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Classical and quantum dynamics of a kicked relativistic particle in a box

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ABSTRACT

We study classical and quantum dynamics of a kicked relativistic particle confined in a one dimensional box. It is found that in classical case for chaotic motion the average kinetic energy grows in time, while for mixed regime the growth is suppressed. However, in case of regular motion energy fluctuates around certain value. Quantum dynamics is treated by solving the time-dependent Dirac equation with delta-kicking potential, whose exact solution is obtained for single kicking period. In quantum case, depending on the values of the kicking parameters, the average kinetic energy can be quasi periodic, or fluctuating around some value. Particle transport is studied by considering spatio-temporal evolution of the Gaussian wave packet and by analyzing the trembling motion.

1. Introduction

Particle dynamics in confined quantum systems has attracted much attention in the context of nanoscale physics [1–3] and quantum chaos theory [4–6]. Usually, studies of confined quantum dynamics within the chaos theory have been focused on two types of problems. First type deals with the analysis of the spectral statistics (so-called quantum chaos) by solving Schrodinger equation in confined geometries (quantum billiards) [6–9]. Second type deals with the quantum dynamics in periodically driven systems by studying average kinetic energy as a function of time [10,11].

Despite the fact that confined quantum systems are widely studied in the literature, most of the researches are mainly focused on the nonrelativistic systems. In this paper we address the problem of delta-kicked relativistic particle confined in a one-dimensional box. Nonrelativistic counterpart of such system have been considered earlier in classical and quantum chaos contexts by considering kicked particle in infinite square well [12–14]. For kicked systems, the main feature of the dynamics is the diffusive growth of the average kinetic energy as a function of time in classical case and its suppression for corresponding quantum system. The latter is called quantum localization of classical chaos [10,11]. The dynamics of the kicked nonrelativistic system is governed by single parameter, product of the kicking strength and kicking period. However, as we will see in the following, the dynamics of the relativistic system is completely different than that of its nonrelativistic counterpart: There is no single parameter which governs the dynamics.

Usually, confined relativistic quantum systems appear in particle physic models such as MIT bag model [15] and the quark potential models [16]. However, recent progress made in fabrication of graphene and studying its unusual properties made possible experimental realization of Dirac particle confined in one- [17,18] and two-dimensional boxes [19–21]. Such condensed matter realization of a Dirac particle in a box can be also realized in graphene nanoribbon ring [24–30] or dot [22,23] which is extensively studied recently both theoretically and experimentally. Graphene nanoribbon is a strip of graphene having different edge geometries. The quasiparticle dynamics in such material is effectively one-dimensional, i.e. can be described by one dimensional Dirac equation [17]. When its length is finite, it becomes “Dirac particle in a 1D box”. “Kicked” version of such system, i.e., kicked Dirac particle in a box can be realized, e.g., by putting it in a standing laser wave. One of such models has been recently studied in [18] by focusing on transport phenomena.

We note that earlier, the Dirac equation for a particle confined in a box was considered in detail in the Refs. [31–36]. Unlike the Schrödinger equation for a box, introducing confinement in the Dirac equation via infinite square well or box faces some difficulties caused by the Klein tunneling and the electron–positron pair creation [37]. To avoid such complication, in the Ref. [34] the authors considered the situation when confinement is provided by a Lorentz-scalar potential, i.e. by a potential coming in the mass term. Such a choice of confinement is often used in MIT bag model [15] and the potential models of hadrons [16]. Another way to

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avoid this complication is to impose box boundary conditions in such a way that they provide zero-current and probability density at the box walls. In the Ref. [31] the types of the box boundary conditions, providing vanishing current at the box walls and keeping the Dirac Hamiltonian as self-adjoint are discussed.

The paper is organized as follows. In the next section we consider classical dynamics of a relativistic particle confined in a one dimensional box. In section 3, following the Ref. [31], we briefly recall the problem of stationary Dirac equation for one dimensional box. In section 4 we treat the time-dependent Dirac equation with delta-kicking potential with the box boundary conditions. In section 5 we discuss wave packet dynamics and trembling motion. Finally, section 6 presents some concluding remarks.

2. Classical dynamics

Classical relativistic particles whose motion is spatially confined may appear in plasma [40] and astrophysical systems [41]. Confinement in such systems can be provided by constant electric or magnetic fields. Hamiltonian of a delta kicked relativistic particle is given by (in the units $m = c = 1$)

$$H = \sqrt{p^2+1} - \varepsilon \cos\left(\frac{2\pi}{\lambda}x\right)\sum_l \delta(t - lT), \tag{1}$$

where λ is the wavelength, ε and T are the kicking strength and period, respectively.

