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Controllable phase transitions and novel selection rules in Josephson junctions with inherent orthogonality

Qiang Cheng [a], Kunhua Zhang [b], Hongyang Ma [a]

[a] School of Science, Qingdao Technological University, Qingdao, Shandong 266520, China
[b] ICQD, Hefei National Laboratory for Physical Sciences at Microscale, University of Science and Technology of China, Hefei, Anhui 230026, China

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ABSTRACT

We propose a new type of Josephson junction consisting of topologically nontrivial superconductors with inherent orthogonality and a ferromagnetic interface. It is found this type of junction can host rich ground states: 0 phase, π phase, $0+\pi$ phase, φ_0 phase and $\varphi_0 \pm \varphi$ phase. Phase transitions can be controlled by changing the direction of the interfacial magnetization. Phase diagrams are presented in the orientation space. Novel selection rules for the lowest order current, $\sin\phi$ or $\cos\phi$, of this kind of junction are derived. General conditions for the formation of various ground states are established, which possess guiding significance to the experimental design of required ground states for practical applications. We construct the succinct form of a Ginzburg–Landau type of free energy from the viewpoint of the interplay between topological superconductivity and ferromagnetism, which can immediately lead to the selection rules. The constructed terms are universally available to the topological Josephson junctions with or without inherent orthogonality reported recently. The spin supercurrent, its selection rules and their relations to the constructed energy are also investigated.

1. Introduction

The ground state of a Josephson junction is called ϕ_0 phase if the value of the ground-state phase difference is ϕ_0 which corresponds to a minimum point of free energy profile. Novel ground states and tunable phase transitions in Josephson junctions play important roles in the fields of memory device design, energy-efficient information processing and the detection of topological superconductivity. For example, the π phase can be used to build qubits and protect them from noise [1,2]; the tunable 0–π transition has direct applications in a superconducting computer with low energy consumption [3]. It is proved possible to write and read bits in cell based on the φ phase, which is a doubly degenerate state with two energy minima at $\phi_0 = \pm\varphi$ [4]. The φ_0 phase, which has a single energy minimum at $\phi_0 \neq 0, \pi$, possesses implications for exploring topological superconductivity [7]. This phase is firstly treated theoretically and experimentally by Mints et al. in Ref. [5,6]. Recent studies show that these novel states can be realized in multi-layer junctions [8,9] or composite structures [10–14]. However, from the angle of practice, how to realize rich ground states and to control phase transitions conveniently in a simple geometry is an urgent issue to be solved in condensed matter physics.

The concise Josephson junctions with inherent orthogonality have attracted continuously growing interest, in which the wave functions of Cooper pairs are orthogonal. The inherent orthogonality means the orthogonality is not caused by outer factors of superconducting systems, such as an external field. According to the origin of the orthogonality, the existing junctions can be divided into two classes. The first is the singlet superconductor|triplet superconductor junction [15–17]. Its orthogonality originates from the spin part of superconducting states since the total spin of the singlet pairs is 0 while that of the triplet ones is 1. The second is the helical superconductor|helical superconductor junction [18]. Its orthogonality originates from the orbital part of the helical superconducting states. The average of the scalar product of **d**-vectors over wavevector is vanishing since the left one is $\mathbf{d}(\mathbf{k}) = (k_x, k_y, 0)$ and the right one is $\mathbf{d}(\mathbf{k}) = (k_y, k_x, 0)$ or $(k_y, -k_x, 0)$. The three-component vector $\mathbf{d}(\mathbf{k})$ is introduced to describe both the spatial part and the spin part of spin-triplet Cooper pairs [19], which is related to the energy matrix by $\hat{\Delta}(\mathbf{k}) = (\mathbf{d}(\mathbf{k}) \cdot \sigma) i\sigma_2$ with the Pauli matrices σ.

The inherent orthogonality itself in the junctions can not lead to multiple ground states. Actually, the second harmonic $\sin 2\phi$ dominates the current–phase relations (CPRs). The lowest order

E-mail address: hongyang_ma@aliyun.com (H. Ma).

