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Effects of variable thermal diffusivity on the structure of convection

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A B S T R A C T

The structure of multiscale convection in a thermally stratified plane horizontal fluid layer is investigated by means of numerical simulations. The thermal diffusivity is assumed to produce a thin boundary sublayer convectively much more unstable than the bulk of the layer. The simulated flow is a superposition of cellular structures with three different characteristic scales. In contrast to the largest convection cells, the smaller ones are localised in the upper portion of the layer. The smallest cells are advected by the larger-scale convective flows. The simulated flow pattern qualitatively resembles that observed on the Sun.

1. Introduction

This study was motivated by the necessity of comprehending the physical factors responsible for the complex spatial structure of solar convection. The magnetic fields in the solar convection zone are dynamically coupled with motions, and the formation of convection patterns is of paramount importance to the dynamics of magnetic fields and, therefore, to the processes of solar activity. As is well known, cellular flow structures of at least three or four types can be identified with certainty on the solar surface and attributed to the phenomenon of thermal convection, viz., granules, mesogranules (whose existence as a physical entity is debatable), supergranules and giant cells (see, in particular, [11,13], and references therein). These different sorts of structures differ widely in their scale. Furthermore, Abramenko et al. [1] reported the detection of mini-granular structures with spatial scales below 600 km. The convective flow in the photospheric and subphotospheric layers is thus a superposition of these differently scaled cellular flows, so that smaller cells are transferred by the motions associated with the larger scales. This multiscale, hierarchical structure of the flow (or scale splitting) is an important feature of solar convection. It has not yet received a convincing explanation, and an adequate hydrodynamic description must be given to both the spatial structure of the flows and the factors responsible for its development.

The multiscale structure of solar convection can be revealed using various observational techniques. However, the power spectra of the velocity field seem to definitely exhibit only two peaks, which correspond to granulation and supergranulation (see, e.g., [8]). The broadband character of the spectrum smears out the other peaks. Nevertheless, the structures forming no pronounced spectral peaks can be identified using other methods, such as local correlation tracking (LCT), wavelet analyses, tracking the motion of supergranules and an LCT-based cork-motion-tracking technique very clearly visualising both supergranules and mesogranules. A further discussion of the pros and cons concerning the existence of mesogranules is given by Rieutord and Rincon [13].

It is known that convection cells in various flows have typically comparable horizontal and vertical sizes. This suggests that convection structures of different types in the solar convection zone should occupy layers of different thicknesses. Since all these structures can be detected at the solar surface, it can readily be understood that they all (except the giant cells filling the whole convection-zone depth) are “suspended” near the upper boundary of the convection zone, while the lower boundaries of layers of different types are located at different depths.

As for hydrodynamic modelling, some “realistic” (or, as termed by Schüssler [14], “comprehensive”) simulations of solar convection (aimed at reproducing the physical processes involved in the solar convective phenomena as closely as possible), which are based on the *MURaM* code [18], demonstrate a gradual increase in the characteristic scale of the flow with depth [15]. According to such simulations, the surface velocity field does not demonstrate any multiscale structure, and only granular-sized cells are clearly noticeable in the computed patterns. Larger-scale structures cannot be detected at the surface and at small depths; at most, they may be too weak to manifest themselves in advecting small-scale

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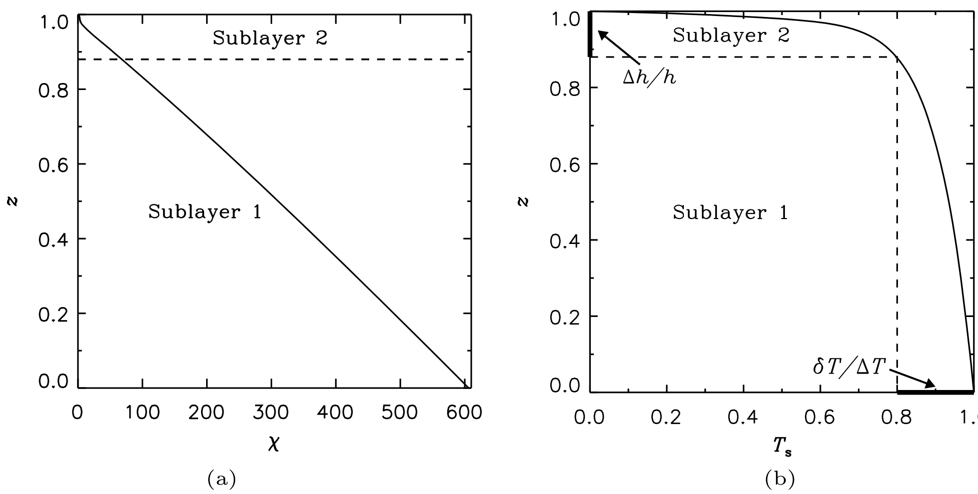

Fig. 1. (a) Static $\chi(z)$ distribution and (b) static temperature profile $T_s(z)$ corresponding to the law (1).

cells. This suggests that some physics responsible, e.g., for supergranulation is not taken into account in the known versions of the "realistic" problems.

