ARTICLE IN PRESS

[Physics Letters A](https://doi.org/10.1016/j.physleta.2018.01.015) ••• (••••) •••-•••

Contents lists available at [ScienceDirect](http://www.ScienceDirect.com/) 1 67

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11 Concretized operay detector for weak random signals via vibrational $\frac{11}{12}$ Generalized energy detector for weak random signals via vibrational $\frac{77}{78}$ 13 resonance 79

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20 ARTICLE INFO ABSTRACT 86

Article history: Received 31 October 2017 Received in revised form 11 January 2018 Accepted 12 January 2018 Available online xxxx Communicated by C.R. Doering *Keywords:*

30 96 Normalized asymptotic efficacy 31 151 32 and the control of the c Generalized energy detector Vibrational resonance Fisher information Non-Gaussian noise

1. Introduction

tical noise models in radar $[1-4]$, outdoor mobile or indoor wireknown that the uniformly most powerful detector usually does not exist [\[1–4,10\].](#page--1-0) Therefore, a feasible approach is the locally optimum (LO) detector that can maximize the detection probability of weak random signals in the asymptotic case of large observations $[1,2,4]$. However, it is shown $[1-4]$ that the structure of the LO detector is closely tied to the second-order derivative of noise distribution. Thus, for non-existent derivatives of noise distributions or unknown noise distributions $[1-4]$, it is attractive to develop the suboptimal but robust detectors in weak signal detections. For instance, the generalized energy (GE) detectors have a low-complexity structure and provide a robust compromise performance for Gaussian noise as well as non-Gaussian (e.g. heavytailed) noise $[1-3]$. The GE detectors attract much attention of many researchers to investigate their performances under various circumstances [\[1–4,11–17\].](#page--1-0)

64 130 <https://doi.org/10.1016/j.physleta.2018.01.015>

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₂₂ Article history: **Article history: In this paper, the generalized energy (GE) detector is investigated for detecting weak random signals as** 23 Received 31 October 2017
23 Beaziund in gruined farm 11 Ianuary 2018 exerved in exposer of the detector, we show that the normalized asymptotic efficacy can
24 Accented 12 January 2018 25 Available online xxxx
that the normalized asymptotic efficacy of the dead-zone-limiter detector, aided by the VR mechanism, 26 Communicated by C.R. Doering
- $\frac{27}{\text{Kewords:}}$ external the symptotic efficacy of dead-zone-limiter detectors can approach a quarter of the $\frac{93}{\text{Kewords}}}$ ²⁸ Generalized energy detector **the second-order Fisher information for a wide range of non-Gaussian noise types.** ⁹⁴ be maximized when the interference intensity takes an appropriate non-zero value. It is demonstrated

