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Randomness in denoised stock returns: The case of Moroccan family business companies

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In this paper, we scrutinize entropy in family business stocks listed on Casablanca stock exchange and market index to assess randomness in their returns. For this purpose, we adopt a novel approach based on combination of stationary wavelet transform and Tsallis entropy for empirical analysis of the return series. The obtained empirical results show strong evidence that their respective entropy functions are characterized by opposite dynamics. Indeed, the information contents of their respective dynamics are statistically and significantly different. Obviously, information on regular events carried by family business returns is more certain, whilst that carried by market returns is uncertain. Such results are definitively useful to understand the nonlinear dynamics on returns on family business companies and those of the market. Without a doubt, they could be helpful for quantitative portfolio managers and investors.

1. Introduction

Since the early works of Mantegna and Stanley [1,2], econophysics has become an attractive field where quantitative concepts and approaches all borrowed from physics are used to analyze and model complex dynamics in financial time series by using Heston model [3–6], visibility graph [7], agent-based approach [8, 9], threshold networks [10], correlation-based network [11], and Weibull distribution with universal shape parameter [12], to name a few. Still, studying stock market efficiency remains a crucial step toward analysis of financial assets behaviour to better build forecasting models that would help in asset evaluation and risk estimation. In this regard, fractal and chaos were widely employed to investigate stochastic nature in world major international stock markets [13], American treasury bills returns and volatilities [14], Casablanca stock market [15], crude oil markets [16], currency exchange rates [17–20], bitcoin market [21], and international financial and commodity markets [22,23].

Likewise, entropy statistic is a meaningful tool to assess randomness in financial time series to better understand their underlying process. Indeed, in recent years, it was extensively employed in several works to examine randomness in international stock markets [24–33], and also in bitcoin market [21] and exchange rate markets [34].

The purpose of this paper is to evaluate randomness in returns of Moroccan family business stocks listed on Casablanca stock exchange. Indeed, the uncertainty of a return is commonly used to describe financial risk in terms of information content randomness. Therefore, measuring randomness of return series by means of entropy statistic is a valuable approach to assess the predictability of such data. For instance, entropy statistic could help quantifying the information content and stability degree in a stock market. In particular, it allows extracting useful information regarding the underlying return series and revealing return closeness to randomness. Thus, one could assess the efficiency of the stock market under study. This kind of investigation is greatly motivated by investors, portfolio managers and analysts to generate high returns.

This paper contributes to the literature as follows. First, disorder in family business return fluctuations is examined. Indeed, previous works found in the literature [24–34] focused on major financial markets; but, no such interest has been given to family business stocks, especially in an emergent market. Bear in mind, family business companies listed on Casablanca stock exchange are major drivers of the Moroccan economy, are actively traded, and generate attractive returns. Thus, it is interesting to examine how random is their underlying process in order to assess their degree of predictability. Second, fluctuations in family business returns are analyzed in a multi-scale framework. In particular, stationary wavelet transform [35,36] is applied to original return series for decomposition and denoising purpose. For instance, by processing the original return series by stationary wavelet transform, approx-

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imation and detail coefficients are obtained. In particular, the purpose is to separate sudden and noisy variations in return series from smoothed long variations. Therefore, we aim to quantify randomness in long variations of returns which is the denoised series to assess efficiency in return series long term dynamics. Indeed, strategic investors, managers, and analysts are more concerned with long term decisions than short term ones. Third, our approach is also applied to the market index for comparison purpose as it is important to discover similarities/dissimilarities between family business stocks and main stock market in terms of randomness. Fourth, entropy-based correlations are used to study dynamic interactions between family business stocks and main market. For instance, the evolution of correlation between time-dependent entropies is examined by using a moving window approach. These entropic correlations are investigated at different time scales to understand how randomness in family business returns are related to randomness in main market returns.

In our study, the well-known Tsallis entropy [37] is employed to measure the degree of randomness in return series. The Tsallis entropy is the generalization of the Boltzmann–Gibbs (BG) statistic. In other words, Tsallis entropy extends the BG entropy to the Tsallis q-entropy where BG entropy is a special case corresponding to the value $q = 1$. Therefore, the q-entropy principal is related to the holistic, multi-scale and globally correlated spatio-temporal structures [38]. Such advantages are attractive and make Tsallis entropy appealing and appropriate to examine stochastic behaviour in financial time series.

The main findings follow. First, the information on regular events carried by family business returns is more certain. Second, the information on regular events carried by market returns is uncertain. Third, the information contents of their respective dynamics are statistically and significantly different. Fourth, the quantity of information on extreme events carried by both markets is uncertain.

This paper is organized as follows: Section 2 briefly describes the wavelet transform and Tsallis entropy. Section 3 presents the empirical results of the Tsallis entropy of Moroccan family business index and that of the main market index. The discussion of the obtained results is provided in Section 4. Finally, Section 5 concludes the paper.

