Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

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Non-autonomous matter-wave solitons in hybrid atomic–molecular Bose–Einstein condensates with tunable interactions and harmonic potential

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ARTICLE INFO

Article history:
Received 13 September 2017
Received in revised form 10 January 2018
Accepted 11 January 2018
Available online xxxx
Communicated by V.A. Markel

Keywords:
Matter-wave solitons
Atomic–molecular Bose–Einstein condensates
Similarity transformation
Harmonic potential

ABSTRACT

In this paper, we investigate matter-wave solitons in hybrid atomic–molecular Bose–Einstein condensates with tunable interactions and external potentials. Three types of time-modulated harmonic potentials are considered and, for each of them, two groups of exact non-autonomous matter-wave soliton solutions of the coupled Gross–Pitaevskii equation are presented. Novel nonlinear structures of these non-autonomous matter-wave solitons are analyzed by displaying their density distributions. It is shown that the time-modulated nonlinearities and external potentials can support exact non-autonomous atomic–molecular matter-wave solitons.

1. Introduction

Recent exciting achievement of creating a molecular Bose–Einstein condensate (BEC) opens the challenge to find new quantum matter states [1–4]. Magnetic Feshbach resonances [5] and photoassociation [6] provide a tool to investigate the many-body properties of ultracold gases when converting atom pairs into molecules.

To describe the atomic–molecular BEC system [7,8], we use the three-dimensional coupled Gross–Pitaevskii (GP) equations based on mean field theory [9]

$$\begin{cases} i\hbar\frac{\partial\Phi_a}{\partial t} = (-\frac{\hbar^2\Delta}{2M_a} + V_a(\mathbf{r},t) + G_{aa}|\Phi_a|^2 + G_{am}|\Phi_m|^2)\Phi_a \\ \qquad\qquad + \chi\Phi_a^*\Phi_m, \\ i\hbar\frac{\partial\Phi_m}{\partial t} = (-\frac{\hbar^2\Delta}{2M_m} + V_m(\mathbf{r},t) + G_{ma}|\Phi_a|^2 + G_{mm}|\Phi_m|^2 \\ \qquad\qquad + \hbar\delta)\Phi_m + \frac{\chi}{2}\Phi_a^2, \end{cases} \tag{1}$$

where $\hbar$ is Planck's constant, $\mathbf{r} = (x, y, z)$ is three-dimensional coordinate, $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$, $\Phi_a = \Phi_a(\mathbf{r}, t)$ and $\Phi_m = \Phi_m(\mathbf{r}, t)$ are the atomic and molecular wave functions, respectively, M_a and M_m are the masses of an atom and a molecule with $M_m = 2M_a$, and G_{aa}, G_{am}, G_{ma} and G_{mm} are the interparticle interaction strengths with $G_{ma} = G_{am}$. Parameter χ is the parametric coupling coefficient between atoms and molecules which is determined by the hyperfine interaction that flips the electronic and nuclear spins of one of the colliding atoms [9]. Parameter δ characterizes Raman detuning for a two-photon resonance. The trap potentials for atoms and molecules are given by $V_a(\mathbf{r},t) = \frac{1}{2}M_a\omega_a(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$ and $V_m(\mathbf{r},t) = \frac{1}{2}M_m\omega_a(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$, respectively, where ω_a is the angular frequency in the transverse direction and $(\omega_x, \omega_y, \omega_z)$ are defined as the ratio between the angular frequency in the (x, y, z)-direction and the angular frequency in the transverse direction. Moreover, the total number of particles is normalized such that $N = \int(\Phi_a\Phi_a^* + 2\Phi_m\Phi_m^*)dr$.

Introducing the scales characterizing the trapping potential, the length, time, and wave functions are scaled as

$$\tilde{\mathbf{r}} = \frac{\mathbf{r}}{a_h}, \quad \tilde{t} = \omega_a t, \quad \tilde{\Phi}_a(\tilde{\mathbf{r}},\tilde{t}) = \frac{a_h^{3/2}}{\sqrt{N}}\Phi_a(\mathbf{r},t),$$
$$\tilde{\Phi}_m(\tilde{\mathbf{r}},\tilde{t}) = \frac{a_h^{3/2}}{\sqrt{N}}\Phi_m(\mathbf{r},t), \tag{2}$$