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 35 36 **1. Introduction 1. Introduction 102 1. Interfer-** 102 ences [\[18\],](#page--1-0) as well as the noise components [\[10,19–26\],](#page--1-0) can be loc 38 Although the Gaussian noise model is valid for a vast range $\frac{104}{\pi}$ rise big to equality the Gaussian conductions. This has the conduction of the subgraph of the subgraph of the subgraph of the subgraph of the s 39 of noisy environments, many areas of engineering show that the $\frac{1018}{2}$ many interference (XD) was existed to the line of the state of t ⁴⁰ Gaussian assumption is far from describing distributions of prac-
Landa and McClintogk [27] where the bigh frequency interferences $\frac{41}{100}$ is also models in the set of the models of index models of index with $\frac{41}{100}$ Landa and McClintock [\[27\],](#page--1-0) where the high-frequency interferences $\frac{42}{100}$ 42 108 less communications [\[5,6\],](#page--1-0) biological systems [\[7–9\],](#page--1-0) etc. For de-⁴³ tecting weak random signals in non-Gaussian noise, it is well $\frac{19,28}{\text{high frequency}}$. It is snown $\frac{18,27,29-32}{\text{light frequency}}$ that the implementation of $\frac{109,27}{\text{light frequency}}$ 44 Income weak tanguar in the substantinum of the second with the product of the second with the second than inject-
110 Income that the uniformly may be a second detector with decay and high-frequency sinusoidal vibration ⁴⁵ 11¹ 46 112 , 112 , 112 , 112 , 112 , 112 112 vations, because the imitation of the mutually independent noise 112 47 I III (LO) detector that can inatulate the detection probability sequences is inconvenient in engineering measurements. Moreover, 113 48 ¹ 14⁴⁸ ¹ 14⁴⁸ ¹ 14⁴⁸ ¹¹⁴ 144⁴⁸ ¹¹⁴ 14⁴ ¹¹⁴ 14⁴ ¹¹⁴ 14⁴ ¹¹⁴ 114⁴ 114 49 valibils [1,2,4]. However, it is shown [1–4] that the structure of rally arises in real devices. Up to now, VR has been confirmed in a 115 $_{50}$ and the LO detector is closely tied to the second-order derivative or number of nonlinearities such as bistable models [\[33–35\],](#page--1-0) neuron $_{116}$ $_{51}$ $_{100}$ and dynamic circuits $_{117}$ $_{117}$ $_{117}$ $_{120}$ $_{130}$ $_{147}$ $_{150}$ $_{160}$ $_{170}$ $_{180}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{190}$ $_{52}$ cributions or unknown noise distributions $[1-4]$, it is attractive to $[30-32]$. Recently, we exploited the VR effect to detect determinis- $_{53}$ develop the suboptimal but robust detectors in weak signal de-
tic weak signals in the presence of strong background noise [\[18\].](#page--1-0) $_{119}$ $_{54}$ rections. For instance, the generalized energy (GE) detectors have $\;$ In this paper, we further explore the VR effect to detect weak ran- $_{55}$ a low-complexity structure and provide a robust compromise per-
dom signals via a GE detector. The rest of this paper is organized in 121 $_{56}$ formance for Gaussian noise as well as non-Gaussian (e.g. heavy- as follows. In Section [2,](#page-1-0) the GE detector based on the VR method 122 $_{57}$ talled) noise [1–3]. The GE detectors attract much attention of is elicited and its normalized asymptotic efficacy is theoretically $_{123}$ $_{58}$ many researchers to investigate their performances under various derived. In Section [3,](#page-1-0) the normalized asymptotic efficacy of the $_{124}$ $_{59}$ circumstances [1-4,11-17]. $_{125}$ and the obtained results demon-60 126 strate that the VR method can be employed in the weak random 61 127 signal detection to enhance the detection probabilities of GE detec-62 ^{*} Corresponding author. The same state of the series of the series of the series tors. Finally, we draw conclusions and present further discussions the series of 63 **I** E-mail address: fabing.duan@gmail.com (F. Duan). **In the Community of the VR method in Section** 4. employed to enhance the performance of the suboptimal detectors. This high-frequency interference enhanced nonlinear phenomenon named vibrational resonance (VR) was originally introduced by play the same positive role as noise does in stochastic resonance [\[19,28\].](#page--1-0) It is shown $[18,27,29-32]$ that the implementation of of the VR method in Section [4.](#page--1-0)

Please cite this article in press as: Y. Ren et al., Generalized energy detector for weak random signals via vibrational resonance, Phys. Lett. A (2018), https://doi.org/10.1016/j.physleta.2018.01.015

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^{*} Corresponding author.

