt.edu.cn (M. Zhang).

Furthermore, the indistinguishability of non-orthogonal quantum states lies in a core position in the quantum cryptography for protecting the information security [2], thus the security of quantum key distribution is vastly investigated under various attacks including the sequential unambiguous state discrimination attack [3–5].

Nowadays, QST and QSD are mainly performed under the projective measurement (PM). Fan [6] analyzes the distinguishability of spatially-separated quantum states under local operations and classical communication, where the measurement method in local operations is PM. Mohseni et al. [7] realize the optimal unbiased discrimination of quantum pure states as well as mixed states under the generalized measurement on the optical system, where PM is still the core process of the generalized measurement. Scott [8] summarizes a kind of informationally complete positive-operator-valued measurement (POVM) and points that this kind of measurement is optimal for quantum cloning and linear QST, where the POVM can be considered as a generalization of the PM.

Although PM plays an important role in QSD, it is still not omnipotent. According to the basic postulate of quantum mechanism about measurement [9], the PM outcomes are eigenvalues of the employed observable operator, and each measurement outcome occurs practically in a certain probability associated with the object state, meanwhile the object state after the measurement collapses into an eigenstate of the observable operator corresponding to the emerged eigenvalue.

As we know, based on the continuity of the projector's eigenvalue, the measured object state can be classified into two cat-

Table 1
Some specifically named quantum measurements under combinations of a few measurement features.

Name	Framework	Interaction strength	Interaction manner	Remark
Projective	standard	strong	deterministic	also called von Neumann measurement [18]
PPS	PPS	strong	deterministic	see Ref. [19]
Continuous	standard	strong	probabilistic	continuum of such measurements and each measurement operator tends to the identity (or vanish) as its time $t \rightarrow 0$ [20]
Weak consecutive	standard	weak	deterministic	continuum of such measurements [21]
Null-value	PPS	strong	probabilistic	see Ref. [22]
Weak-value	PPS	weak	deterministic	see Ref. [23]

egories: the continuous-variable state [10,11] and the discrete-variable state. One typical state for the continuous-variable one is the coherent state of an optical field [12–14], which is described with a complex number. This kind of coherent state is usually discriminated through the homodyne detection [11,13,15], which can be considered as a PM on the phase of the state-describing complex number, achieved by adjusting the phase of the local oscillator in the homodyne detection setup. A typical state for the discrete-variable one is the two-dimensional qubit state. The available projection basis is limited by the measurement devices for QSD, so the unambiguous state discrimination [9,16] with a certain probability of no determined decision under specific PMs is usually used to discriminate qubit states.

Let us consider a kind of QSD task for different planar laser-energy distributions [17] where the specified discriminated states are described as finite and high-dimensional superposition of planar-position basis states whose complex-number probability amplitudes have the same magnitude but different phases, the detailed description will be given in Sec. 2. For this specified kind of QSD task, the available PMs are toward the bases of planar-position states and can only provide the squared modules of the magnitudes of the complex-number probability amplitudes. It should be underlined that the homodyne detection is not suitable for this specified kind of QSD task. In order to accomplish the specified QSD task where the PM can not work, we propose a scheme with a different quantum measurement method in this Letter.

The measurement on a quantum system is one of the postulates of quantum mechanics, which is described only with measurement operators performed on the target quantum system. Another target-probe interaction model of quantum measurement is needed when both target and probe are quantum systems. In such model the standard framework consists of three steps [18]: (i) preparing initial states for both systems independently, (ii) making the probe interact with the target, and (iii) measuring the probe for a readout related to the target. Another measurement framework has been introduced by Aharonov, Bergmann, and Lebowitz [19], and pre- and post-selected (PPS) steps are included in their measurement framework where the preparing step is considered as an initial state pre-selection and a final state post-selection is performed on the target between the steps of interacting and measuring. In both frameworks the interaction strength can be either strong (at the maximum strength) or weak (below the maximum strength), and the interaction manner can be either deterministic or probabilistic. Some combinations of these measurement features are no other than those specifically named quantum measurements, listed in Table 1.

Here we try to adopt the weak-value measurement (WVM) to accomplish the aforementioned specified QSD task. Although QSD via weak measurements is not a new idea, e.g. previously discussed in papers [20–22], the WVM method is not utilized in these papers. In addition Jordan et al. [24] utilizes the weak-value

amplification effect to explain the Heisenberg scaling with weak measurement from a QSD point of view, however the targets under discrimination are two-dimensional qubits, which is also different from our specified QSD task. Nonetheless, these papers provide us with important reference on establishing a QSD scheme.

The weak-value is first proposed by Aharonov, Albert, and Vaidman [23] in 1988, which is different from PM outcomes, and it appears when averaging multiple outcomes of repeated pre- and post-selected measurements under a weak deterministic interaction. Recently with the development of WVM experiments [25, 26], it is realized that the new quantum-object-describing variable “weak-value” is worthy of thorough research. Bin Ho and Imoto [27] generally express the relation between the weak value and the spin-operator modular value. Sokolovski shows the meaning of “anomalous weak values” in quantum and classical theories [28] and relates this meaning to some paradoxes [29,30]. Lee and Tsutsui [31] present an inequality of uncertainty relations in metrology and analyze its feature with the help of weak values. Moreover, the QST via WVM is also extensively investigated [17, 32–41].

In this Letter we first investigate the feasibility of the WVM scheme for the above-mentioned specified QSD task, then discuss the different roles of WVM in QSD and QST tasks. In the design of this new scheme, we consider similar features with the QST via WVM, meanwhile notice the specific difference in the task goal and in the required *a priori* information about the target state. Moreover, the experience on the WVM shows that there is a steady relation, existing in both QSD and QST, among the quantity of the collected experimental data, the accuracy of the WVM outcome and the confidence level in statistics. Therefore, the contribution of this research may enrich the WVM application in the QSD task and deepen the understanding about the role of WVM in different tasks, in addition the above mentioned relation may provide a suggestion in principle on the experiment parameter setting.

The rest of this Letter is organized as follows. Sec. 2 presents the general framework of the proposed new scheme and demonstrates it with an example, in which the particular *a priori* information about the target state is utilized in the design of this example scheme, and its process is theoretically analyzed. In Sec. 3 the difference between QSD and QST via WVM is investigated through numerical simulation on the task goal, the number of times for a WVM procedure, and the required execution time. In Sec. 4 the relation among the quantity of the collected experimental data, the accuracy of the WVM outcome and the confidence level in statistics is discussed. At last, Sec. 5 summarizes the Letter.

2. The quantum state discrimination scheme via weak-value measurement

Since QSD is conducted after holding the *a priori* information about the target state, and such information is various under differ-

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