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Effects of entanglement in an ideal optical amplifier

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ABSTRACT

In an ideal linear amplifier, the output signal is linearly related to the input signal with an additive noise that is independent of the input. The decoherence of a quantum-mechanical state as a result of optical amplification is usually assumed to be due to the addition of quantum noise. Here we show that entanglement between the input signal and the amplifying medium can produce an exponentially-large amount of decoherence in an ideal optical amplifier even when the gain is arbitrarily close to unity and the added noise is negligible. These effects occur for macroscopic superposition states, where even a small amount of gain can leave a significant amount of which-path information in the environment. Our results show that the usual input/output relation of a linear amplifier does not provide a complete description of the output state when post-selection is used.

1. Introduction

A linear optical amplifier multiplies the input signal by a constant gain g while adding noise that is independent of the input [1–8]. It is generally assumed that all of the degradation of a quantum state that occurs during amplification is due to the addition of quantum noise. Here we show that entanglement between the input signal and the amplifying medium in an ideal optical amplifier will generate "which-path" information that can produce an exponentially-large amount of decoherence even when the gain is arbitrarily close to unity and the added noise is negligibly small. This situation occurs for inputs that are macroscopic superposition states, such as a Schrodinger cat, where even a small amount of gain can result in a significant amount of which-path information left in the environment. Our results show that the usual linear input/output relation of an optical amplifier does not completely describe the output state when post-selection techniques are used to analyze the output.

To be more precise, the output of any linear optical amplifier in the Heisenberg picture is given by [1–8]

$$\hat{x}_{out} = g\hat{x}_{in} + \hat{N}. \tag{1}$$

Here $\hat{x} \equiv (\hat{a} + \hat{a}^\dagger)/\sqrt{2}$ is one of the phase quadratures of the signal field, where $\hat{a}$ is the corresponding photon annihilation operator and $\hat{x}_{in}$ and $\hat{x}_{out}$ describe the input and output of the amplifier. $\hat{N}$ is a noise operator that commutes with $\hat{x}_{in}$, and a similar equation describes the other phase quadrature $\hat{p} \equiv (\hat{a} - \hat{a}^\dagger)/\sqrt{2}i$. The statistical properties of the quantum noise $\hat{N}$ have been analyzed in detail [8] and it is generally considered to be the limiting factor in the performance of an ideal optical amplifier.

In the limit where $g \to 1$, $\hat{N} \to 0$ for an ideal amplifier and Eq. (1) would seem to imply that there should be no significant difference between the input and output fields. That is not the case for macroscopic superposition states as will be shown below. Although Eq. (1) is mathematically correct, $\hat{x}_{out}$ from Eq. (1) cannot be used to calculate the variance and other higher-order moments when post-selection is used. More generally, we will show that the Heisenberg picture approach of Eq. (1) is not equivalent to using the Schrodinger picture when non-unitary transformations are applied, as is the case in post-selection.

An optical parametric amplifier (OPA) is a commonly-used example of a linear amplifier, and Caves et al. [8] showed that any ideal (phase-insensitive) linear amplifier can be modeled by an OPA. It is well known that an OPA produces entanglement between the output signal and another optical mode known historically as the idler, as illustrated in Fig. 1. For an OPA, $\hat{N} = -\sqrt{g^2 - 1}\hat{q}_{in}$, where $\hat{q}_{in}$ is the $\hat{x}$ quadrature in the idler mode. The same results apply to any ideal linear amplifier, including those based on an inverted atomic medium [9]. We will use an OPA to illustrate the effects of entanglement on a quantum signal.

The linear relationship of Eq. (1) is only valid in the limit of a strong pump, where the effects of saturation and fluctuations in the pump field are negligible. We will assume throughout that this condition is satisfied and that the pump can be treated classically. A number of earlier papers [10–16] have investigated nonlinear

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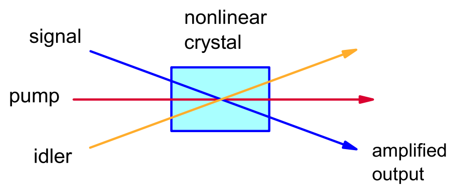

Fig. 1. Amplification of a signal by a parametric amplifier. The Hamiltonian corresponds to the annihilation of a photon from the pump beam accompanied by the emission of a photon in both the signal and idler modes. The which-path information produced by the entanglement between the signal and idler modes can produce an exponentially-large amount of decoherence even in the limit of small gain and negligible added noise.

phenomena that can occur when the pump is sufficiently weak that saturation and fluctuations in the pump power are significant, but those effects are unrelated to the decoherence of interest here, which can occur even in an ideal linear amplifier in the limit of a strong pump.

As an example of these effects, we consider the amplification of Schrodinger cat states in the next section. We show that an ideal amplifier can greatly reduce the visibility of the quantum interference between the two terms in a cat state even when the quantum noise is negligible. This situation is analyzed in more detail in Section 3 using the Husimi–Kano Q-function, which allows the visibility of the quantum interference to be calculated analytically. The results from the Q-function calculation in the Schrodinger picture are compared with the results of the Heisenberg picture in Section 4. The implications of these results are discussed in Section 5 along with our conclusions.

2. Decoherence of Schrodinger cat states

The decoherence of a Schrodinger cat state by a parametric amplifier will be considered in this section, where the most interesting results correspond to the limit of $g \to 1$. The decoherence of the cat state can be measured using the interferometer arrangement shown in Fig. 2. Earlier studies of the amplification of cat states [4,17–28] did not consider the limit of $g \to 1$ or the interferometer approach of Fig. 2.