Classical dynamics of a relativistic particle in one-dimensional space is governed by Hamiltonian equations which are given as

$$\frac{dp}{dt} = \frac{\partial H}{\partial x}, \qquad \frac{dx}{dt} = -\frac{\partial H}{\partial p}. \tag{2}$$

Assuming that the motion of the particle is confined within the box of size L and solving Eqs. (2) by imposing box-boundary conditions, one can analyze the classical dynamics of a relativistic particle confined in a 1D box. Nonrelativistic counterpart of this problem was studied in the Refs. [13,14], where the map describing phase-space evolution of the system is derived. Unlike the kicked rotor, classical dynamics of a kicked particle in a box depends not only on the product of the kicking strength and period, but also on the number of pulse waves in the box, i.e. on the ratio of the wavelength to the box size [13,14]. In other words, kicked particle confined in a box has much larger parametric space than that for kicked rotor. Relativistic generalization of the map derived in [13] can be written as

$$x_{n+1} = \left(L + (-1)^{B_n}\left(x_n + Tp_n \Big/ \sqrt{p_n^2+1} - \mathrm{Sgn}(p_n)LB_nA\right)\right) mod\ L,$$
$$p_{n+1} = (-1)^{B_n}p_n + \frac{2\pi\varepsilon T}{\lambda}\sin\left(\frac{2\pi}{\lambda}x_{n+1}\right), \tag{3}$$

where $B_n = [\mathrm{Sgn}(p_n)(x_n + p_n)/L]$; [...] is the number of bounces of the particle between the walls during the interval between nth and $(n+1)$th kick and Sgn(...) stand for integer part and sign of the argument respectively. It is clear that this map (as the equations of motion themselves) is Lorentz invariant, since it is a discretized version of Eqs. (2).

Fig. 1 presents phase-space portraits (a) and the average kinetic energy (b) of a kicked relativistic particle in a 1D box for those values of the kicking parameters at which the dynamics of the particle is regular. For this case the average kinetic energy does

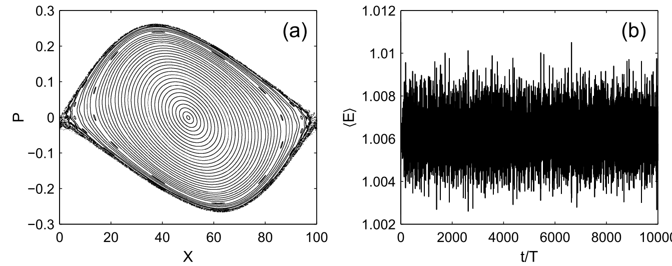

Fig. 1. Phase-space portrait (a) and the time-dependence of the average energy vs the number of kicks (b) for the classical system. The kicking strength $\varepsilon = 0.0159$ and the kicking period $T = 100$.

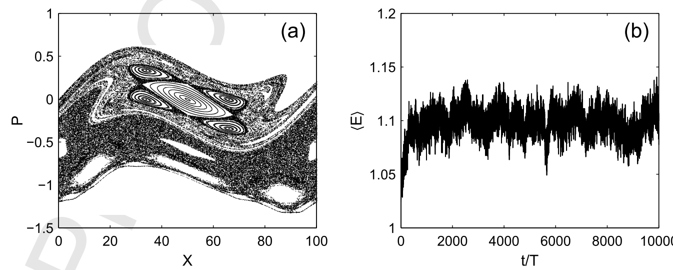

Fig. 2. The same as in Fig. 1 for $\varepsilon = 0.0637$ and $T = 100$.

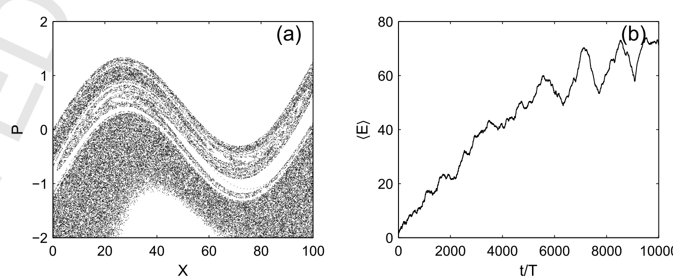

Fig. 3. The same as in Fig. 1 for $\varepsilon = 0.1751$ and $T = 99.99327$.

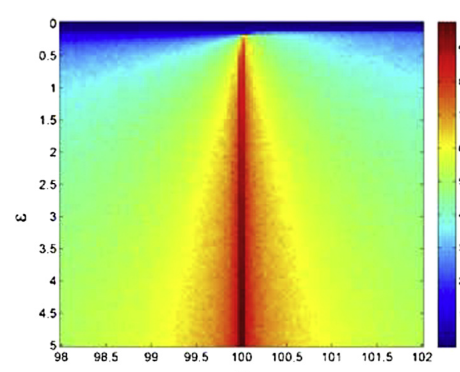

Fig. 4. (Color online.) Average kinetic energy as a function of the kicking parameters at the 1000th kick (logarithmic scale).

not grow in time (unlike the nonrelativistic case) and fluctuates around some fixed value. In Fig. 2 similar plot is presented for the values of the kicking parameters causing mixed dynamics. The energy in Fig. 2 (b) grows in time and the growth is suppressed after the certain number of kicks. In Fig. 3 phase-space portrait (a) and time-dependence of the average kinetic energy (b) are plotted for fully chaotic case. The energy grows almost monotonically for this case in the considered time period. This regime can be considered as an acceleration mode. Existence of the acceleration modes can be clearly seen from the Fig. 4, where the average kinetic energy is plotted vs kicking parameters, ε and T. Maximum values of $E(t)$