Physics Letters A ••• (••••) •••–•••

Contents lists available at ScienceDirect

Physics Letters A

www.elsevier.com/locate/pla

Generalized energy detector for weak random signals via vibrational resonance

Yuhao Ren, Yan Pan, Fabing Duan *

Institute of Complexity Science, Qingdao University, Qingdao 266071, PR China

ARTICLE INFO

Article history:
Received 31 October 2017
Received in revised form 11 January 2018
Accepted 12 January 2018
Available online xxxx
Communicated by C.R. Doering

Keywords:
Generalized energy detector
Vibrational resonance
Normalized asymptotic efficacy
Fisher information
Non-Gaussian noise

ABSTRACT

In this paper, the generalized energy (GE) detector is investigated for detecting weak random signals via vibrational resonance (VR). By artificially injecting the high-frequency sinusoidal interferences into an array of GE statistics formed for the detector, we show that the normalized asymptotic efficacy can be maximized when the interference intensity takes an appropriate non-zero value. It is demonstrated that the normalized asymptotic efficacy of the dead-zone-limiter detector, aided by the VR mechanism, outperforms that of the GE detector without the help of high-frequency interferences. Moreover, the maximum normalized asymptotic efficacy of dead-zone-limiter detectors can approach a quarter of the second-order Fisher information for a wide range of non-Gaussian noise types.

1. Introduction

Although the Gaussian noise model is valid for a vast range of noisy environments, many areas of engineering show that the Gaussian assumption is far from describing distributions of practical noise models in radar [1–4], outdoor mobile or indoor wireless communications [5,6], biological systems [7–9], etc. For detecting weak random signals in non-Gaussian noise, it is well known that the uniformly most powerful detector usually does not exist [1–4,10]. Therefore, a feasible approach is the locally optimum (LO) detector that can maximize the detection probability of weak random signals in the asymptotic case of large observations [1,2,4]. However, it is shown [1–4] that the structure of the LO detector is closely tied to the second-order derivative of noise distribution. Thus, for non-existent derivatives of noise distributions or unknown noise distributions [1–4], it is attractive to develop the suboptimal but robust detectors in weak signal detections. For instance, the generalized energy (GE) detectors have a low-complexity structure and provide a robust compromise performance for Gaussian noise as well as non-Gaussian (e.g. heavy-tailed) noise [1–3]. The GE detectors attract much attention of many researchers to investigate their performances under various circumstances [1–4,11–17].

It is interesting to note that the high-frequency interferences [18], as well as the noise components [10,19–26], can be employed to enhance the performance of the suboptimal detectors. This high-frequency interference enhanced nonlinear phenomenon named vibrational resonance (VR) was originally introduced by Landa and McClintock [27], where the high-frequency interferences play the same positive role as noise does in stochastic resonance [19,28]. It is shown [18,27,29–32] that the implementation of high-frequency sinusoidal vibrations is often easier than injecting pseudo-noise sequences that obey a distribution into observations, because the imitation of the mutually independent noise sequences is inconvenient in engineering measurements. Moreover, the high-frequency vibration or the deterministic jitter often naturally arises in real devices. Up to now, VR has been confirmed in a number of nonlinearities such as bistable models [33–35], neuron systems [29,36–38], optical systems [39,40], and dynamic circuits [30–32]. Recently, we exploited the VR effect to detect deterministic weak signals in the presence of strong background noise [18]. In this paper, we further explore the VR effect to detect weak random signals via a GE detector. The rest of this paper is organized as follows. In Section 2, the GE detector based on the VR method is elicited and its normalized asymptotic efficacy is theoretically derived. In Section 3, the normalized asymptotic efficacy of the proposed GE detector is analyzed, and the obtained results demonstrate that the VR method can be employed in the weak random signal detection to enhance the detection probabilities of GE detectors. Finally, we draw conclusions and present further discussions of the VR method in Section 4.