The first step in this process is to generate a Schrodinger cat state by passing a coherent state $|\alpha_0\rangle$ (laser beam) with complex amplitude α_0 in the signal mode through a single-photon interferometer that contains a Kerr medium [29] K in one path, as illustrated by the state-preparation box on the left-hand side of Fig. 2. A constant phase shift is applied in such a way that a net phase shift of $\pm\phi$ will be applied to the coherent state depending on the path taken by the single photon γ_1, as illustrated in Fig. 3b. We post-select on those events in which γ_1 is detected in the detector labeled D_1 in Fig. 2, which produces a Schrodinger cat state [30–32] given by

$$|\psi\rangle = \left(\left|e^{i\phi}\alpha_0\right\rangle + \left|e^{-i\phi}\alpha_0\right\rangle\right)|0_i\rangle/\sqrt{2}. \tag{2}$$

Here we have assumed that the idler mode of the OPA is initially in its vacuum state $|0_i\rangle$. The normalization of Eq. (2) also assumes that ϕ is sufficiently large that there is negligible overlap between the two coherent-state components.

The signal mode is then amplified using an OPA with a gain $g = 1 + \varepsilon$, which will increase the amplitude of the signal by a relatively small amount for $\varepsilon << 1$ as illustrated in Fig. 3c. The amplification will also displace the idler mode in accordance with the relations [7,8]

$$\begin{aligned}\hat{q}_{out} &= g\hat{q}_{in} - \sqrt{g^2-1}\,\hat{x}_{in}\\ \hat{\pi}_{out} &= g\hat{\pi}_{in} + \sqrt{g^2-1}\,\hat{p}_{in},\end{aligned} \tag{3}$$

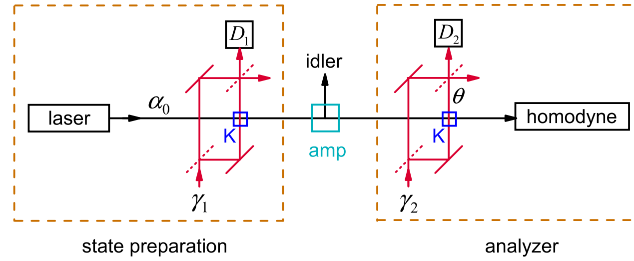

Fig. 2. Testing the properties of a parametric amplifier by generating a quantum state, passing it through an amplifier, and then analyzing the properties of the output state. Here a Schrodinger cat state is first produced by passing a coherent state through a single-photon interferometer with a Kerr medium K in one path. After amplification, a second single-photon interferometer will produce quantum interference between the two components of the cat state when a homodyne measurement indicates a net phase shift near zero, as illustrated in Fig. 3. This allows a measurement of the amount of decoherence due to entanglement between the signal and idler modes in the amplifier, which can occur even when the quantum noise is negligibly small. Here γ_1 and γ_2 represent single photons, D_1 and D_2 are single-photon detectors, and the pump beam for the parametric amplifier is not shown.

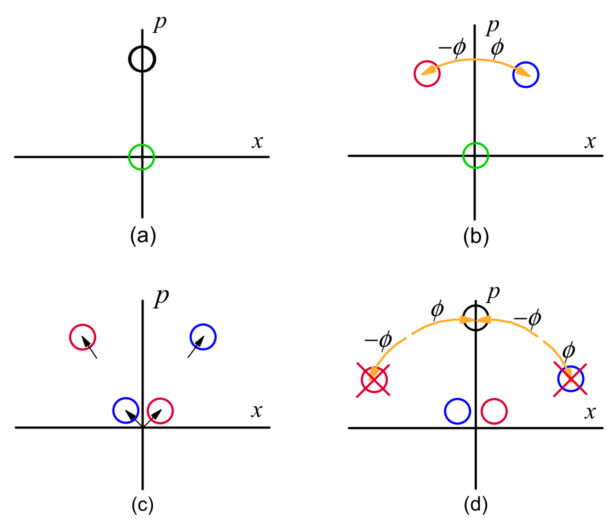

Fig. 3. Qualitative phase-space description of the interferometer system of Fig. 2, where x and p correspond to the two quadratures of the fields. (a) The initial state in which the signal mode is a coherent state with amplitude α_0 as represented by the upper (black) circle, while the idler mode is in its vacuum state represented by the lower (green) circle. (b) The state of the system after the first single-photon interferometer, where there are equal probability amplitudes that the signal mode has been shifted in phase by $\pm\phi$. (c) Entangled state created by the parametric amplifier of Fig. 1, where the signal and idler modes have been displaced in correlated directions. (d) The state of the system after the final single-photon interferometer, where a second phase shift of $\pm\phi$ can recombine the two components of the Schrodinger cat. The visibility of the quantum interference between these two probability amplitudes is reduced exponentially by the remaining entanglement with the idler mode.

where $\hat{\pi}$ is the other phase quadrature for the idler. This creates entanglement between the signal and the idler modes, since the idler mode is displaced in different directions in phase space for the two Schrodinger cat state components, as illustrated by the red and blue colors in Fig. 3c. It is important to note that the change in the idler can be much larger than the change in the signal, since $\sqrt{g^2-1} \sim \sqrt{2\varepsilon} >> \varepsilon$ for $\varepsilon << 1$.

After the amplifier, the coherence properties of the output signal are analyzed using another single-photon interferometer with a Kerr medium as shown in the analyzer box on the right-hand side of Fig. 2. This process also applies a phase shift of $\pm\phi$, where we post-select on single photon γ_2 having been detected in D_2. The net phase shift after the second interferometer will be either 0 or $\pm 2\phi$ as indicated by the arrows in Fig. 3d. The phase of the