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# Electromagnetic radiation in a semi-compact space

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ABSTRACT

In this note, we investigate the electromagnetic radiation emitted from a revolving point charge in a compact space. If the point charge is circulating with an angular frequency $\omega_0$ on the $(x, y)$-plane at $z = 0$ with boundary conditions, $x \sim x + 2\pi R$ and $y \sim y + 2\pi R$, it emits radiation into the $z$-direction of $z \in [-\infty, +\infty]$. We find that the radiation shows discontinuities as a function of $\omega_0 R$ at which a new propagating mode with a different Fourier component appears. For a small radius limit $\omega_0 R \ll 1$, all the Fourier modes except the zero mode on $(x, y)$-plane are killed, but an effect of squeezing the electric field totally enhances the radiation. In the large volume limit $\omega_0 R \to \infty$, the energy flux of the radiation reduces to the expected Larmor formula.



## 1. Introduction

An accelerating charged particle emits electromagnetic radiation. If some of the spaces are compact and bounded by material walls, the radiation behaves differently. Some examples are microwaves propagating inside a compact waveguide, light propagating in an optical fiber, or black body radiation in a finite volume. A similar but slightly different situation appears in the string theory with higher dimensional spaces; $d = 9$ spaces among which six-dimensional sub-spaces are compactified with periodic boundary conditions to describe our three-dimensional spaces. In string theory, we often consider D-branes, localized objects charged under the so called Ramond–Ramond (RR) fields (see [1,2] for reviews). There are various types of D-branes: a Dp-brane is a $p$-dimensional object. In the brane world scenario, our universe is described by a D3-brane whose motion in the six-dimensional compact space is supposed to describe the early universe [3–5]. In such a situation, radiations of gravitational and RR fields in a compact space are important to be investigated [6–8].

In this short note, motivated by the studies of radiations from D-branes in motion, we study electromagnetic radiations from a revolving point charge in a compact space.[1] When the size of the compact spaces is smaller than the typical wave length of radiation, one may expect that the radiation will be suppressed. The purpose of the note is to check whether it is correct or not. In the next section we introduce our setup and provide useful formulas to calculate the radiation. We especially evaluate the retarded Green's function in compact spaces. In section 3 we calculate energy flux of radiation which is defined at far infinity away from the revolving charge in the non-compact direction. The radiation has a discontinuous behavior as the size of the compact directions $R$ is varied. It is also necessary to regularize divergences associated with resonances in the compact space, which also cause the discontinuities. We summarize the results in section 4. In Appendix, we list exact expressions of the electric and magnetic fields without using an approximation $d \ll R$ where $d$ is the radius of the circular motion of a charged particle.

## 2. Setup and Green's function

The setup studied in this note is as follows. We consider four-dimensional space–time with time $t$ and space coordinates $x^i$, $i = 1, 2, 3$. The first and the second spacial directions are compactified by imposing the periodic boundary conditions with radius $R$,

$$x^1 \sim x^1 + 2\pi R, \qquad x^2 \sim x^2 + 2\pi R, \tag{1}$$

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[1] Unruh radiation in a compact space has been discussed in [9].

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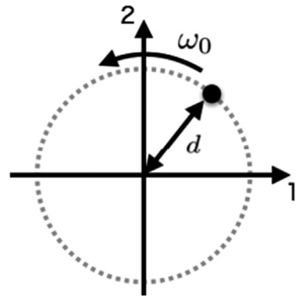


**Fig. 1.** The motion of the point charge.

and the other direction $z$ is extended to infinity. We introduce a point charge $q$ revolving in the $x^1$–$x^2$ plane with a constant angular frequency $\omega_0$ (see Fig. 1). The motion of the point charge is described by a vector $\mathbf{z}(t)$ with the components

$$z^1(t) = d\cos(\omega_0 t), \qquad z^2(t) = d\sin(\omega_0 t), \qquad z^3(t) = 0, \tag{2}$$

where $d$ is the radius of the orbit. We assume that the motion is non-relativistic. The energy flux of radiation is defined at $x^3 = +\infty$ (or $-\infty$) as

