Physics Letters A 382 (2018) 968–972

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Stabilization of the Peregrine soliton and Kuznetsov–Ma breathers by means of nonlinearity and dispersion management

J. Cuevas-Maraver [a,b,*], Boris A. Malomed [c,d], P.G. Kevrekidis [e], D.J. Frantzeskakis [f]

[a] Grupo de Física No Lineal, Departamento de Física Aplicada I, Universidad de Sevilla, Escuela Politécnica Superior, C/ Virgen de África, 7, 41011 Sevilla, Spain
[b] Instituto de Matemáticas de la Universidad de Sevilla (IMUS), Edificio Celestino Mutis, Avda. Reina Mercedes s/n, 41012 Sevilla, Spain
[c] Department of Physical Electronics, School of Electrical Engineering, Faculty of Engineering, Tel Aviv University, Tel Aviv 69978, Israel
[d] ITMO University, St. Petersburg 197101, Russia
[e] Department of Mathematics and Statistics, University of Massachusetts, Amherst, MA 01003-9305, USA
[f] Department of Physics, National and Kapodistrian University of Athens, Panepistimiopolis, Zografos, Athens 15784, Greece

ARTICLE INFO

Article history:
Received 22 November 2017
Received in revised form 31 January 2018
Accepted 6 February 2018
Available online 9 February 2018
Communicated by A.P. Fordy

Keywords:
Nonlinear Schrödinger equation
Rogue waves
Modulational instability
Dispersive shock waves
Dark solitons

ABSTRACT

We demonstrate a possibility to make rogue waves (RWs) in the form of the Peregrine soliton (PS) and Kuznetsov–Ma breathers (KMBs) effectively stable objects, with the help of properly defined dispersion or nonlinearity management applied to the continuous-wave (CW) background supporting the RWs. In particular, it is found that either management scheme, if applied along the longitudinal coordinate, making the underlying nonlinear Schrödinger equation (NLSE) self-defocusing in the course of disappearance of the PS, indeed stabilizes the global solution with respect to the modulational instability of the background. In the process, additional excitations are generated, namely, dispersive shock waves and, in some cases, also a pair of slowly separating dark solitons. Further, the nonlinearity-management format, which makes the NLSE defocusing outside of a finite domain in the transverse direction, enables the stabilization of the KMBs, in the form of confined oscillating states. On the other hand, a nonlinearity-management format applied periodically along the propagation direction, creates expanding patterns featuring multiplication of KMBs through their cascading fission.

1. Introduction

The nonlinear Schrödinger equation (NLSE) and its variants are well known as universal models for nonlinear waves and solitons, as well as relevant phenomenology, in many areas of physics including water waves, plasmas, nonlinear optics, Bose–Einstein condensates, and so on [1–9]. Among various solutions of these equations, a class of unstable but physically meaningful ones represent rogue waves (RWs), which can spontaneously emerge on top of continuous-wave (CW) modulationally (alias Benjamin–Feir [10,11]) unstable states, and then disappear. RWs were originally identified in terms of water waves in the ocean [12]. Later, this concept was extended to nonlinear fiber optics [13–17] and other areas (see, e.g., Refs. [18–21]). Recently, the pioneering work of [22] argued that the so-called Peregrine solitons (PSs) are a generic byproduct of a phenomenon called gradient catastrophe arising at the level of the semi-classical form of the NLSE. Moreover such solutions also emerged in the context of interactions of dispersive shock waves [23]. An overview of the current state of the studies of RWs can be found in Ref. [24,25].

The classical integrable NLSE with the cubic self-focusing nonlinearity, in terms of the spatial-domain propagation (or with the anomalous group-velocity dispersion (GVD), in terms of fiber optics [2]) gives rise both to the CW states subject to the modulational instability, and to exact RW solutions, the most fundamental ones being the Peregrine soliton (PS) [26], the Kuznetsov–Ma breather (KMB) [27,28], and the Akhmediev breather [29]. The PS is a state of an instanton type built on top of the CW background, i.e., it is localized both in the longitudinal and transverse coordinates (if the NLSE is considered as a model of a planar waveguide in the spatial domain). The KMB, on the other hand, is localized in the transverse direction, and periodically oscillates in the longitudinal one, while the Akhmediev breather [29], is periodic in the transverse direction and self-localized along the propagation distance. Due to the fact that all these states are supported by the modulationally unstable background, they are unstable too, which poses a limitation to their physical realizations; even when they are carefully realized experimentally [17], the modulational instability of the background cannot be avoided. On the other hand, the

* Corresponding author.
E-mail address: jcuevas@us.es (J. Cuevas-Maraver).

https://doi.org/10.1016/j.physleta.2018.02.013
0375-9601/

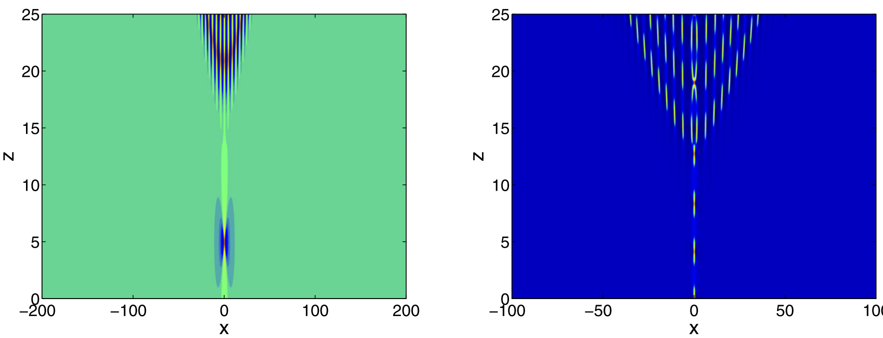

Fig. 1. Density plots illustrating the evolution of a Peregrine soliton (left) and a Kuznetsov–Ma breather with $\omega = 1.5$ (right) in the framework of the constant-coefficient NLSE (1), which does not include any management.