* Corresponding author.
E-mail address: fabing.duan@gmail.com (F. Duan).

https://doi.org/10.1016/j.physleta.2018.01.015
0375-9601/

2. Generalized energy detector

Consider the vector $X = (X_1, X_2, \cdots, X_n)$ with n observation components, and each real-valued component X_i at time index i can be written as [1–4]

$$X_i = \theta S_i + W_i, \; i = 1, 2, \cdots, n. \tag{1}$$

Here, the random signal sequence $S = \{S_i\}$ is a white stationary stochastic process with its probability density function f_s. θ is the signal intensity, and S has zero-mean and variance $\sigma_s^2 = 1$. The background noise components W_i are independent and identically distributed (i.i.d.) random variables with zero-means, unity variances and the common density function f_{w}. The noise and signal processes are independent [1]. This random signal detecting problem can be treated as a binary hypothesis test of $H_0 : \theta = 0$ and $H_1 : \theta \neq 0$, and the corresponding joint probability densities of X can be expressed as [1–4]

$$H_0 : \; f_{\mathrm{X}}(\mathrm{x}) = \prod_{i=1}^{n} f_{\mathrm{w}}(x_i), \; \theta = 0,$$

$$H_1 : \; f_{\mathrm{X}}(\mathrm{x}) = \mathrm{E}_{\mathrm{s}}\left\{\prod_{i=1}^{n} f_{\mathrm{w}}(x_i - \theta s_i)\right\}, \; \theta \neq 0,$$

with the expectation operator $\mathrm{E}_{\mathrm{s}}(\cdot) = \int \cdot f_s(\mathrm{x}) d\mathrm{x}$. For the weak signal intensity $\theta \to 0$ and the zero-mean vector of signal sequence S, we are interested in the performance for the alternative hypothesis H_1 close to the null hypothesis H_0 [1,2]. Thus, the slope of the detection power at $\theta = 0$ is expected to be maximized [1–4]. From the generalized Neyman–Pearson theorem [1] and the zero first-order derivative of the detection power, the maximum second-order derivative of the detection power leads to a test with the LO statistic $T_{\mathrm{LO}}(\mathrm{X}) = \sum_{i=1}^{n} f_{\mathrm{w}}''(X_i)/f_{\mathrm{w}}(X_i)$ for $f_{\mathrm{w}}''(x) = d^2 f_{\mathrm{w}}(x)/dx^2$. Then, the LO detector based on the statistic T_{LO} has a largest slope of the detection power at the origin of $\theta = 0$ among the detectors that have a given false-alarm probability P_{F} [1,2].

However, it is seen that the LO statistic T_{LO} is closely tied to the density function f_{w}. When the derivative of the density function f_{w} does not exist or f_{w} is unknown, the statistic T_{LO} is unavailable. In practice, some suboptimal but robust GE detectors

$$T_{\mathrm{GE}}(\mathrm{X}) = \sum_{i=1}^{n} e(X_i) \underset{H_0}{\overset{H_1}{\gtrless}} \gamma, \tag{2}$$

are usually adopted with the memoryless nonlinearity e and the decision threshold γ [1,2]. We note that this GE detector in Eq. (2) is just the LO detector as $e(x) = f_{\mathrm{w}}''(x)/f_{\mathrm{w}}(x)$. When the observation size n is sufficiently large, the random variable $T_{\mathrm{GE}}(\mathrm{X})$ asymptotically follows Gaussian distribution according to the central limit theorem [1,2,4]. With some regularity conditions [1], the detection probability of the GE detector is proportional to the normalized asymptotic efficacy for a given false-alarm probability P_{F} [1,2,4,18], and the normalized asymptotic efficacy ξ_{GE} of a GE detector is defined in Eq. (A.1) (cf. Appendix A).