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currents (LOCs) $\sin\phi$ and $\cos\phi$, the ingredient for novel ground states [20,21], is absent due to the presence of orthogonality. However, the introduction of a magnetic potential will change the situation [22–24]. For example, the φ_0 phase can be formed in the first class junction with a ferromagnetic interface [25]. The selection rules of LOC for the junction have been derived in the frame of the tunneling Hamiltonian theory or the quasiclassical Green's function formalism [26,27]. The second class junction with a ferromagnetic barrier can host more phases due to the presence of $\sin\phi$ and $\cos\phi$ in CPRs. The selection rules of LOC show an intimate association with free energy [18].

In this paper, we propose a new class of junctions with inherent orthogonality which originates from the perpendicular **d**-vectors of triplet superconductors, i.e., the scalar product of the vectors is directly equal to zero. The superconducting system we consider is quasi two-dimensional and the wavevector only has two components, i.e., k_x and k_y. The **d**-vector of the left superconductor is chosen as $k_x\hat{z}$, $k_y\hat{z}$ or the chiral state $(k_x + ik_y)\hat{z}$, and the vectors of the right are chosen as the helical ones $k_x\hat{x} \pm k_y\hat{y}$ or $k_y\hat{x} \pm k_x\hat{y}$. All of these states are topologically nontrivial but belong to different topological classes [28,29]. The chiral state and the helical ones are fully gapped. The former is characterized by the winding number ω_{2d} and the latter by the Z_2 number. The index characterizing the states $k_x\hat{z}$ and $k_y\hat{z}$ is ω_{1d} due to the existence of nodal lines. On the other hand, the **d**-vectors on the left side bear a common character that they all obey the spin-rotation symmetry about the z-axis whereas the helical states on the right break the full spin-rotation symmetry [30]. It is reasonable to expect anomalous Josephson effects in the new junctions when ferromagnetism (spin-polarization) is introduced in the interfacial barrier. However, which phases the junctions can host, what the selection rules LOC satisfies and how ferromagnetism and topological superconductivity interact remain questions to be answered.

In the present work, we systematically study the newly proposed junctions with a ferromagnetic interface using the method of quasiclassical Green's functions with generalized Riccati parametrization [31]. On basis of the numerical results, rich ground states are found which are 0 phase, π phase, $0+\pi$ phase, φ_0 phase and $\varphi_0 \pm \varphi$ phase. The phase transitions between the ground states can be controlled by rotating the direction of the interfacial magnetization. The phase diagrams are presented in the orientation space when the interfacial non-magnetic potential is vanishing or of finite magnitude. A general relation between the formation of ground states and CPRs is discussed, which provides the fundament to the construction of requisite ground states for real applications. Peculiar selection rules of LOC are summarized. To understand the rules physically, we establish a Ginzburg–Landau type of free energy with **d**-vectors and magnetization, which can explain the results of topological Josephson junctions with or without the inherent orthogonality reported recently. The free energy directly expresses the coupling manners between ferromagnetism and various topological superconductivity. The CPRs and selection rules for the spin supercurrent and their intimate relations with the constructed energy are also included.

2. Model and formalism

We consider the Josephson junctions in the clean limit as shown schematically in Fig. 1. The interface, located at $x = 0$ and along the y-axis, is modeled by a delta function $U(x) = (U_0 + \mathbf{M}\cdot\hat{\sigma})\delta(x)$ with U_0 the non-magnetic potential and $\mathbf{M}\cdot\hat{\sigma}$ the ferromagnetic one. The magnetization $\mathbf{M} = M(\sin\theta_m\cos\phi_m, \sin\theta_m\sin\phi_m, \cos\theta_m)$ is specified by the polar angle θ_m and the azimuthal angle ϕ_m. For the superconductor on the left side, we consider the states with the following **d**-vectors,