concept of the dispersion and nonlinearity management [6,30] suggests a possibility to stabilize RWs by making the GVD and/or local nonlinearity coefficients functions of the propagation distance or transverse coordinate. This way, the solitons and breathers would have enough room to emerge in areas where the NLSE is self-focusing, and, on the other hand, the background may be globally stabilized by making the NLSE self-defocusing outside of the area reserved for the formation of the RWs. The objective of the present work is to demonstrate the "proof of principle" as regards these possibilities for the effective stabilization of the PS and KMBs, applying the schemes of both the dispersion and nonlinearity management. While our focus here is on numerical experiments, the existence [30] and earlier experimental implementation [30–32] of related schemes suggests their potential realization in (near-)future optical and related physical systems.

The paper is organized as follows. The model and numerical methods used for its analysis are presented in Section 2. The results obtained for the stabilization of the PS and KMBs, under the action of the management, are reported, respectively, in Sections 3 and 4 (while both the dispersion and nonlinearity management are applied to the PS, only the latter scheme is considered for the KMBs). Finally, the paper is concluded by Section 5.

2. The model and numerical scheme

The NLSE which we use for the stabilization of the PSs and KMBs is taken as

$$iu_z + \frac{1}{2}D(z)u_{xx} + \gamma(x,z)|u|^2u = 0. \quad (1)$$

In the spatial domain, which corresponds to the light propagation in a planar waveguide, the diffraction coefficient is constant, $D(z) \equiv 1$, while the local nonlinearity coefficient may be modulated as a function of the propagation and transverse coordinates, z and x [6]. In the temporal domain, corresponding to the light propagation in an optical fiber, x is actually the reduced time, $\tau \equiv t - z/V_{\mathrm{gr}}$ (t is time proper, and V_{gr} is the group velocity of the carrier wave), the relevant fiber's model has $\gamma(x,z) \equiv 1$, while the GVD coefficient, $D(z)$ may be made a function of the propagation length, using known techniques of the GVD management [15,30].

The integrable version of the NLSE, i.e., Eq. (1) with $D(z) \equiv 1$ and $\gamma(x,z) \equiv 1$, gives rise to the exact PS [26] and KMB [27,28] solutions:

$$u_{\mathrm{PS}}(x,z) = \left[1 - \frac{4(1+2\mathrm{i}z)}{1+4x^2+4z^2}\right]\mathrm{e}^{\mathrm{i}z}, \quad (2)$$

$$u_{\mathrm{KMB}}(x,z) = \left[1 + \frac{2(1-2a)\cos(\omega z) - \mathrm{i}\omega\sin(\omega z)}{\sqrt{2a}\cosh(bx) - \cos(\omega z)}\right]\mathrm{e}^{\mathrm{i}z}, \quad (3)$$

where $a \equiv \left(1+\sqrt{\omega^2+1}\right)/4$ and $b \equiv 2\sqrt{2a-1}$, while ω is an arbitrary frequency of the KMB oscillations. As explained in the Introduction, both solutions are supported by the CW background, $\exp(iz)$, which is prone to the modulational instability.

To demonstrate effects of management, we present here results of numerical simulations of Eq. (1) with initial condition:

$$u(x,0) = u_{\mathrm{PS}}(x,z_0), \; z_0 = -5, \quad (4)$$

when dealing with PS (the choice of $z_0 = -5$ is appropriate for demonstrating both the growth and the decay phase of the wave structure). In the case of KMBs, the input is taken as:

$$u(x,0) = u_{\mathrm{KMB}}(x,0). \quad (5)$$

In the latter case, we set $\omega = 1.5$ here, as this value was found to be appropriate for representing the generic situation. Note that, as RW solutions possess relatively steep peaks, the present version of the NLSE is a mildly stiff equation for simulations, in these cases. To handle it, we have used the exponential time differencing fourth-order Runge–Kutta numerical algorithm [33]. The discretization of the second derivative was performed by dint of the Fourier spectral collocation, implying periodic boundary conditions imposed on the integration domain, $-L < x < +L$. Here we report results produced for $L = 200$, and a discretization spacing $\Delta x = 25/256 \approx 0.10$, as well as a time step $\Delta t = (\Delta x)^2/4$. These parameters ensure the stability of the numerical integration.

Fig. 1 shows the outcome of the simulations performed for the NLSE (1) in the absence of management, $D = \gamma \equiv 1$, using the above-mentioned PS and KMB wave forms as initial conditions. The onset of the modulational instability, seeded by truncation errors of the numerical algorithm, is clearly observed at the center of the domain. It is natural that this occurs there, as the presence of the PS amplifies growing perturbations on top of the unstable background. Notice that, recently, the instability of the KMB – and by extension of the PS in the limit of vanishing frequency – was analyzed via Floquet theory in Ref. [34].

3. The management of Peregrine solitons

First, we test the effects of the management applied to the PS. For this purpose, we have performed simulations of Eq. (1) with either $D \equiv 1$ and z-dependent nonlinearity $\gamma(z)$, or vice versa. As we show below, in both cases outcomes are quite similar. The nonlinearity management is implemented as:

$$\gamma(x,z) = \begin{cases} 1 & \text{at } z < z_1 \\ -1 & \text{at } z \geq z_1 \end{cases}, \qquad D \equiv 1, \quad (6)$$

i.e., the originally focusing nonlinearity switches to defocusing at $z = z_1$, while the dispersion management can be introduced as