3 Consider the vector $X = (X_1, X_2, \dots, X_n)$ with *n* observation $X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\frac{4}{3}$ components, and each real-valued component X_i at time index *i* $\frac{3}{8}$ $\begin{bmatrix} 5 & \text{can be written as } [1-4] & \begin{bmatrix} 1 & -4 \end{bmatrix} \end{bmatrix}$

$$
X_i = \theta S_i + W_i, \quad i = 1, 2, \cdots, n. \tag{1}
$$

9 Here, the random signal sequence $S = \{S_i\}$ is a white stationary $\begin{bmatrix} S_i & S_i \end{bmatrix}$ is the set of the stationary $\begin{bmatrix} S_i & S_i \end{bmatrix}$ is the stationary $\begin{bmatrix} S_i & S_i \end{bmatrix}$ is the stationary $\begin{bmatrix} S_i & S_i \end{bmatrix}$ is the s 10 stochastic process with its probability density function f_s . *θ* is the $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ is the $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ is the $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$ is the $\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \end{$ 11 signal intensity, and *S* has zero-mean and variance $\sigma_s^2 = 1$. The $\sigma_s^2 = \frac{1}{\sigma_s^2}$ $\sigma_s^2 = \frac{1}{\sigma_s^2}$ $\sigma_s^2 = \frac{1}{\sigma_s^2}$ ¹² background noise components W_i are independent and identically the state of the stat ¹³ distributed (i.i.d.) random variables with zero-means, unity vari-
¹³ distributed (i.i.d.) random variables with zero-means, unity vari-¹⁴ ances and the common density function f_w . The noise and signal $\frac{-2}{x}$ ances and $\frac{9}{x}$ 15
processes are independent [\[1\].](#page--1-0) This random signal detecting prob-16 **Fig. 1.** LO detector functions of the normalized generalized Gaussian distribution for 82 17 $\frac{17}{17}$ $\$ $H_1: \theta \neq 0$, and the corresponding joint probability densities of *X* in this figure, the reader is referred to the web version of this article.) $\frac{19}{19}$ can be expressed as $\frac{11}{11}$ 85 lem can be treated as a binary hypothesis test of $H_0: \theta = 0$ and can be expressed as $[1-4]$

$$
H_0: f_X(x) = \prod_{i=1}^n f_w(x_i), \ \theta = 0,
$$

$$
H_1: f_X(x) = E_s \Big\{ \prod_{i=1}^n f_w(x_i - \theta s_i) \Big\}, \ \theta \neq 0,
$$

²⁷ with the expectation operator $E_s(\cdot) = \int \cdot f_s(x) dx$. For the weak sig-
are updated as $X_{ji} = \theta S_i + W_i + \eta_{ji}$. Based on the observations X, 28 nal intensity $\theta \to 0$ and the zero-mean vector of signal sequence *S*, a new GE detector is proposed as θ ²⁹ we are interested in the performance for the alternative hypothe-³⁰ sis *H*₁ close to the null hypothesis *H*₀ [\[1,2\].](#page--1-0) Thus, the slope of the $T_{\text{tot}}(\hat{Y}) = \sum \left[\frac{1}{N} \sum_{\ell} \rho(\hat{Y}_{\text{tot}})\right]_{\geq 1}^{H_1}$ 31 detection power at $\theta = 0$ is expected to be maximized [\[1–4\].](#page--1-0) From $\left\{ H_1(x) - \sum_{i=1}^n \left[m_i(x) \right] \right\}$ ³² the generalized Neyman–Pearson theorem [\[1\]](#page--1-0) and the zero first-
³² the generalized Neyman–Pearson theorem [1] and the zero first-³³ order derivative of the detection power, the maximum second-
In Appendix A, we derive the mean and variance of the proposed $\,^{99}$ ³⁴ order derivative of the detection power leads to a test with the LO GE detector under two hypotheses of H_0 and H_1 . Then, the nor-³⁵ statistic $T_{\text{LO}}(X) = \sum_{i=1}^n f''_w(X_i)/f_w(X_i)$ for $f''_w(x) = d^2 f_w(x)/dx^2$. malized asymptotic efficacy ξ_{GE} of the proposed GE detector can ¹⁰¹ ³⁶ Then, the LO detector based on the statistic T_{LO} has a largest slope be calculated by Eq. (A.1). 37 of the detection power at the origin of $\theta = 0$ among the detectors 103 ³⁸ that have a given false-alarm probability P_F [\[1,2\].](#page--1-0) **3. VR effects in GE detectors 3. 104**