2. Methods

The stationary wavelet transform [35,36] is used to analyze a given signal $s(x)$ in the time-frequency domain. In other words, it is a powerful multi-resolution technique for time-scale representation of the original signal. For instance, it employs a scaling (father) function $\phi(x)$ and a mother wavelet function $\Psi(x)$ to decompose the original signal respectively into approximation $\tilde{c}_{j+1,k}$ and details $\tilde{d}_{j+1,k}$ coefficients expressed as follows [36]:

$$\tilde{c}_{j+1,k} = \left\langle s(x), \frac{1}{2^{j/2}} \phi\left(\frac{x-k}{2^j}\right)\right\rangle \quad (1)$$

$$\tilde{d}_{j+1,k} = \left\langle s(x), \frac{1}{2^{j/2}} \Psi\left(\frac{x-k}{2^j}\right)\right\rangle \quad (2)$$

where $j = 1, 2, \ldots, J$ and $k = 1, 2, \ldots, N/2^j$. In the wavelet transform framework, time and frequency resolutions are obtained by changing respectively k and j. By using integration operator, Eq. (1) and Eq. (2) can also respectively be expressed as Eq. (3) and Eq. (4) given by:

$$\tilde{c}_{j,k} = \int \phi_{j,k}(x) s(x) dx \quad (3)$$

$$\tilde{d}_{j,k} = \int \psi_{j,k}(x) s(x) dx \quad (4)$$

In our study, the popular Daubechies-4 wavelet is employed for analysis of the financial time series at first, second, and third level of decomposition to obtain three approximation series (coefficients). Indeed, it is chosen thanks to its orthogonal and compact support abilities, and capability to better smooth the original signal at different levels of decomposition. Particularly, Daubechies-4 wavelet uses overlapping windows so as the spectrum of high frequency coefficients reflects all high frequency changes. Thus, the resulting approximation coefficients do not include any high frequency component. In this regard, they are well smoothed and effectively represent long term variations of the original signal under study. Recall that $\tilde{c}_{j+1,k}$ and detail $\tilde{d}_{j+1,k}$ coefficients are respectively used to represent original signal trend and short variations. In other words, $\tilde{c}_{j+1,k}$ (Eq. (3)) represent the denoised series from which randomness will be quantified by means of Tsallis entropy. In other words, Tsallis entropy is computed from time series $\tilde{c}_{j+1,k}$ (Eq. (3)). In general, Tsallis entropy is given by:

$$S_q = \frac{k}{q-1}\left(1 - \sum_i p_i^q\right) \quad (5)$$

where $q > 1$ is a real parameter called entropic-index, k is a normalization constant, p_i is a discrete probability such that $\sum_i p_i = 1$. The entropic-index q is used to quantify the degree of nonextensivity of the entropy. In particular, high order-q privileges salient events [39]. In addition, it describes the deviations of Tsallis entropy from the standard Boltzmann–Gibbs or Shannon one. When q asymptotically tends to unity, the Tsallis q-entropy is reduced to the simple form of the Boltzmann–Gibbs (BG) entropy denoted S_{BG} and given by:

$$S_{BG} = -k \sum_i p_i \log(p_i) \quad (6)$$

In this study, k is set to unity and the entropic-index q varies between 2 and 20.

3. Results

A set of daily prices from sixteen family business stocks listed on Casablanca stock exchange (CSE) is collected from 25 March 2013 to 22 March 2016. All data is gathered from the CSE official website [40] in local currency. Then, the sixteen family business stocks are used to form an equally weighted index denoted FBI: family business index. Then, FBI returns series r are defined as the first differences of the natural logarithmic price levels; for example, $r_t = \log(P_t) - \log(P_{t-1})$ where P is the closing price level of FBI and t is the time script. Similarly, we used data from Casablanca stock exchange most active shares index (MADEX) for comparison against the FBI. For illustration purpose, return series of family business index (FBI) and MADEX are plotted in Fig. 1, and approximation series are plotted in Fig. 2 and Fig. 3 respectively for FBI and MADEX. In addition, the probability distribution functions (PDFs) of wavelet-based approximation coefficients from family business index (FBI) returns and those from general market index MADEX returns are exhibited in Fig. 4 for comparison purpose.

The relationship between Tsallis entropy and the entropic-index q ($2 \le q \le 20$) is depicted in Fig. 5 for both indices and at each wavelet decomposition level. As shown in Fig. 5, for family business index (FBI), Tsallis entropy S_q increases with entropy-index q at all levels of wavelet analysis. In contrary, for Casablanca stock exchange most active shares index (MADEX), Tsallis entropy S_q decreases with entropy-index q at all levels of stationary wavelet analysis. In addition, both S_q of family business index and market

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