It is seen [1,2,4,12–18] that the GE detector in Eq. (2) with a certain nonlinearity has a good compromise detection performance for Gaussian noise as well as non-Gaussian noise, provided that the detector parameters are tuned appropriately. Here, we argue that the normalized asymptotic efficacy ξ_{GE} of a GE detector might benefit from the VR mechanism. Consider m high-frequency sinusoidal interferences

$$\eta_{ji} = A_\eta \sin(2\pi f_j i/f_{sa}), \; j = 1, 2, \cdots, m, \tag{3}$$

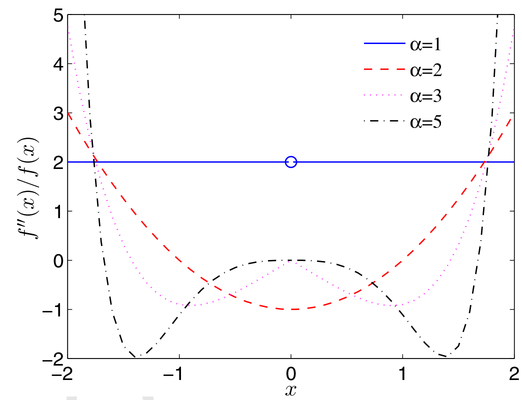

Fig. 1. LO detector functions of the normalized generalized Gaussian distribution for different exponents $\alpha = 1, 2, 3$ and 5. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

with the same amplitude A_η but different frequencies f_j. Here, in the discrete-time implementation, the sample frequency $f_{sa} > 2f_j$ and interference frequencies $f_j \neq f_k$ for $j \neq k$ $(j, k = 1, 2, \cdots, m)$. We emphasize that, in the following parts, the frequencies f_j are randomly assigned and sorted from $f_{sa}/70$ to $f_{sa}/180$. Next, m high-frequency sinusoidal interferences are artificially injected into the observation vector X, and then, at time i, the observations are updated as $\hat{X}_{ji} = \theta S_i + W_i + \eta_{ji}$. Based on the observations $\hat{X}$, a new GE detector is proposed as

$$T_{\mathrm{HF}}(\hat{X}) = \sum_{i=1}^{n} \left[\frac{1}{m} \sum_{j=1}^{m} e(\hat{X}_{ji})\right] \underset{H_0}{\overset{H_1}{\gtrless}} \gamma. \tag{4}$$

In Appendix A, we derive the mean and variance of the proposed GE detector under two hypotheses of H_0 and H_1. Then, the normalized asymptotic efficacy ξ_{GE} of the proposed GE detector can be calculated by Eq. (A.1).

3. VR effects in GE detectors

The above analyzes of Section 2 are applicable to an arbitrary GE detector of Eq. (4). In this Section, for illustration, we consider the piecewise nonlinearity

$$e(x) = \begin{cases} g(x), & |x| \leq \Theta, \\ C, & |x| > \Theta, \end{cases} \tag{5}$$

which is a square-law-limiter with $g(x) = x^2$ or a dead-zone-limiter with $g(x) = 0$ [1]. Here, $C > 0$ and $\Theta \geq 0$ are tunable detector parameters. The random signal S is assumed to be a zero-mean white Gaussian stochastic process, and the background generalized Gaussian noise has the density function $f_{\mathrm{w}}(x) = c_1 \exp(-c_2 |x|^\alpha)$ with $c_1 = \frac{\alpha}{2}\Gamma^{\frac{1}{2}}\left(\frac{3}{\alpha}\right)/\Gamma^{\frac{3}{2}}\left(\frac{1}{\alpha}\right)$ and $c_2 = \left[\Gamma\left(\frac{3}{\alpha}\right)/\Gamma\left(\frac{1}{\alpha}\right)\right]^{\frac{\alpha}{2}}$. $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$ is the gamma function and the exponent $\alpha > 0$ controls the decay rate of tails [1,2,4]. This normalized distribution model of the generalized Gaussian noise density f_{w} allows us to conveniently study the effect of non-Gaussian noise types on detector performances. The corresponding LO detector functions $f_{\mathrm{w}}''/f_{\mathrm{w}}$ are drawn in Fig. 1 for different exponents α. For the exponent $\alpha = 1$ (Laplacian noise), the LO detector function can be achieved by the nonlinearity in Eq (5) with $g(x) = 1$ and a small constant Θ [1], as indicated in Fig. 1 (the blue solid line).

It is illustrated in Fig. 2 (a) that the normalized asymptotic efficacy ξ_{GE} is a resonant-like function of the interference intensity A_η for the GE detector with nonlinear element numbers $m = 1, 2, 5, 10$ and 20. Here, the square-law-limiter nonlinearity