³⁹ However, it is seen that the LO statistic $T_{\rm LO}$ is closely tied to the 105 ⁴⁰ density function f_w . When the derivative of the density function The above analyzes of Section 2 are applicable to an arbitrary ¹⁰⁶ ⁴¹ f_w does not exist or f_w is unknown, the statistic T_{10} is unavail- GE detector of Eq. (4). In this Section, for illustration, we consider ¹⁰⁷ ⁴² able. In practice, some suboptimal but robust GE detectors the piecewise nonlinearity 43 109 density function *f*w. When the derivative of the density function f_w does not exist or f_w is unknown, the statistic T_{LO} is unavail-

$$
T_{GE}(X) = \sum_{i=1}^{n} e(X_i) \underset{H_0}{\geq} \gamma, \tag{5}
$$

are usually adopted with the memoryless nonlinearity *e* and the decision threshold γ [\[1,2\].](#page--1-0) We note that this GE detector in Eq. (2) is just the LO detector as $e(x) = f''_w(x)/f_w(x)$. When the observation size *n* is sufficiently large, the random variable $T_{GE}(X)$ asymptotically follows Gaussian distribution according to the central limit theorem $[1,2,4]$. With some regularity conditions $[1]$, the detection probability of the GE detector is proportional to the normalized asymptotic efficacy for a given false-alarm probability P_F [\[1,2,4,18\],](#page--1-0) and the normalized asymptotic efficacy $ξ_{GE}$ of a GE detector is defined in Eq. [\(A.1\)](#page--1-0) (cf. [Appendix A\)](#page--1-0).

 $\frac{1}{58}$ 11 is seen [1,2,4,12-16] that the GE detector in Eq. (2) with a condetector performances. The corresponding LO detector functions $\frac{124}{58}$ *f fw f f are drawn in Fig. 1 for different exponents <i>α*. For the ex-
for Coussian neice se well as non Coussian neice assuided that the <i>fw / *fw* are drawn in Fig. 1 for different exponents *α*. For the ex-
 $_{60}$ for Gaussian hoise as well as non-Gaussian hoise, provided that the ponent $\alpha=1$ (Laplacian noise), the LO detector function can be 126 61 defector parameters are tuned appropriately. Here, we argue that achieved by the nonlinearity in Eq (5) with $g(x) = 1$ and a small 127 62 the normalized asymptotic efficacy *ξ*_{GE} of a GE detector might ben-constant Θ [1], as indicated in Fig. 1 (the blue solid line). 128 ⁶³ efit from the VR mechanism. Consider *m* high-frequency sinusoidal light is illustrated in Fig. 2 (a) that the normalized asymptotic ¹²⁹ It is seen $[1,2,4,12-18]$ that the GE detector in Eq. (2) with a certain nonlinearity has a good compromise detection performance for Gaussian noise as well as non-Gaussian noise, provided that the detector parameters are tuned appropriately. Here, we argue that interferences

$$
\eta_{ji} = A_{\eta} \sin(2\pi f_j i / f_{sa}), \ j = 1, 2, \cdots, m,
$$
\n(3)

different exponents $\alpha = 1, 2, 3$ and 5. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

 $_{20}$ $_{n}$ $_{n}$ $_{86}$ 21 $H_0: f_y(y) = \prod_{i=1}^{n} f_{xy}(y_i)$ $\theta = 0$ the discrete-time implementation, the sample frequency $f_{sa} > 2f_j$ as and interference frequencies $f_j \neq f_k$ for $j \neq k$ ($j, k = 1, 2, \cdots, m$). as 23 89 We emphasize that, in the following parts, the frequencies *f ^j* 24 $H = f(x) = E \left[\prod_{i=1}^{n} f(x_i - \theta_i) \right]$ and θ are randomly assigned and sorted from $f_{sa}/70$ to $f_{sa}/180$. Next, so 25 \cdots \cdots \cdots \cdots $\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{bmatrix}$, \cdots \cdots 26 92 into the observation vector *X*, and then, at time *i*, the observations are updated as $\hat{X}_{ji} = \theta S_i + W_i + \eta_{ji}$. Based on the observations \hat{X}_i , a new GE detector is proposed as

$$
T_{\text{HF}}(\hat{X}) = \sum_{i=1}^{n} \left[\frac{1}{m} \sum_{j=1}^{m} e(\hat{X}_{ji}) \right]_{H_0}^{H_1} \gamma.
$$
 (4)