$$W = \left\langle \int_0^{2\pi R} dx^1 dx^2 \lim_{x^3 \to \infty} (E_1 B_2 - E_2 B_1) \right\rangle, \tag{3}$$

where $\mathbf{E} = (E_1, E_2, E_3)$ and $\mathbf{B} = (B_1, B_2, B_3)$ are the electric and magnetic fields respectively, and the bracket means time averaging. The quantity $E_1 B_2 - E_2 B_1$ is nothing but the third component of the Poynting vector. We are using the unit system of $\epsilon_0 = \mu_0 = 1$ and $c = 1$. In a non-compact case, this energy flux is given by

$$W_{\text{nonc}} = \frac{\omega_0^4 q^2 d^2}{12\pi}, \tag{4}$$

which is one half of the radiation of the Larmor formula [10]. In the following we focus on the ratio $f := W/W_{\text{nonc}}$ to represent the effect of the compactness of the space.

The electromagnetic field can be obtained by solving the Maxwell's equations in the Lorenz gauge condition $\partial\phi/\partial t + \nabla \cdot \mathbf{A} = 0$,

$$\Box \phi(t, \mathbf{x}) = \rho(t, \mathbf{x}), \tag{5}$$

$$\Box \mathbf{A}(t, \mathbf{x}) = \mathbf{i}(t, \mathbf{x}), \tag{6}$$

where $\phi$ and $\mathbf{A}$ are scalar and vector potentials. The charge density $\rho(t, \mathbf{x})$ and the current density $\mathbf{i}(t, \mathbf{x})$ are described as

$$\rho(t, \mathbf{x}) = q\delta^3(\mathbf{x} - \mathbf{z}(t)), \tag{7}$$

$$\mathbf{i}(t, \mathbf{x}) = q\frac{d\mathbf{z}(t)}{dt}\delta^3(\mathbf{x} - \mathbf{z}(t)), \tag{8}$$

for a non-relativistically moving charged particle. In the following of this section, we derive a general expression of the Green function in a semi-compact space with the periodic boundary condition. Namely we do not specify the concrete settings of $\rho$ and $\mathbf{i}$ in (7) and (8) for the moment. Solutions of the wave equations, eqs. (5) and (6), are obtained by using the retarded Green's function $G_{\text{ret}}(t, \mathbf{x})$,

$$\phi(t, \mathbf{x}) = \int dt' \int d^3\mathbf{x}'\, G^{\text{ret}}(t - t', \mathbf{x} - \mathbf{x}')\, \rho(t', \mathbf{x}'), \tag{9}$$

$$\mathbf{A}(t, \mathbf{x}) = \int dt' \int d^3\mathbf{x}'\, G^{\text{ret}}(t - t', \mathbf{x} - \mathbf{x}')\, \mathbf{i}(t', \mathbf{x}'). \tag{10}$$

The electric and magnetic fields are obtained by

$$\mathbf{E} = -\nabla\phi - \frac{\partial}{\partial t}\mathbf{A}, \qquad \mathbf{B} = \nabla \times \mathbf{A}, \tag{11}$$

respectively.

Now we obtain the retarded Green's function with the boundary conditions (1). In the non-compact case, the retarded Green's function is given by

$$G^{\text{ret}}_{\text{nonc}}(t, \mathbf{x}) = \frac{1}{4\pi}\frac{1}{|\mathbf{x}|}\delta(t - |\mathbf{x}|)\theta(t) = \int \frac{d\omega}{2\pi} e^{-i\omega t} \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{1}{|\mathbf{k}|^2 - \omega^2 - i\epsilon \cdot \text{sgn}(\omega)} e^{i\mathbf{k}\cdot\mathbf{x}}, \tag{12}$$

which satisfies

$$\Box G^{\text{ret}}_{\text{nonc}}(t, \mathbf{x}) = \delta(t)\delta^3(x). \tag{13}$$

Here, $\text{sgn}(\omega)$ is the sign function of $\omega$ and $\epsilon > 0$ is an infinitesimal number which specifies the integration contour.