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https://doi.org/10.1016/j.physleta.2018.01.014
0375-9601/

with $a_h = \sqrt{\hbar/M_a\omega_a}$. After the tilde is removed, the three-dimensional coupled GP equation (1) is reduced to a dimensionless form as

$$\begin{cases} i\frac{\partial\Phi_a}{\partial t} = (-\frac{\Delta}{2} + V(\mathbf{r},t) + g_{aa}|\Phi_a|^2 + g_{am}|\Phi_m|^2)\Phi_a \\ \qquad\qquad + \gamma\Phi_a^*\Phi_m, \\ i\frac{\partial\Phi_m}{\partial t} = (-\frac{\Delta}{4} + 2V(\mathbf{r},t) + g_{am}|\Phi_a|^2 + g_{mm}|\Phi_m|^2 + \epsilon)\Phi_m \\ \qquad\qquad + \frac{\gamma}{2}\Phi_a^2, \end{cases} \tag{3}$$

where the external potential is $V(\mathbf{r},t) = \frac{1}{2}(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2)$, parameters $\epsilon = \delta/\omega_a$, $\gamma = \sqrt{N}\chi/(\hbar\omega_a a_h^{3/2})$ and $g_{ij} = NG_{ij}/(\hbar\omega_a a_h^3)$ with $i, j = a, m$.

Assume the ratios between the angular frequency in the (y,z)-direction and the angular frequency in the transverse direction are $\omega_y = \omega_z = 1$. In the limit of highly elongated traps, i.e. $\omega_x \ll 1$, the tight confinement ensures that no excited states are available in the (y,z)-direction and thus the dynamics takes place along the axial direction. That is to say the atomic–molecular BEC is confined strongly by a harmonic potential with frequency ω_a and the harmonic length $a_h = \sqrt{\hbar/M_a\omega_a}$ in the transverse direction [10,11]. Introducing the transformations $\Phi_a(\mathbf{r},t) = \psi_a(t,x)\pi^{-1/2}e^{-\frac{y^2+z^2}{2}}$ and $\Phi_m(\mathbf{r},t) = \psi_m(t,x)(\pi/2)^{-1/2}e^{-(y^2+z^2)}$, the model for the three-dimensional atomic–molecular BEC system (3) is reduced to the quasi-one-dimensional coupled GP equation

$$\begin{cases} i\frac{\partial\psi_a}{\partial t} = (-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x,t) + \lambda_{aa}|\psi_a|^2 + \lambda_{am}|\psi_m|^2)\psi_a \\ \qquad\qquad + \theta\psi_a^*\psi_m, \\ i\frac{\partial\psi_m}{\partial t} = (-\frac{1}{4}\frac{\partial^2}{\partial x^2} + 2V(x,t) + \lambda_{am}|\psi_a|^2 + \lambda_{mm}|\psi_m|^2 + \epsilon)\psi_m \\ \qquad\qquad + \frac{\theta}{2}\psi_a^2, \end{cases} \tag{4}$$

where the external potential is $V(x,t) = \frac{1}{2}\omega^2x^2$ with $\omega = \omega_x$, parameters $\theta = (2\pi)^{-1/2}\gamma$, $\lambda_{aa} = \frac{1}{2\pi}g_{aa}$, $\lambda_{am} = \frac{1}{3\pi}g_{am}$ and $\lambda_{mm} = \frac{1}{\pi}g_{mm}$.

In the previous work, Zhang et al. [12] and Liu et al. [13] have examined analytically the exact solutions of the coupled GP equations with time-independent potential by similarity transformation. Recently, studies on the exact solutions of the GP equations with time-dependent potential have also been done [14–17]. Moreover, in a recent experiment [18], the ^{7}Li BEC has been excited by the time-periodic modulation of the interaction strength via Feshbach resonance. Then theoretical study on time-periodic modulation of the interaction strength is further done to analyze the one-component mean-field GP equation [19]. Motivated by these work [12–19], in this paper, we consider the exact solutions of the quasi-one-dimensional coupled GP equation (4) with time-modulated frequency of external potential and nonlinearities, i.e. the parameter coefficients $\omega, \lambda_{aa}, \lambda_{am}, \lambda_{mm}, \epsilon$ and θ are functions of time t. In this case, the exact solutions that we obtain are non-autonomous matter-wave soliton solutions. In the past years, much work has been done to investigate various non-autonomous solitons [20–22] in one-component nonlinear Schrödinger systems [23]. However, in the present letter we have found some non-autonomous matter-wave solitons in coupled atomic–molecular BEC system.

The paper is organized as follows. In Section 2, the exact non-autonomous matter-wave soliton solutions of the quasi-one-dimensional coupled GP equation are obtained. In Section 3, the novel nonlinear structures of the non-autonomous matter-wave solitons are investigated by analyzing their density distributions. We summarize our results in Section 4.