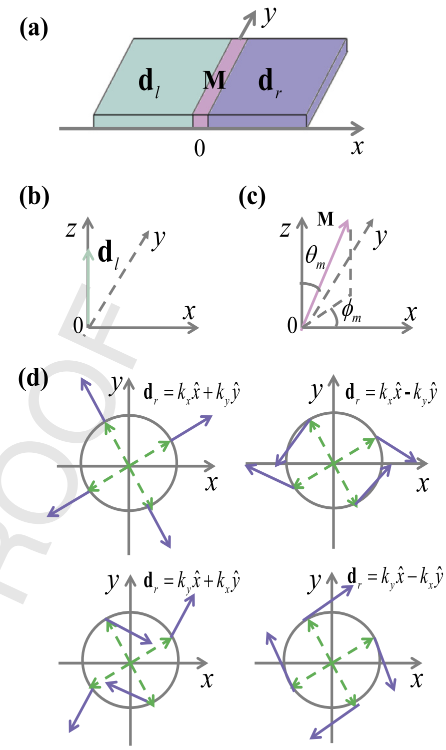

Fig. 1. (Color online) (a) Schematic illustration of junctions with orthogonal **d**-vectors. The $x(y)$-axis is defined by the crystallographic $a(b)$-axis. (b) The direction of the **d**-vector in the left superconductor which is along the z-axis. (c) The magnetization **M** in the ferromagnetic interface specified by the polar angle θ_m and the azimuthal angle ϕ_m. (d) Four types of wavevector-dependent **d**-vectors in the right superconductor. The green dashed arrows denote wavevectors. The purple solid lines denote directions of **d**-vectors.

$$\mathbf{d}_l(\mathbf{k}) = \begin{cases} \Delta_1(k_x + ik_y)\hat{z}, \\ \Delta_2 k_x\hat{z}, \\ \Delta_3 k_y\hat{z}, \end{cases} \tag{1}$$

with $\mathbf{k} = (k_x, k_y)$.

For the superconductor on the right side, we consider the following helical states,

$$\mathbf{d}_r(\mathbf{k}) = \begin{cases} \Delta_0(k_x\hat{x} + k_y\hat{y}), \\ \Delta_0(k_x\hat{x} - k_y\hat{y}), \\ \Delta_0(k_y\hat{x} + k_x\hat{y}), \\ \Delta_0(k_y\hat{x} - k_x\hat{y}). \end{cases} \tag{2}$$

The temperature-dependent gap magnitudes Δ_0, Δ_1, Δ_2 and Δ_3 are determined by the Bardeen–Cooper–Schrieffer type equation. The orthogonality of our junctions is expressed as $\mathbf{d}_l(\mathbf{k})\cdot\mathbf{d}_r(\mathbf{k}) = 0$.

The retarded Green's function $\hat{g}^R$ in superconductors can be written in terms of Riccati parameterization as

$$\hat{g}^R = -i\pi \begin{pmatrix} (1-\gamma\tilde{\gamma})^{-1}(1+\gamma\tilde{\gamma}) & (1-\gamma\tilde{\gamma})^{-1}2\gamma \\ (1-\gamma\tilde{\gamma})^{-1}(-2\tilde{\gamma}) & -(1-\gamma\tilde{\gamma})^{-1}(1+\tilde{\gamma}\gamma) \end{pmatrix} \tag{3}$$

with the coherence functions γ and $\tilde{\gamma}$ which describe the probability amplitudes for conversion of a hole to a particle and a particle to a hole, respectively. In this work, we will use the units such that $\hbar = k_B = 1$. The coherence functions obey the Riccati type transport equations in the real energy representation

$$(iv_{Fx}\partial_x + 2\varepsilon)\gamma = \gamma\tilde{\Delta}\gamma - \Delta, \tag{4}$$

$$(iv_{Fx}\partial_x - 2\varepsilon)\tilde{\gamma} = \tilde{\gamma}\Delta\tilde{\gamma} - \tilde{\Delta}, \tag{5}$$

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