In [Appendix A,](#page--1-0) we derive the mean and variance of the proposed GE detector under two hypotheses of H_0 and H_1 . Then, the normalized asymptotic efficacy $ξ$ ^{*GE*} of the proposed GE detector can be calculated by Eq. [\(A.1\).](#page--1-0)

3. VR effects in GE detectors

the piecewise nonlinearity

$$
e(x) = \begin{cases} g(x), & |x| \leq \Theta, \\ C, & |x| > \Theta, \end{cases}
$$
 (5)

⁴⁷ are usually adopted with the memoryless poplinearity e and the which is a square-law-limiter with $g(x) = x^2$ or a dead-zone- 48 decision threshold ν [1.2] We note that this GE detector in Eq. (2) limiter with $g(x) = 0$ [\[1\].](#page--1-0) Here, $C > 0$ and $\Theta \ge 0$ are tunable detec-
decision threshold ν [1.2] We note that this GE detector in Eq. (2) $\frac{49}{15}$ is inst the 10 detector as $e(x) = f''(x)/f(x)$. When the observation is assumed to be a zero-mean the sero-mean the $\frac{1}{50}$ 116 $\frac{1}{50}$ 51 1000 size *h* is suincicitly farge, the random variable $T_{GE}(x)$ asymp-
totically follows Gaussian distribution according to the central limit Gaussian noise has the density function $f_w(x) = c_1 \exp(-c_2 |x|^\alpha)$ 117 52 concentry follows determined the extraction according to the extraction in the set of (2) $3/16$ $5/16$ $7\frac{9}{16}$ 118 **b** theorem [1,2,4]. With some regularity conditions [1], the detection with $c_1 = \frac{\alpha}{2} \Gamma^{\frac{1}{2}} \left(\frac{3}{\alpha}\right) / \Gamma^{\frac{3}{2}} \left(\frac{1}{\alpha}\right)$ and $c_2 = \left[\Gamma\left(\frac{3}{\alpha}\right) / \Gamma\left(\frac{1}{\alpha}\right) \right]^{\frac{\alpha}{2}}$. Γ(α) = 119 ₅₄ probability of the GE detector is proportional to the normalized
34 asymptotic efficacy for a given false-alarm probability P_E [1.2.4.18] $\int_0^\infty x^{\alpha-1}e^{-x}dx$ is the gamma function and the exponent $\alpha > 0$ 120 $\frac{121}{25}$ 121 $\frac{121}{25}$ 121 $\frac{121}{25}$ controls the decay rate of tails [\[1,2,4\].](#page--1-0) This normalized distribu- $\frac{12}{26}$ tion model of the generalized Gaussian noise density f_w allows $\frac{122}{2}$ $\frac{1}{25}$ incurred in Eq. (An) (B, Appendix A). The CO and the state of conveniently study the effect of non-Gaussian noise types $\frac{123}{25}$ constant Θ [\[1\],](#page--1-0) as indicated in Fig. 1 (the blue solid line).

64 130 efficacy *ξG E* is a resonant-like function of the interference in- ϵ ₅ 131 tensity *A_{<i>η*} for the GE detector with nonlinear element numbers 131 $m_{jij} = A_{jj} \sin(2\pi f_j i / f_{sa}), j = 1, 2, \cdots, m,$ (3) $m = 1, 2, 5, 10$ and 20. Here, the square-law-limiter nonlinearity 132 It is illustrated in [Fig. 2](#page--1-0) (a) that the normalized asymptotic

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