2. Non-autonomous matter-wave solitons

In this section, the similarity transformation is adopted to map the quasi-one-dimensional coupled GP equation (4) onto the coupled constant-coefficient nonlinear Schrödinger equation (NLS) of atomic–molecular type, and some non-autonomous matter-wave solitons in the hybrid atomic–molecular BECs are given. Firstly, the complex wave functions ψ_a and ψ_m can be written as

$$\begin{cases} \psi_a = \phi_a(T,X)\,e^{\alpha_1 i + \beta_1}, \\ \psi_m = \phi_m(T,X)\,e^{\alpha_2 i + \beta_2}, \end{cases} \tag{5}$$

where $X = b_1x + b_2$, $\alpha_2 = 2\alpha_1 = 2(F_3 + F_2x - F_1x^2)$, and $T, b_1, b_2, F_1, F_2, F_3, \beta_1$ and β_2 are functions of time t. The wave functions $\phi_a(T,X)$ and $\phi_m(T,X)$ solve the following coupled constant-coefficient NLS equation [12]

$$\begin{cases} i\frac{\partial\phi_a}{\partial T} = (-\frac{1}{2}\frac{\partial^2}{\partial X^2} + g_{aa}|\phi_a|^2 + g_{am}|\phi_m|^2)\phi_a + \alpha\phi_a^*\phi_m, \\ i\frac{\partial\phi_m}{\partial T} = (-\frac{1}{4}\frac{\partial^2}{\partial X^2} + g_{am}|\phi_a|^2 + g_{mm}|\phi_m|^2 + \sigma)\phi_m + \frac{\alpha}{2}\phi_a^2, \end{cases} \tag{6}$$

where $g_{aa}, g_{am}, g_{mm}, \sigma$ and α are constants.

Substituting Eq. (5) into the quasi-one-dimensional coupled GP equation (4) and letting functions $\phi_a(T,X)$ and $\phi_m(T,X)$ satisfy Eq. (6), the terms $T, b_1, b_2, F_1, F_2, F_3, \beta_1$ and β_2 should obey a set of ordinary differential equations (ODEs). With the aid of symbolic computation, solving these ODEs, we have

$$\begin{aligned} &\theta = \alpha\frac{dT}{dt}e^{-\beta_1},\quad \lambda_{aa} = g_{aa}\frac{dT}{dt}e^{-2\beta_1},\quad \lambda_{am} = g_{am}\frac{dT}{dt}e^{-2\beta_1}, \\ &\lambda_{mm} = g_{mm}\frac{dT}{dt}e^{-2\beta_1},\quad \epsilon = \sigma\frac{dT}{dt}, \\ &T = C_5^2\int e^{4\int F_1dt}dt + C_4,\quad F_2 = C_6e^{\int 2F_1dt},\quad b_1 = C_5e^{\int 2F_1dt}, \\ &F_3 = -\frac{1}{2}C_6^2\int e^{4\int F_1dt}dt + C_2,\quad b_2 = -C_5C_6\int e^{4\int F_1dt}dt + C_1, \\ &\beta_2 = \beta_1 = \int F_1dt + C_3, \end{aligned} \tag{7}$$

where C_i $(i = 1, 2, \cdots, 6)$ are arbitrary constants and F_1 solves the following Riccati equation

$$\frac{dF_1}{dt} - 2F_1^2 = \frac{1}{2}\omega^2. \tag{8}$$

The transformation in Eq. (5) connects the non-autonomous matter-wave soliton solutions of the coupled GP equation (4) with the exact solutions of the coupled constant-coefficient NLS equation (6). Thus it is important to solve the coupled constant-coefficient NLS equation (6) and the Riccati equation (8). The varying of the scattering length and frequency of external potential in the coupled GP equation (4) may result in interesting dynamical behaviors in the hybrid atomic–molecular BECs. The amplitude and width of the non-autonomous matter-wave solitons vary with time, which provides us a possible way to control the matter-wave solitons by changing experimental parameters. A recent work on space–time mappings [24] has been done to map two quantum field evolutions onto each other exactly, where they also consider the case of time-modulated interaction strength. The time transformation technique in [24] is relevant to time transformation in this paper.

2.1. Three types of special external potentials

In what follows, three types of special external potentials are considered to investigate the dynamics of non-autonomous matter-wave solitons in hybrid atomic–molecular BECs. From Eqs. (7)–(8)