It is natural to believe that the structure of convection should generally be controlled by the particularities of the fluid-layer stratification. Our aim is to investigate the role of certain physical mechanisms that can give rise to a multiscale structure of convection – first of all, by producing static temperature profiles of particular shapes in the fluid layer. At this stage of research, we do not pursue the aim of closely reproducing the physical conditions in the solar convection zone and the convection patterns actually present on the Sun but seek for physical factors capable of producing the scale-splitting effect.

Specifically, we consider here the effects of temperature-dependent thermal diffusivity. We assume this quantity to vary in such a way that the static temperature gradient $\mathrm{d}T_s/\mathrm{d}z$ is small in the bulk of the layer but jumpwise changes to high absolute values in a thin sublayer near the top boundary of the layer. It is important that this gradient is negative at any height; therefore, the thermal stratification is everywhere convectively unstable. This means that the phenomenon of penetrative convection has nothing to do with the subject of our study. Such a thermal-gradient jump resembles (although does not reproduce) a partial-ionisation layer in the solar convection zone, where the enhanced specific heat reduces the adiabatic thermal gradient and the enhanced opacity increases the radiative thermal gradient. The convective instability of such a layer is therefore especially high [17,12]. The most pronounced jump of the vertical entropy gradient due to partial ionisation is located at depths of order 1 Mm below the solar photosphere. Here, we do not claim to propose a model of the solar convection zone but merely consider the physical effects of a certain stratification peculiarity under idealised conditions, for an incompressible fluid. Not only may this problem be of help in seeking structure-forming factors for solar convection but it is also interesting from a purely hydrodynamic standpoint.

Linear problems of convective stability in layers with similar static temperature profiles were considered by Getling [3,4]. In the framework of the incompressible-fluid models, indirect evidence for possible scale splitting was detected. The development of small-scale convective motions near the surface of the layer was found to require very sharp gradient jumps and very thin high-gradient sublayers. These expectations were partially substantiated by two-dimensional nonlinear numerical simulations of convection [7]. Recently, we studied the two-dimensional problem more extensively [6,16].

Here, we present the results of our simulations of three-dimensional convection under conditions similar to those assumed in [16] in terms of the special form of the temperature dependence of thermal diffusivity. We shall demonstrate that, even in the framework of a model based on an extended Boussinesq approximation, variable thermal diffusivity can produce a multiscale flow in which at least three cell types are present.

Our present study is hydrodynamic rather than astrophysical. Further steps toward adequately describing the flow structure in the solar convection zone should, in our opinion, include the consideration of different structure-forming processes in parallel with successively taking into account more physics involved.

2. Formulation of the problem and numerical technique

We consider a rectangular box $[0, L_x] \times [0, L_y] \times [0, h]$ of a plane horizontal layer of a viscous, incompressible fluid (in our computations described here, $L_x = L_y = 15h$). Let the bottom and the top boundary of the layer to be perfect thermal conductors, whose temperatures are maintained constant and equal to $T_{\mathrm{bot}} \equiv \Delta T > 0$ and $T_{\mathrm{top}} = 0$, respectively. Also let the sidewalls of the region be thermally insulated. We specify the no-slip impermeability conditions at the bottom and side boundaries of the region. The top boundary is also assumed to be rigid.

We choose the temperature dependence of thermal diffusivity in the form

$$\chi(T) = 1 + 5T + 600T^{10}, \tag{1}$$

with $\chi(T_{\mathrm{top}})/\chi(T_{\mathrm{bot}}) = 1/606$; from here on, we use ΔT as the unit temperature and the layer thickness h as the unit length. In these dimensionless variables, the distributions of $\chi(z)$ and $T(z)$ for a motionless fluid due to the law (1) are shown in Figs. 1a and 1b, respectively. The static temperature T_s varies little (by a dimensional quantity $\delta T \ll \Delta T$) across the main portion of the layer (Sublayer 1) of a dimensional thickness $h - \Delta h$, where $\Delta h \ll h$, while the most part of the temperature difference, $\Delta T - \delta T$, corresponds to Sublayer 2 with a small thickness Δh, near the upper surface. The kink near $z = h - \Delta h$ in the temperature profile specified in this way qualitatively resembles (although does not reproduce) the transition from the bulk of the solar convection zone, where the stratification is weekly superadiabatic, to the overlying strongly unstable layer with a depth of order 1 Mm.

We use an extended Boussinesq approximation, which admits thermal-diffusivity variations (see, e.g., [5], for a discussion of different versions of this approximation). If, in addition to the above-specified variables, we choose the characteristic time of viscous